Addendum to Hydrogen Atom

Force Due to a magnetic field B on a length *s* of wire carrying current *I*: $F = BIs$

Torque on square loop with side length *s* $\tau = 2 \cdot s/2\ BIs \sin\theta$

Magnetic dipole moment $\mu = Is^2$

$$\vec{\mu} = Ia\,\hat{n}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Work $dW = \tau d\theta$

Let initial orientation be at $\theta = 90$

Work done = $\mu B \int_{90}^{\theta_{final}} \sin\theta\, d\theta$

Potential energy stored in the loop, $V_{pot} = -\mu B \cos\theta_{final} \Rightarrow H_{int} = -\vec{\mu}.B$

If the magnetic field is inhomogeneous;

Force = $-\vec{\nabla} V_{pot} = (\vec{\mu} \cdot \vec{\nabla})\vec{B}$

Consequence: Stern-Gerlach apparatus which can measure the angular momentum component along the z-direction determined by the field direction.

Magnetic dipole moment of a single charge *q* orbiting at fixed radius *r* with velocity *v*:

$$\mu = \frac{qv}{2\pi r} r^2 = \frac{qvr}{2} = \frac{q}{2mc} L$$

Interaction Hamiltonian is given by,

$$\boldsymbol{H}_{int} = -\vec{\mu}.B$$

$$= -\frac{q}{2mc}\vec{J}.B$$

Electron orbital angular momentum : $\boldsymbol{H}_{int} = -\frac{q}{2mc}\vec{L} \cdot \vec{B}$

For magnetic field along the z-direction: $\mathbf{H}_{int} = \mu_B\, m_l B_z$. Here $\mu_B = \frac{e\hbar}{2mc}$

Hence can have a breaking of the degeneracy of different *m* quantum numbers in an atom exposed to an external (or internal) magnetic field (Zeeman effect).