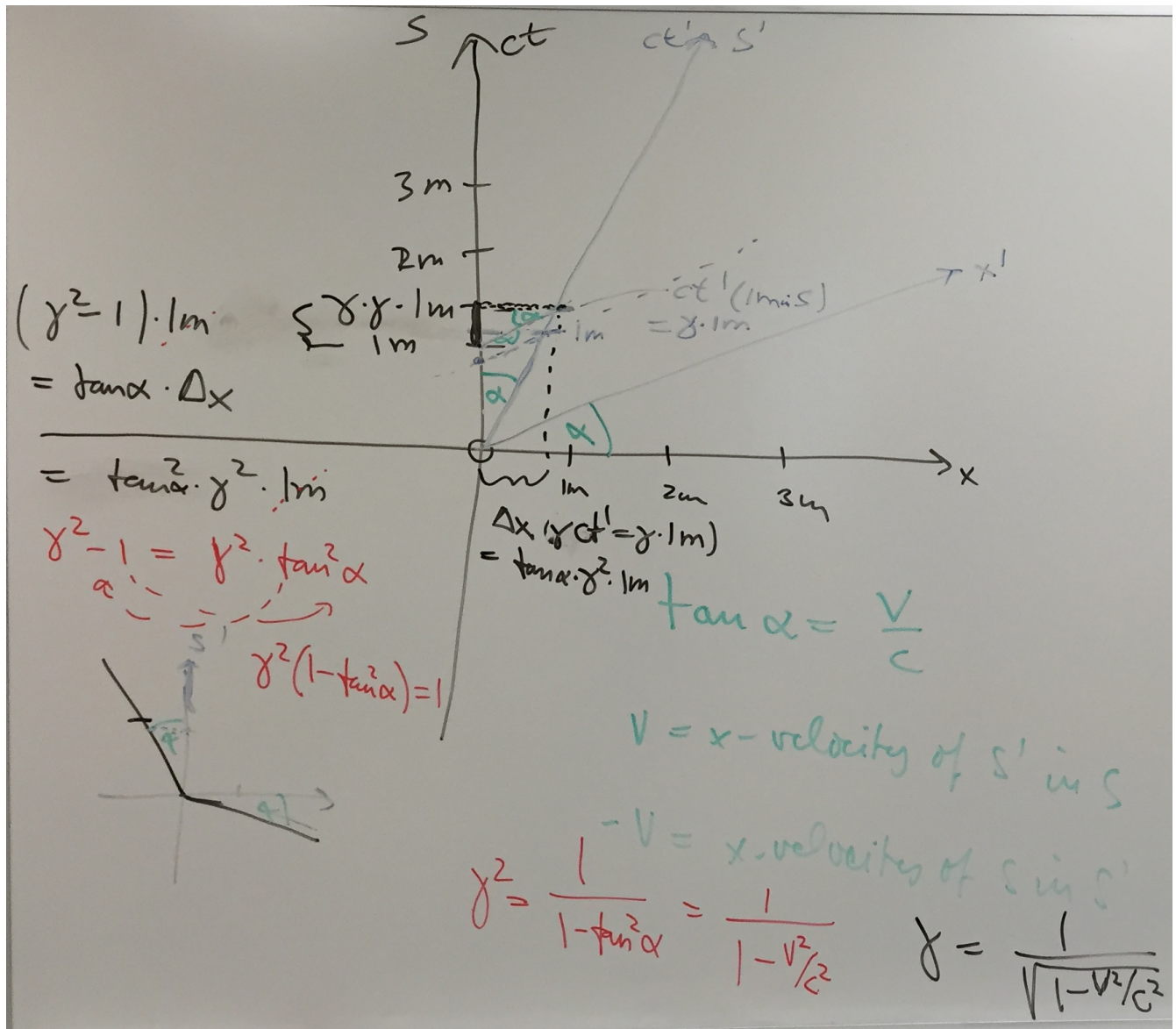


No absolute reference frame: from inertial reference frame S inertial reference frame S' appears to be moving towards the right (main drawing). From inertial reference frame S' inertial reference frame S appears to be moving towards the left (insert at bottom left).

No absolute "simultaneity" – events that all happen at the same time ($ct' = 0$) in S' appear to happen later in S the further away in x they are (blue axis labeled x').

No absolute time scale: The event " $ct' = 1$ m" as read off the clock of S' at its origin occurs at time $ct = \gamma$ in S ("S' clock is going slow").



$v = c \tan \alpha \rightarrow \frac{v}{c} = \beta = \tan \alpha$ Everything is symmetric with respect to c . γ represents relative scale of time and

space. Can be used to calculate time dilation and length contraction. $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Example:

$$v = 0.8c$$

$$\gamma = \frac{1}{\sqrt{1 - 0.8^2}}$$

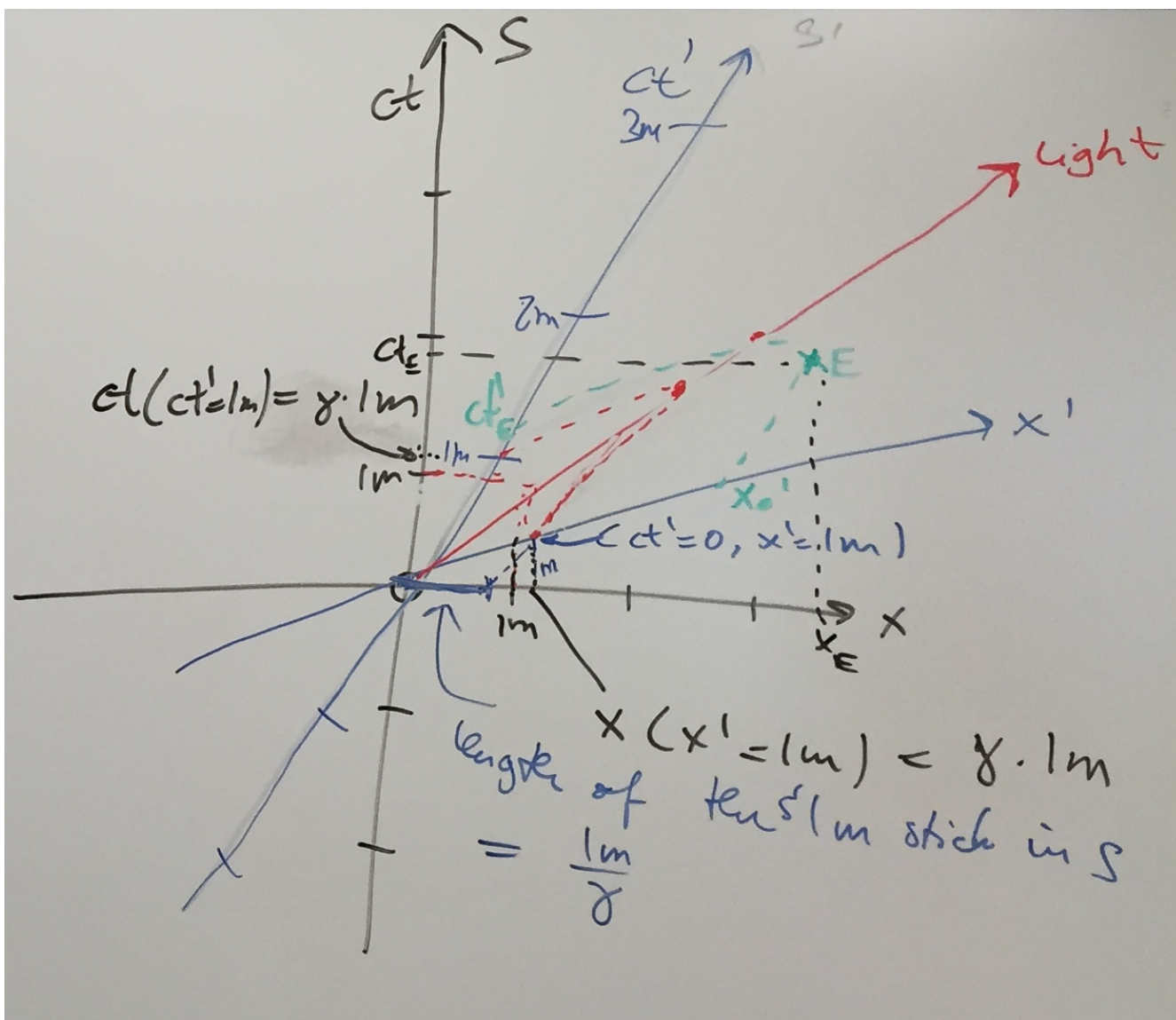
$$= \frac{1}{0.6}$$

$$\approx 1.6667$$

Clock in S' goes slow by factor γ as measured by S
 Clock in S goes slow by factor γ as measured by S'

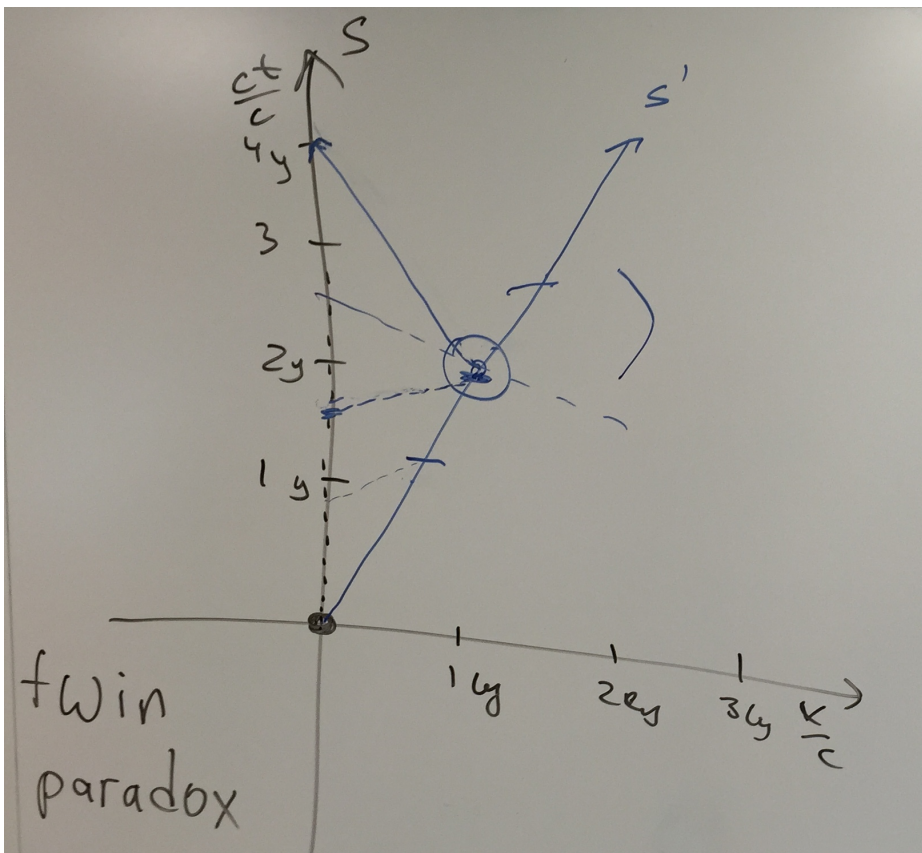
The point ($ct' = 0, x' = 1 \text{ m}$) in S' lies $\gamma \text{ m}$ to the right of the origin in S (at $x = \gamma \text{ m}$). On the other hand, the point $x = 1 \text{ m}$ lies $\gamma \text{ m}$ to the right of the origin in S'.

Because of the different definition of simultaneity, the length of the 1 m stick at rest in S' appears to be only $1/\gamma$ (shortened) in S ("length contraction").



Equations for calculating time dilation and length contraction between reference frames are called Lorentz Transformations:

$$x = \gamma(x' + \frac{v}{c}ct') \quad x' = \gamma(x - \frac{v}{c}ct) \quad ct = \gamma(ct' + \frac{v}{c}x') \quad ct' = \gamma(ct - \frac{v}{c}x)$$



Two twins, both same age at origin. Alice travels in a spaceship (in +x-direction) at 80% the speed of light. Bob stays back on Earth. After 2 years have passed on Earth, Alice is 1.6 light-years away and has aged only $2/\gamma = 1.2$ years according to Bob. However, because her different definition of “right now”, she thinks that at the point where she reached 1.2 years, Bob has only aged $1.2/\gamma = 0.72$ years. Of course, she also thinks she is only 1.2 years times $0.8c = 0.96$ light years away from Earth!

At this point, she turns around and flies back to Earth with the same speed (-x-direction). According to her own clock, she ages another 1.2 years during her trip back, and Bob also should age only 0.72 more years, for a total of 1.44 years. Yet from Bob’s perspective, he ages another 2 years and is now 4 years older since Alice left, or 1.6 years older than Alice. Somebody must be wrong, because when Alice returns, she can stop her space ship and they can compare clocks to find out who has really aged more.

In fact, they find out that indeed Bob aged more. This seems to violate the rule that in Special Relativity, all coordinate systems are equally valid and therefore Alice’s description should be just as right as Bob’s. However, this is only true for INERTIAL systems, and Alice’s system is not an inertial system for the WHOLE trip, since she does have to turn around (i.e., first decelerate and then re-accelerate in the opposite direction - her velocity is NOT constant!). It is during this brief phase of turning around that her definition of what is happening “right now” on Earth changes dramatically – before the deceleration, she thinks “right now” on Earth is only 0.72 years after her departure. However, after she turns around, “right now” on Earth is suddenly 2.56 years later – it appears to her as if her brother has aged 2.56 years nearly in an instant. So when she returns, she is not surprised to find that her brother is $0.72y + 2.56y + 0.72y = 4$ years older, in agreement with his own clock.

Similar paradoxa discussed: Muons have an average life time of only $2 \mu\text{s}$. They get produced about 9000 m up in the atmosphere when high-energy cosmic rays slam into air molecules. Even if they were going nearly with the speed of light (300 m in $1 \mu\text{s}$), they should need $30 \mu\text{s}$ to reach the ground, so (nearly) all of them should have decayed before that. Yet, there are lots of muons which do reach the ground! Why? Because in their own frames, they only “age” by $30 \mu\text{s}/\gamma$ before hitting the ground, which can be much smaller than $2 \mu\text{s}$ if γ is large enough (ultra-relativistic speeds very close to c). But how can THEY explain THEMSELVES how they can travel 9000 m in less than $2 \mu\text{s}$? Easy, because 9000 m are length-contracted to $9000/\gamma$ m which can be small.