PHYSICS 621 - Fall Semester 2013 - ODU

Graduate Quantum Mechanics - Problem Set 2

Problem 1)

Do continuous functions defined on the interval $[0\ldots L]$ and that vanish at the end points $x = 0$ and $x = L$ form a vector space? How about periodic functions obeying $f(L) = f(0)$? How about all functions with $f(0)=4$? If the functions do not qualify, list the things that go wrong.

Problem 2)

Consider the vector space **V** spanned by real 2x2 matrices. What is its dimension? What would be a suitable basis? Consider three example "vectors" from this space:

$$|1\rangle = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad |2\rangle = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}; \quad |3\rangle = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix}$$

Are they linearly independent? Support your answer with details.

Problem 3)

Consider the two vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 2\hat{i} - 6\hat{j}$ in the 2-dimensional space of the x-y plane. Do they form a suitable set of basis vectors? (Explain.) Do they form an orthonormal basis set? If not, follow the construction in the book (p.15) and the lecture to turn them into an othomormal set.

Problem 4)

Prove the triangle inequality $\left\| |V\rangle + |W\rangle \right| \le \left\| V\rangle \right| + \left\| W\rangle \right|$ for arbitrary vectors in any vector space with an inner product. You may use the Schwarz Inequality $\left| \langle V|W\rangle \right| \le \left\| V\rangle \right| \cdot \left\| W\rangle \right|$.

Problem 5)

Assume the two operators Ω and Λ are Hermitian. What can you say about

i) $\Omega\,\Lambda$;
ii) $\Omega\,\Lambda + \Lambda\,\Omega$;
iii) $[\Omega\,,\Lambda] = \Omega\,\Lambda - \Lambda\,\Omega$;
iv) $i\,[\Omega\,,\Lambda]$?

Problem 6)

Prove that the matrix $\begin{bmatrix} \cos\varphi & i\sin\varphi \\ i\sin\varphi & \cos\varphi \end{bmatrix}$ is unitary.