DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
OLD DOMINION UNIVERSITY
PH.D. DIAGNOSTIC EXAM
Spring 2018

ODU HONOR PLEDGE

I pledge to support the Honor system of Old Dominion University. I will refrain from any form of academic dishonesty or deception, such as cheating or plagiarism. I am aware that as a member of the academic community, it is my responsibility to turn in all suspected violators of the Honor Code. I will report to a hearing if summoned.

Student Signature: _____________________________
Student Name (BLOCK CAPITALS): ___________________________________
UIN Number: _________________________________________________

Please turn in this examination document with the pledge above signed and with one answer book for each solved problem.

1. This examination contains 25 problems from the following six areas:

   A. MATH (At most 3 problems can be answered from the Math area) A1 A2 A3 A4
   B. CIRCUITS & ELECTRONICS B1 B2 B3
   C. SYSTEMS, SIGNAL AND IMAGE PROCESSING C1 C2 C3 C4 C5 C6
   D. PHYSICAL ELECTRONICS I D1 D2 D3
   E. PHYSICAL ELECTRONICS II E1 E2 E3
   F. COMPUTER SYSTEMS F1 F2 F3 F4 F5 F6

2. You must answer eight problems (no more than three from the MATH group).
3. Answer in the blue books provided. Use a separate book for each problem. Put the title and problem number on the front of each book (eg., MATH A-1)
4. Return all the 25 problems.
5. You will be graded on your answers to eight problems only.
6. The examination is “closed-book;” only blue books, exam problems and a scientific calculator are allowed. No formula sheet is allowed. Some problems include reference formulas. No material shall be shared without prior permission of the proctor(s).
7. You have four hours to complete this examination.
PROBLEM A1 – MATH

In an RLC circuit, a switch is turned on at t=0. How does the current change after t=0?

Hint: We know the current equation from KVL:

\[ Ri + L \frac{di}{dt} = V \quad t > 0 \]

Solve the inhomogeneous equation.
Vector Analysis

Consider the function $f(x) = x^2 + y^2$.

a) Make a sketch of $f$. You do not need to be numerically accurate, but you need to capture the qualitative shape of $f$.

b) Compute the vector field $A = \nabla f$.

c) Make a sketch of $A$. Again, you do not need to be numerically accurate, but capture $A$ qualitatively.

d) Compute $\text{curl}(A)$.

e) It is possible to compute $\text{curl}(\nabla f)$ even without knowing $f$? What is the result, and how can it be interpreted?
PROBLEM A3 – MATH

**Linear Algebra**

Let \( A = [a_{ij}] \) be an \( n \times n \) symmetric matrix.

1. State the definition of a symmetric matrix along with two necessary and sufficient conditions to test that matrix \( A \) is positive definite.
2. Assume that \( A \) is symmetric and positive definite and show that the quadratic form

\[
P(x) = \frac{1}{2} x^T A x - x^T b,
\]

where \( x \) and \( b \) are \( n \)-dimensional vectors, reaches its minimum at the point where \( A x = b \).

3. Determine the minimum value \( P_{\min} \) of \( P(x) \).

4. Define the Rayleigh quotient for a symmetric and positive definite matrix \( A \).

5. Show the value of the Rayleigh quotient for matrix \( A \) cannot go below its minimum eigenvalue and cannot exceed its maximum eigenvalue.
Probability

Three points are selected at random and independently on the unit circle centered at the origin. They are then connected to form a triangle. What is the probability that the origin is in the interior of this triangle?
PROBLEM B1 – CIRCUITS AND ELECTRONICS

Sinusoidal Steady State Analysis

A. Find the circuit shown in the figure is operating at a frequency of 10 rad/s. Assume $\alpha$ is real and lies between -10 and +10, that is, $-10 \leq \alpha \leq 10$.
   a) Find the value of $\alpha$ so that the Thevenin impedance looking into the terminals a, b is purely resistive.
   b) What is the value of the Thevenin impedance for the $\alpha$ found in Part a?
   c) Can $\alpha$ be adjusted so that the Thevenin impedance equals $500 - j500 \Omega$?
   d) What is the value of $\alpha$ so that the Thevenin impedance equals $500 - j500 \Omega$?
   e) For what values of $\alpha$ will the Thevenin impedance be inductive?

B. For the following figure:
   a) Find the average power dissipated in the line in the figure if $V_g = 415 \angle 0 V$ (rms).
   b) Find the capacitive reactance that when connected in parallel with the load will make the load look purely resistive.
   c) What is the equivalent impedance of the load in Part b?
   d) Find the average power dissipated in the line when the capacitive reactance is connected across the load.
   e) Express the power loss in Part D as a percentage of the power loss found in Part A.
PROBLEM B2 – CIRCUITS AND ELECTRONICS

Laplace Application to Circuit Analysis

A. Given that \( F(s) = \mathcal{L}\{f(t)\} \), show that

\[
(1)^n \frac{d^n F(s)}{ds^n} = \mathcal{L}(t^n f(t))
\]

B. The switch in the circuit seen in the figure has been in position a for a long time before moving instantaneously to position b at \( t=0 \).

![Circuit Diagram]

a) Find \( V_1(s) \) and \( V_2(s) \)

b) Find \( v_1(t) \) and \( v_2(t) \)
Electronic Circuit

For a BJT circuit with $v_i=2.0\,\text{V}$ and $\beta=50$.

(a) Which BJT is off? Explain it clearly

(b) Calculate $V_C$, $V_E$, $I_B$, $I_C$, $I_E$
The two purely black and white images shown above are each 80 x 80 pixels. Each image contains an equal number of black and white pixels, which are represented using 8-bit grayscale intensity levels. The black lines around the boarder signify the image edges and are not pixels. Answer the following questions. *Clearly label all axes.*

a) Sketch the histogram of each image, a and b. (1pt)

b) Suppose each image is smoothed using a 3 x 3 uniform filter having coefficients that sum to 1. Disregarding the pixels that are affected by the image boundaries, determine all unique pixel intensity values that will be present in each smoothed image. (4pts)

c) Roughly sketch the two resulting histograms from part b. (2pts)

d) Roughly sketch the 2-dimensional Discrete Fourier Transform (DFT) of each original image (prior to smoothing). (3pts)
PROBLEM C2 – SYSTEMS, SIGNALS AND IMAGE PROCESSING

(10 points in total, 2 points each)

Consider a causal LTI system described by the following difference equation:

\[ y[n] - \frac{1}{2} y[n-1] = x[n] \]

(a). Determine the system function \( H[z] \) for the system.

(b). Find the impulse response \( h[n] \) for the system. Is the system stable?

(c). Determine the output \( y[n] \), using convolution, if \( x[n] = (1/4)^n u[n] \).

(d). Repeat (c) using Z transform.

(e). Find the response of the system for the input

\[ x(n) = 5 + 4 \cos(\pi n + \frac{\pi}{3}) \], for all \( n \)
An analog signal, $x(t)$, has a spectrum as shown below.

-20k  20k  30k Hz

A

a) What is the bandwidth of the signal? What is the Nyquist rate for $x(t)$? (2 points)

b) Assume that you sampled the analog signal, $x(t)$, using a sampling frequency of 60k Hz and obtained a discrete-time signal $x[n]$, draw the spectrum of $x[n]$. (4 points)

c) If we generate another analog signal by the following equation,
   $y(t) = x(t)^2$
   draw the spectrum of $y(t)$. (2 points)

d) What is the bandwidth of $y(t)$? (2 points)
PROBLEM C4 – SYSTEMS, SIGNALS AND IMAGE PROCESSING

Control Systems

Consider the block diagram of a closed-loop position control system in Figure 1 where the transfer function of the plant is

\[ G_p(s) = \frac{2}{s(s+5)}. \]

Figure 1. Block diagram of a closed-loop system.

a) (4 points) If a proportional controller is used, that is, \( G_c(s) = K \), find the following two transfer functions: \( T_{rc}(s) = \frac{C(s)}{R(s)} \) and \( T_{ru}(s) = \frac{U(s)}{R(s)} \). If the closed-loop system is asymptotically stable, find the initial and final values of the unit step responses corresponding to \( c(t) \) and \( u(t) \).

b) Let \( G_c(s) = K \), that is, consider a proportional controller.
   i. (1 point) Analytically find the range of values for the gain \( K \) that make the closed-loop system asymptotically stable.
   ii. (2 points) Draw the root locus and explain if it would be possible to select a controller gain \( K \) to achieve a settling and rise time arbitrarily small, that is, to be able to design a very fast closed-loop system.
   iii. (2 points) If the closed-loop is asymptotically stable, determine the possible gain and phase margins. Hint: Draw the Nyquist plot.
   iv. (1 point) Set up the equation to determine the proportional controller gain that results in a critically damped step response.

REVIEW

For a prototype second order open-loop transfer function \( G(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2) \) the following unit step response relations are useful:

- percent overshoot = \( 100 \exp(-\zeta\pi / \sqrt{1 - \zeta^2}) \)
- \( 2\% \) settling time \( \approx 4 / (\zeta\omega_n) \)

Continued on next page
Suppose that the loop gain of the closed-loop system can be written as $KG(s)$ with

$$G(s) = K_G \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)}$$

where $K$ is the gain of the controller that needs to be determined, $G(s)$ represents the loop gain when $K=1$, and the loop gain has $m$ zeros at $z_i$ and $n$ poles at $p_j$. The magnitude condition of root locus states that

$$|K| = \frac{\prod_{j=1}^{n}|s-p_j|}{KG\prod_{i=1}^{m}|s-z_i|}, \text{ whenever a closed-loop pole }$$
Communications

Discuss canonical representation of bandpass signals by answering the following specific questions:

1. State the definition of a bandpass signal \( x(t) \) and write the expression of its corresponding pre-envelope signal \( x_A(t) \).
2. Use the pre-envelope signal \( x_A(t) \) to define the complex lowpass equivalent signal \( \tilde{x}(t) \), and write the expression of the bandpass signal \( x(t) \) in terms of \( \tilde{x}(t) \).
3. Use the complex lowpass equivalent signal \( \tilde{x}(t) \) to define the inphase and quadrature components, \( x_I(t) \) and \( x_Q(t) \), respectively, and write the expression of the bandpass signal \( x(t) \) in terms of \( x_I(t) \) and \( x_Q(t) \).
4. Use the complex lowpass equivalent signal \( \tilde{x}(t) \) to define the expressions of the envelope and phase, \( a(t) \) and \( \theta(t) \), respectively, and write the expression of the bandpass signal \( x(t) \) in terms of \( a(t) \) and \( \theta(t) \).

NOTES:
- For full credit you must include all relevant details.
- Euler’s formula: \( e^{j\pi} = \cos(x) + j \sin(x) \).
- Transfer function of the Hilbert transformer: \( H(f) = -j \text{sgn}(f) \), where \( \text{sgn}(.) \) denotes the sign function.
1. (5 pts) Consider a signal transmitted over a 20KHz channel. If the SNR is 20 dB, what is the maximum achievable rate?

2. (5 pts) Node A needs to send a frame to node B. There are two links between node A and node B. On the first link, the frame can be dropped with probability $p$. On the second link, the frame can be dropped with probability $q$. Node A retransmits the frame if it is lost. Suppose the ACK from node B is never lost. What is the mean number of transmissions required to successfully deliver the frame to node B?
PROBLEM D1 – PHYSICAL ELECTRONICS I

Very low frequency electromagnetic waves were used for submarine communications in history. We assume seawater has $\varepsilon = 81\varepsilon_0$, $\mu = \mu_0$, and conductivity $\sigma = 4$ mho/m. Let a radio signal, in the form of 30 kHz electromagnetic wave, was transmitted vertically down into seawater, as shown in the figure below. The electric field strength measured just beneath the surface of the seawater is 1 V/m.

(a) Find the complex permittivity $\varepsilon$ and complex intrinsic impedance $\eta$ of seawater for the given electromagnetic wave.

(b) If the submarine is 100 m below the water surface, how long does it take for the signal to travel from the water surface till it reaches the submarine?

(c) Assume the detection system on the submarine can detect the minimal power density of 1 mW/m$^2$, how deep the submarine can go in the seawater before the communication is lost?

Fig. Submarine communication in seawater

\[ \vec{E} \quad \vec{S} \]
PROBLEM D2 – PHYSICAL ELECTRONICS I

Electromagnetics

The electric field of the an electromagnetic wave is given by

\[ E_x = E_0 \cos(kz - \omega t) + E_0 \cos(kz - \omega t) \]

a) What is the associated magnetic field \( B(x,y,z,t) \)?

b) What is the energy per unit area per unit time transported by this wave?

c) What is the time average of the Poynting \( <S> \) vector?
PROBLEM D3 – PHYSICAL ELECTRONICS I

Optical Fiber Communications

1. a) The number of modes supported by a fiber is 200. Core diameter is 50 \( \mu \text{m} \) and cladding diameter is 125 \( \mu \text{m} \). Incident wavelength is 0.8\( \mu \text{m} \). Calculate maximum angle which permits launching of ray into the fiber.

Formulas:

1. Number of modes, \( M = \frac{V^2}{2} \), where \( V \) is the normalized frequency parameter.
2. \( V = \frac{2na}{\lambda} NA \), where \( a \) is core radius and \( \lambda \) is incident wavelength.
3. \( NA = n_0 \sin \theta_{\text{max}} \), \( n_0 \) is refractive index of air.

1. b) \( \theta_{\text{max}} \)

\( \theta_{\text{max}} \) is from 1(a). Refractive index of core \( n_{core} \) is 1.46. Calculate the angle with which the ray internally reflects within the core.
PROBLEM E1 - PHYSICAL ELECTRONICS II

Consider a p-n junction similar to the one depicted in the schematic p-n junction below but with \( N_A = 10^{18} \text{ cm}^{-3} \) and \( N_D = 10^{16} \text{ cm}^{-3} \). At room temperature:

a) calculate the built-in potential \( \Phi_{bi} \) of this junction in [V].

b) calculate the extend of the depletion region \( x_p \) on the p side in [nm]

c) calculate the extend of the depletion region \( x_n \) on the n side in [\( \mu \text{m} \)]

d) calculate the magnitude of the electric field at the metallurgical junction in [V/cm]. The metallurgical junction depth \( x_j \) is the location where the semiconductor changes from n-type to p-type.

e) Calculate the potential difference \( \Phi \) in [V] between the n-side and the p-side with an applied forward bias of \( V = 0.6 \text{ V} \).

f) With the same forward bias as above calculate the excess minority carrier concentration at the edges of the depletion region.

g) For the case of a junction area of 10 \( \mu \text{m}^2 \) calculate the junction capacitance in [fF] for this diode at room temperature with an applied forward bias of \( V = 0.6 \text{ V} \).

h) Calculate the current density \( J \) in [A/cm\(^2\)] flowing through this diode under an applied forward bias of \( V = 0.6 \text{ V} \).
PROBLEM E2 – PHYSICAL ELECTRONICS II

Physical Electronics

1. Draw a (011) plane and a (111) plane for crystalline silicon

2. An ideal silicon junction has \( N_A = 2 \times 10^{19} \text{ cm}^{-3} \) and \( N_D = 4 \times 10^{15} \text{ cm}^{-3} \). Calculate the depletion layer width, the maximum field and the junction capacitance at zero volt and at reverse bias of 3V (\( T = 300 \text{K} \)).

A list of equations and data is provided to you below. Please note that not all equations and data should be used.

\[
\begin{align*}
J_p &= q\mu_p p \left( \frac{1}{q} \frac{dE_i}{dx} \right) - kT \mu_p \frac{dp}{dx} , \quad \frac{d^2\psi}{dx^2} = - \frac{d\phi}{dx} = - \frac{p_s}{\epsilon_s} - \frac{q}{\epsilon_s} (N_D - N_A + p - n) . \\
V_{bi} &= \psi_n - \psi_p = kT \ln \left( \frac{N_A N_D}{n_i^2} \right) , \quad N_A x_p = N_D x_n . \quad W = x_p + x_n . \\
\varepsilon_m &= \frac{qN_D x_n}{\varepsilon_s} = \frac{qN_A x_p}{\varepsilon_s} , \quad V_{bi} = \frac{1}{2} \varepsilon_m W . \quad W = \sqrt{\frac{2\varepsilon_s \left( N_A + N_D \right)}{q \left( N_A N_D \right) \varepsilon_{bi}}} V_{bi} . \\
\varepsilon(x) &= - \varepsilon_m + qN_B x , \quad \varepsilon_m = \frac{qN_B W}{\varepsilon_s} , \quad C_j = \frac{\varepsilon_s}{W} = \frac{q\varepsilon_s N_B}{2(V_{bi} - V)} . \\
V_{bi} &= \frac{kT}{q} \ln \frac{p_{no} n_{no}}{n_i^2} = \frac{kT}{q} \ln \frac{n_{no}}{n_{po}} , \quad n_{no} = n_{po} e^{qV_n/kT} . \\
p_{po} &= p_{no} e^{qV_{bi}/kT} . \quad n_n = n_p e^{q(V_n - V)/kT} , \quad n_p = n_{po} e^{qV/kT} . \\
J &= J_p (x_n) + J_n (-x_p) = J_s (e^{qV/kT} - 1) . \quad J_s = \frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n} ,
\end{align*}
\]

Silicon (300 K): \( N_C = 2.86 \times 10^{19} \text{ cm}^{-3} \); \( N_V = 2.66 \times 10^{19} \text{ cm}^{-3} \); \( n_i = 9.65 \times 10^9 \text{ cm}^{-3} \)

\( m_p = 1 m_0; \, m_n = 0.19 m_0; \, m_0 = 0.91 \times 10^{-30} \text{ kg}; \, k = 1.38 \times 10^{-23} \text{ J/K}; \, q = 1.6 \times 10^{-19} \text{ C} \)
Plasma Science and Discharges

The electron impact excitation cross section for the mercury $^3P_1$ state can be approximated by the curve below. For a Maxwellian electron distribution

$$f(\nu) d\nu = 4\pi \nu^2 \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} d\nu$$

Drive the reaction rate $<\sigma \nu>$ for electron impact excitation of the $^3P_1$ state.
1. (2 points)
   Describe the difference between a maskable and non-maskable interrupt. Give an example why using a non-maskable interrupt would be preferable over a maskable interrupt.

2. (5 points)
   A free running timer is a counter that increments every clock cycle. When the maximum count is reached, it rolls over. Assuming a 32-bit free running counter is mapped to memory location 0x8000000 and the clock frequency is 50 MHz, give the Nios2 assembly code\(^1\) necessary to measure the time in microseconds between two events. Store your result in memory location 0x01000000. Assume that an event occurs when the contents of memory location 0x80000001 is 0xFFFFFFFF. When there is no event, the contents of this location is 0x00000000. Note any limitations on the precision or range of your method.

3. (3 points)
   Solve part two except assume you have an interrupt driven timer that generates interrupts every microsecond. Assume you have the ability to turn the timer on & off and interrupts on & off to suit your approach. Furthermore, you may assume any basic functions are available to support your code provided you make it clear the function required of each.

\(^1\) If you are not familiar with the Nios2 processor, use the assembly language platform you are familiar with, *but tell me what it is.*
<table>
<thead>
<tr>
<th>Category</th>
<th>Instruction</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arithmetic</strong></td>
<td><strong>add</strong> rB, rA, imm</td>
<td>rB ← rA + imm</td>
</tr>
<tr>
<td></td>
<td><strong>add</strong> rC, rA, rB</td>
<td>rC ← rA + rB</td>
</tr>
<tr>
<td></td>
<td><strong>sub</strong> rC, rA, rB</td>
<td>rC ← rA - rB</td>
</tr>
<tr>
<td></td>
<td><strong>muli</strong> rC, rA, rB</td>
<td>rC ← (rA × imm₃₀)</td>
</tr>
<tr>
<td></td>
<td><strong>mul</strong> rC, rA, rB</td>
<td>rC ← (rA × rB)₃₀</td>
</tr>
<tr>
<td></td>
<td><strong>mulxuu</strong> rC, rA, rB</td>
<td>rC ← ((unsigned)rA × (unsigned)rB)₆₃₉₉</td>
</tr>
<tr>
<td></td>
<td><strong>and</strong> rC, rA, rB</td>
<td>rC ← rA and rB</td>
</tr>
<tr>
<td></td>
<td><strong>andi</strong> rB, rA, imm</td>
<td>rB ← rA and imm</td>
</tr>
<tr>
<td></td>
<td><strong>or</strong> rC, rA, rB</td>
<td>rC ← rA or rB</td>
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<tr>
<td></td>
<td><strong>ori</strong> rB, rA, imm</td>
<td>rB ← rA or imm</td>
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<tr>
<td></td>
<td><strong>xor</strong> rC, rA, rB</td>
<td>rC ← rA xor rB</td>
</tr>
<tr>
<td></td>
<td><strong>xori</strong> rB, rA, imm</td>
<td>rB ← rA xor imm</td>
</tr>
<tr>
<td></td>
<td><strong>nor</strong> rC, rA, rB</td>
<td>rC ← rA nor rB</td>
</tr>
<tr>
<td><strong>Comparator</strong></td>
<td><strong>cmpgei</strong> rB, rA, imm</td>
<td>rB ← (rA ≥ imm₃₀) ? 1 : 0</td>
</tr>
<tr>
<td></td>
<td><strong>cmplti</strong> rB, rA, imm</td>
<td>rB ← (rA &lt; imm₃₀) ? 1 : 0</td>
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<tr>
<td></td>
<td><strong>cmpnei</strong> rB, rA, imm</td>
<td>rB ← (rA ≠ imm₃₀) ? 1 : 0</td>
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<td><strong>cmpeqi</strong> rB, rA, imm</td>
<td>rB ← (rA = imm₃₀) ? 1 : 0</td>
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<td><strong>cmpgeui</strong> rB, rA, imm</td>
<td>rB ← (rA ≥ imm₃₀) ? 1 : 0</td>
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<td><strong>cmpltui</strong> rB, rA, imm</td>
<td>rB ← (rA &lt; imm₃₀) ? 1 : 0</td>
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<td>rC ← (rA &lt; rB) ? 1 : 0</td>
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<td></td>
<td><strong>cmpne</strong> rC, rA, rB</td>
<td>rC ← (rA ≠ rB) ? 1 : 0</td>
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<td></td>
<td><strong>cmpeq</strong> rC, rA, rB</td>
<td>rC ← (rA = rB) ? 1 : 0</td>
</tr>
<tr>
<td></td>
<td><strong>cmpgeu</strong> rC, rA, rB</td>
<td>rC ← (rA ≥ rBu) ? 1 : 0</td>
</tr>
<tr>
<td></td>
<td><strong>cmpltu</strong> rC, rA, rB</td>
<td>rC ← (rA &lt; rBu) ? 1 : 0</td>
</tr>
<tr>
<td><strong>Shifts</strong></td>
<td><strong>sll</strong> rC, rA, rB</td>
<td>rC ← rA &lt;&lt; rB₄₁</td>
</tr>
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<td></td>
<td><strong>slli</strong> rC, rA, imm</td>
<td>rC ← rA &lt;&lt; imm₄₁</td>
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<td><strong>srl</strong> rC, rA, rB</td>
<td>rC ← rA₮ &gt;&gt; rB₄₁</td>
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<td></td>
<td><strong>srli</strong> rC, rA, imm</td>
<td>rC ← rA₮ &gt;&gt; imm₄₁</td>
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<td></td>
<td><strong>sra</strong> rC, rA, rB</td>
<td>rC ← rA₮ &gt;&gt; rB₄₁</td>
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<tr>
<td></td>
<td><strong>srai</strong> rC, rA, imm</td>
<td>rC ← rA₮ &gt;&gt; imm₄₁</td>
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<td><strong>rol</strong> rC, rA, rB</td>
<td>rC ← rA rol rB₄₁</td>
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<td></td>
<td><strong>ror</strong> rC, rA, rB</td>
<td>rC ← rA ror rB₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>rolr</strong> rC, rA, imm</td>
<td>rC ← rA rol imm₄₁</td>
</tr>
<tr>
<td><strong>Memory</strong></td>
<td><strong>ldw</strong> rB, imm (rA)</td>
<td>rB ← MEM[imm₄₁+rA]</td>
</tr>
<tr>
<td></td>
<td><strong>stw</strong> rB, imm (rA)</td>
<td>MEM[imm₄₁+rA] ← rB</td>
</tr>
<tr>
<td><strong>Branch</strong></td>
<td><strong>br</strong> imm</td>
<td>goto PC+4+imm₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>bge</strong> rA, rB, imm</td>
<td>if(rA ≥ rB) goto PC+4+imm₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>blt</strong> rA, rB, imm</td>
<td>if(rA &lt; rB) goto PC+4+imm₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>bne</strong> rA, rB, imm</td>
<td>if(rA ≠ rB) goto PC+4+imm₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>beq</strong> rA, rB, imm</td>
<td>if(rA = rB) goto PC+4+imm₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>bgeu</strong> rA, rB, imm</td>
<td>if(rBu ≥ rB) goto PC+4+imm₄₁</td>
</tr>
<tr>
<td></td>
<td><strong>bltu</strong> rA, rB, imm</td>
<td>if(rBu &lt; rB) goto PC+4+imm₄₁</td>
</tr>
<tr>
<td><strong>Jump</strong></td>
<td><strong>call</strong> imm</td>
<td>goto imm &lt;&lt; 2 ; ra ← PC+4</td>
</tr>
<tr>
<td></td>
<td><strong>callr</strong> rA</td>
<td>goto ra ; rac ← PC+4</td>
</tr>
<tr>
<td></td>
<td><strong>ret</strong></td>
<td>goto ra</td>
</tr>
<tr>
<td></td>
<td><strong>jmp</strong> rA</td>
<td>goto rA</td>
</tr>
<tr>
<td></td>
<td><strong>jmpi</strong> imm</td>
<td>goto imm &lt;&lt; 2</td>
</tr>
</tbody>
</table>
Digital Systems (10 points total)

Part A (3 points)
For the SM chart provided, give the SM table for the indicated state assignment.

Part B (3 points)
Give the next state and output equations by inspection (no K-maps). The state variables are A, B, and C. Do not simplify your next state or output equations.
**Part C (3 points)**
Modify your SM Chart as needed to make it suitable for a one address microcoded controller. The previous SM chart, omitting the original state assignments, is provided for your convenience. Annotate the modified SM chart with the new state assignment. In addition, give the microinstruction format, the exact configuration of the test multiplexer, and the contents of the microprogram ROM.

![Diagram](image)

**Part D (1 point)**
The state machine modeled by the SM chart is actually a sequence detector that detects the sequence "10101" on X1 when X2='0', and "10110" when X2='1' (overlaps and switching of X2 on the fly included). Will your modified SM chart detect the sequence in exactly the same way in all circumstances? Why or why not?
PROBLEM F3 – COMPUTER SYSTEMS

Computer Architecture

1. What is the range of addresses in the instruction memory for a jump instruction (e.g., j: exit)? You need to explain why.

2. Assume that there are no pipeline (a single cycle datapath) and that the breakdown of executed instructions is as follows:

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>add</td>
<td>20%</td>
</tr>
<tr>
<td>Sub</td>
<td>10%</td>
</tr>
<tr>
<td>beq</td>
<td>25%</td>
</tr>
<tr>
<td>lw</td>
<td>25%</td>
</tr>
<tr>
<td>sw</td>
<td>10%</td>
</tr>
<tr>
<td>j</td>
<td>10%</td>
</tr>
</tbody>
</table>

In what fraction of all cycles in the input of the sign-extended circuit needed? Why?
Computer Algorithms

In string matching, you are determining whether a pattern (short array of characters) is present in a passage of text (long array of characters).

Part 1 (4 points)
Give the pseudocode for an algorithm that counts the number of occurrences of a particular pattern. Include the possibility of overlapping patterns.

Part 2 (4 points)
For your algorithm, give the step-by-step trace for the following input

   text=1312121213
   pattern=121

Part 3 (2 points)
Determine the time and space complexity for your algorithm.
PROBLEM F5 – COMPUTER SYSTEMS

Data Structure

1. A stack bStack contains the following items

```
7
8
-3
14
5
```

What is the output to the screen of the following code?

```java
int x;
while (!bStack.isEmpty()){
    bStack.pop(x);
    if (x>0 && !bStack.isEmpty())
        bStack.pop();
    cout << x << endl;
}
```

2. Please provide pseudo code or diagram (explanations) for following questions

Given the input A (2, 8, 7, 2 0, 1, 1, 6),

2.1 Construct a binary search tree according to the input A sequence.

2.2 Add a node, 5, into this binary search tree?

2.3 Delete a node, 0, from this binary search tree?
PROBLEM F6 – COMPUTER SYSTEMS

Logic Design

1. Given the following truth table,

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y (output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) (3 pts) write the Boolean equation in sum-of-products form.

(b) (4 pts) Simplify the Boolean equation using Karnaugh map.

(c) (3 pts) Draw a logic diagram using ‘and’, ‘not’ and ‘or’ gates, to represent this function.