Graduate Quantum Mechanics - Problem Set 5

Problem 1)

An operator **A**, corresponding to a physical observable, has two normalized eigenstates $|\phi_1\rangle$ and $|\phi_2\rangle$ with non-degenerate eigenvalues a_1 and a_2, respectively. A second operator **B**, corresponding to a different physical observable, has normalized eigenstates $|\chi_1\rangle$ and $|\chi_2\rangle$, with eigenvalues b_1 and b_2, respectively. The two sets of eigenstates are related by:

$$|\phi_1\rangle = \frac{2|\chi_1\rangle + 3|\chi_2\rangle}{\sqrt{13}} \text{ and } |\phi_2\rangle = \frac{3|\chi_1\rangle - 2|\chi_2\rangle}{\sqrt{13}}.$$

The physical observable corresponding to **A** is measured and the value a_1 is obtained. Immediately afterwards, the physical observable corresponding to **B** is measured, and again immediately after that, the one corresponding to **A** is remeasured (independently from the result of the 2nd measurement). What is the probability of obtaining a_1 a second time?

Problem 2)

The ammonia molecule NH_3 has two different possible configurations: One (which we will call $|1\rangle$) where the nitrogen atom is located above the plane spanned by the three H atoms, and the other one (which we will call $|2\rangle$) where it is below. (These two states span the Hilbert space in our simple example). In both states, the expectation value of the energy $\langle n|\mathbf{H}|n\rangle$ is the same, E (n = 1,2). On the other hand, the two states are not eigenstates of the Hamiltonian; in fact, we have $\langle 2|\mathbf{H}|1\rangle = \langle 1|\mathbf{H}|2\rangle = -V$ (where V is some positive value).

[Comment: V tells us how strongly "coupled" the two states are – as you can see by looking at the Schrödinger equation, if you start out in either basis state, over time you will "tunnel" into the other one – and back…]

1) Write down the Hamiltonian in matrix form
2) Find both eigenvalues and (normalized) eigenstates of the Hamiltonian
3) The latter are the stable configurations of the system. Interpret their physical meaning. How can they be distinguished, and what do you think will be the ground state of ammonia?
4) Comment on the behavior of these two eigenstates under the Parity operation (which, in this case, simply interchanges "up" with "down"). Are they eigenstates of the operation? With what eigenvalues (called "parity")?

Problem 3)

Consider the following three operators (representing physical observables) on the 2-dimensional Hilbert space of complex-valued column vectors (we will learn shortly which physical system they represent):

$$\mathbf{L}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \mathbf{L}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad \mathbf{L}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

1) What are the possible values one can obtain when $\mathbf{L}_z$ is measured?
2) Assume L_z is measured and one finds the value -1. Afterwards, what are $\langle L_x \rangle, \langle L_x^2 \rangle$ and ΔL_x ?
3) Find the normalized eigenstates and eigenvalues of L_x.
4) What are the possible values one could measure for L_x and what are their probabilities if you start out with the same state as in 2)? Show that you get the same answer as in 2) if you calculate the "classical (probabilistic) expectation value" and "RMS" for the probability distribution of possible measurement results.
5) Calculate the commutators between any two of the three operators above (all 3). Do you see a pattern? Is it possible to find simultaneous eigenstates for any two of the operators? Therefore, is it possible to prepare a state of the system with well-defined values for all three?