Summary

1.) postulates — $|\psi\rangle_t$ = state

$\mathcal{O}$ = observable; es = basis $|o\rangle$

or set $|o, \lambda, \ldots\rangle$ commuting operators

$\langle o, \lambda \ldots | \psi\rangle$ representation;

countable dimensions → column vector

~~∞~~ dimensions: complex function

$$\mathbb{P}_o = \sum_\lambda |o,\lambda\rangle\langle o,\lambda|$$

~~c~~ count. dim: $\mathrm{Prob}(o) = \langle\psi|\mathbb{P}_o|\psi\rangle$

$\aleph_1$ " : $d\,\mathrm{Prob}(o \ldots o+do) = \langle\psi|\mathbb{P}_o|\psi\rangle do$

Expectation value: $\langle\mathcal{O}\rangle = \langle\psi|\mathcal{O}|\psi\rangle$

2.) Special operators: X, P, bases $|x\rangle$, $|p\rangle$

$\psi(x) = \langle x|\psi\rangle$ $X = \int dx\, |x\rangle x \langle x|$, $P = \int dp\, |p\rangle p \langle p|$ but

$\tilde\psi(p) = \langle p|\psi\rangle$ $\langle x|P = \frac{\hbar}{i}\frac{\partial}{\partial x}\langle x|$

Fourier transform $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{i\vec p\cdot\vec x/\hbar}$

Collapse: $|\psi'\rangle = \frac{\mathbb{P}_o|\psi\rangle}{|\mathbb{P}_o|\psi\rangle|}$ $e^{-i\Delta\vec a\cdot\vec P/\hbar}$ translation by ...

H: $i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$ $e^{-i\Delta t H/\hbar}$ propagator " in time

Generators of transformation: $\delta\langle\mathcal{O}\rangle = \frac{1}{i\hbar}\langle[\mathcal{O}, G]\rangle$ $\hat{}$ $P, H, \vec L, \ldots$

Heisenberg: $\Delta\Omega\,\Delta\Lambda \geq \frac{1}{2}\langle[\Omega,\Lambda]\rangle$

~~E~~ Eigenstates of H: $H|E,\ldots\rangle = E|E,\ldots\rangle$

$H|\psi\rangle = \sum_E \langle E|\psi\rangle E|E\rangle \Rightarrow |\psi\rangle(t) = \cdots$

Probability + Probability current

3.) Motion in 1D: Gaussian W.P.,

particle in a box, HO

↳ $A\sin\left(n\pi\frac{x-x_0}{L}\right)$,

$E_n = \frac{\hbar^2 n^2\pi^2}{2mL^2}$

Solutions: $\psi(x) = \left(\frac{m\omega}{\hbar}\right)^{1/4}\sqrt{\frac{1}{\sqrt{\pi}2^n n!}}\, H_n(\hat x)\, e^{-\hat x^2/2}$

$\hat x = x\sqrt{\frac{m\omega}{\hbar}}$

$\hat p = P/\sqrt{m\omega\hbar}$ ladder operators

All you need to know: comm., $\left\{ a = \frac{1}{\sqrt 2}(\hat x + i\hat p),\ a^\dagger \right.$

$|n\rangle \to |n\pm1\rangle$, $\hat x, \hat p$ matrix elements, EV of H ... $\hat H = \frac{H}{\hbar\omega} = aa^\dagger - \frac{1}{2}\mathbb{1} = a^\dagger a + \frac{1}{2}\mathbb{1}$

$nD: \ |x\rangle \rightarrow |x\rangle \otimes |y\rangle \otimes |z\rangle = |x,y,z\rangle$

$\vec{L} = \vec{R} \times \vec{P}$ $\qquad = |r,\theta,\varphi\rangle_s$

$\qquad e^{-i\vec{\alpha}\cdot\frac{\vec{L}}{\hbar}} \qquad = |\rho,\varphi,z\rangle_c$

Any vector: rotation by $e^{-i\vec{\alpha}\cdot\frac{\vec{L}}{\hbar}}$: or 2D

$$[\vec{V}, \vec{L}\cdot\hat{n}] = i\hbar\, \hat{n} \times \vec{V}$$

$$\Rightarrow [L_i, L_j] = i\hbar \sum \varepsilon_{ijk} L_k$$

angular mom.

$\langle\varphi|L_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi}$ and more complicated for

$L_x, L_y, L_+, L_-, \vec{L}^2$

Common ES: $|\beta, \ell, m\rangle$ with $\vec{L}^2|\ell,m\rangle = \hbar^2\ell(\ell+1)$ $\quad \ell = 0,1,\ldots$

$\langle\theta,\varphi|\ell,m\rangle = Y_{\ell m}(\theta,\varphi)$ $\qquad L_z|\ell,m\rangle = \hbar m \quad m=-\ell\ldots$

Full solutions to spherically symm. H ($[H,\vec{L}]=0$, H contains only $\vec{P}^2$, $|\vec{R}|$ or $\vec{R}^2$ …

Hydrogen atom $\Rightarrow \langle r,\theta,\varphi|E,\ell,m\rangle = \frac{u_{E,\ell}(r)}{r} Y_{\ell m}(\theta,\varphi)$

$$\boxed{\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}(E-V)\right]u(r) = 0}$$

$\hookrightarrow j_0 = \frac{\sin kr}{kr}$, $\quad n_0 = -\frac{\cos kr}{kr}$

$\mu = \frac{m_1 m_2}{m_1+m_2}$

electronic atom

$R_y = \frac{m_e e^4}{2\hbar^2} = \frac{m_e c^2 \alpha^2}{2} \approx 13.6\,eV$, $\quad E_n = -\frac{m_2}{m_e+m_2}\frac{R_y}{n^2}Z^2$

$a_0 = \frac{\hbar c}{m_e c^2 \alpha} \approx 0.053\,nm$, $\quad a = a_0 \frac{m_e+m_2}{m_2}\frac{n}{Z}$

$R_{n,\ell}(r) \sim (r/a)^\ell L^{2\ell+1}_{n-\ell-1}\left(\frac{2r}{a}\right) e^{-r/a}$

Magnetic μ w/ $\vec{L}$

$$H_{int} = -\vec{\mu}\cdot\underbrace{\vec{B}}_{[C]_{GS}} = -\frac{q}{2M}\vec{L}\cdot\vec{B}/[c] \qquad \vec{B} = B_z\hat{z}$$

$$\Rightarrow \langle \ell,m|H_{int} = \mu_B m_\ell B_z \qquad \mu_B = \frac{e\hbar}{2m[c]}$$

~~Larg~~ force in inhomogeneous magnetic field

Stern Gerlach