Graduate Quantum Mechanics - Problem Set 11

Problem 1)

Starting with the ground state $\psi_{1,0,0}(\vec{r})$ (Eq. 13.1.27, Shankar p. 357) of the hydrogen atom in configuration space, calculate the corresponding wave function $\psi_{1,0,0}(\vec{p})$ in momentum space via Fourier Transform: $\psi(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \iiint d^3\vec{r} \exp(-i\vec{p}\cdot\vec{r}/\hbar)\, \psi(\vec{r})$

Problem 2)

A particle of reduced mass μ = 470 MeV/c^2 is moving in a spherical potential well of range *a* and depth V_0 = -76.73 MeV, i.e. $V(\vec{r}) = V_0$ for $|\vec{r}| = r \le a$ and = 0 else. The particle is bound in the 1s (*l*=0) ground state with binding energy $E = -2.225$ MeV. (This is supposed to be a very simple model of the deuteron). Note: $\hbar c = 197.327$ MeV·fm.

a. Solve the Schrödinger equation for both $r \le a$ and for $r > a$.
b. Using the boundary conditions at $r = a$, extract the size of the "potential range" a.
c. Determine the absolute normalization constant of the wave function by requiring that it is normalized to 1 over all space.
d. From your results above, calculate the probability that a measurement of r will find $r > a$, i.e. the particle is "outside the range" of the potential (which is of course forbidden classically).