Notes 11/25/14 Forrest Miller

The Boltzman Distribution.

N (large number) $\underbrace{\wedge}_{\text{discrete energy lvls}}$ $n(E_i) = C \cdot g(E_i) \cdot e^{-E_i/kT}$

Less likely to find particles at higher energies.

$\langle E \rangle$ = Average energy per particle $= \dfrac{\sum n(E_i) \cdot E_i}{N = \sum n(E_i)} = \dfrac{\# dof}{2} \cdot kT$

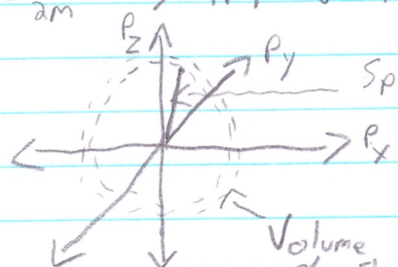
Has to be modified for continuous energies:

$$\Delta n(E \ldots E + \Delta E) = C \cdot g(E) \cdot e^{-E/kT} \cdot \Delta E$$

$g(E)\,\Delta E$ = number of available 1-particle states $E \ldots E + \Delta E$

$g(E)\,\Delta E \rightarrow$ Volume in $\vec{p}$-space

$E = \dfrac{\vec{p}^{\,2}}{2m} \Rightarrow |\vec{p}| = \sqrt{2mE}$



Sphere w/ constant radius $R = \sqrt{2mE}$

Volume $= \dfrac{4\pi}{3}(2mE)^{3/2}$

Volume of shell $= 4\pi(2mE) \cdot \Delta P$
$= 4\pi m (2mE)^{1/2} \cdot \Delta E$

$\Delta E = \dfrac{P}{M} \cdot \Delta P$

$\Rightarrow \Delta P = \dfrac{m}{p}\Delta E = \dfrac{m}{\sqrt{2mE}} \cdot \Delta E$

$g(E)\Delta E = 4\pi m (2mE)^{1/2} \Delta E = 4\pi p^2 \cdot \Delta P$

$\langle E \rangle \Rightarrow$ can be used to derive;

$P \cdot V = n_{moles} \cdot N_A \cdot kT$ & $E = \frac{3}{2} n_{moles} N_A \cdot kT$

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New way to write Boltzmann Distribution

B.D: $n(E_i) = \frac{g(E_i)}{e^{-\mu/kT} \cdot e^{E/kT}} = \frac{g(E_i)}{e^{(E-\mu)/kT}}$

μ = chemical potential

$C = \frac{1}{e^{-\mu/kT}}$

Total wave function of 2 particles
$$\begin{cases} \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) + \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) & \text{Boson; completely symmetric} \\ \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) - \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) & \text{Fermions; completely Anti-Symmetric} \end{cases}$$

What is the consequence of this?

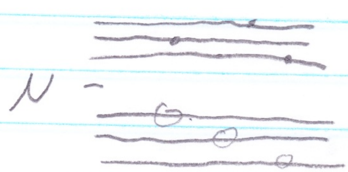
- It is impossible to have more then one electron in one Eigenstate

This affects the Boltzman Distribution.

Fermions: - $n(E_i) = \frac{g(E_i)}{e^{(E-\mu)/kT} + 1}$ <- Fermi-Dicac Distribution

always $> 1 \Rightarrow n(E_i) \leq g(E_i)$ as req'd

If $E < \mu$: $\left[e^{(E-\mu)/kT} + 1\right]$ is still > 1, but $\to 1$ as $T \to 0$



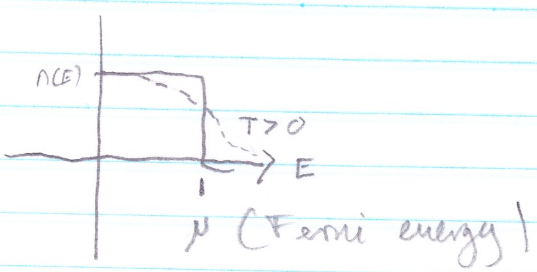
CB empty for $E < \mu$ w/ small T

VB filled for $E < \mu$ w/ small T

T=0

with Large Temp: electrons move from VB -> CB

FD:



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For Bosons: $\dfrac{g(E_i)}{e^{(E-\mu)/kT} - 1}$ Bose-Einstein Distribution

Bosons are the opposite of Fermions in the sense that they "want" to be together.

for $E > \mu$: $[e^{(E-\mu)/kT} - 1]$ is > 0

for $E = \mu$ or $T = 0$ Formula doesn't work.

B.E.D. works for "reasonable" Energies & Temperatures

for the lowest occupational level
$n(E_i)$ is Large

for upper occupational levels
$n(E_i)$ is small

This causes Bosons to condense into the lowest energy state

= phase transition (Bose-Einstein condensate)

Examples: Superfluid ^4He
Ultra cold ($\sim nK$) atoms

Fermions can couple to Bosons $(s = \frac{1}{2} + \frac{1}{2} = 1, 0)$
$\Rightarrow$ Superfluid ^3He
electron Cooper Pairs $\rightarrow$ superconductivity