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$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137.036}, \quad e = 1.6\ E{-19}\ C$$

$$Ry = \alpha^2 \frac{M_e c^2}{2} = 13.606\ eV$$

$$a_0 = \frac{\hbar}{\alpha M_e c} = 0.053\ nm$$

$\hbar c = 197.33\ eV \cdot nm$

If you have e^-, p, you can make Hydrogen H.

Possible eigen states for the Hamiltonian of the hydrogen atom : $\Psi_{n,\ell,m_\ell}(r,\theta,\varphi) \hat{=} |\Psi_{n,\ell,m_\ell}\rangle$

so;

$H_h |\Psi_{n,\ell,m_\ell}\rangle = E_n |\Psi_{n,\ell,m_\ell}\rangle$, where $E_n = -Ry \frac{1}{n^2}$ $= -13.6\ eV$

<u>Refinements:</u>

1) electrons have a degree of freedom called <u>spin</u>. (really a vector operator, $\vec{S}_{oper}$)

$S = \frac{1}{2} \rightarrow \vec{S}^2_{oper}|\Psi_{elec.\ state}\rangle = \hbar^2 s(s+1)|\Psi_{elec.\ state}\rangle$, $s = \frac{1}{2}$; $\Rightarrow \frac{3}{4}\hbar^2$ only possible Eigen value for $\vec{S}^2_{oper}$

$\therefore$ Particles w/spins $= \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$ <u>Fermions</u>

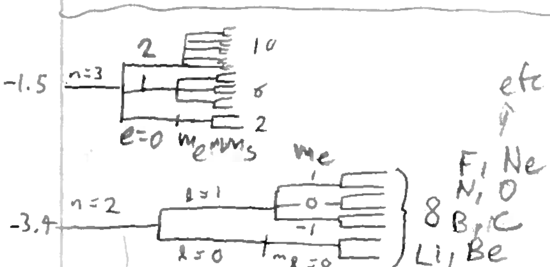
$\dots 0, 1, 2, \dots$ <u>Bosons</u>

m_s, the z-component of spin: $= -\frac{1}{2}, \frac{1}{2}$

$S_z|\Psi_{elec}\rangle = \begin{matrix} +\frac{1}{2}\hbar \\ \text{or} \\ -\frac{1}{2}\hbar \end{matrix} |\Psi_{elec}\rangle$

• now, diagram becomes: $n=1 \mid \ell=0 \mid m_\ell = 0 \mid m_s = +\frac{1}{2}$ / $m_s = -\frac{1}{2}$

Eigenstates for Hamiltonian: $|\Psi_{n,\ell,m_\ell,m_s}\rangle$



decay, $\Delta E = Ry\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \approx 10.2\ eV$

$hf = \Delta E$ $\quad \hbar 2\pi c \sim 1240\ nm\ eV$

$f \cdot \lambda = c \Rightarrow \lambda = \frac{c}{f} = \frac{hc}{\Delta E}$

-13.6 $n=1 \mid \ell=0 \mid m_\ell=0 \mid m_s = +\frac{1}{2}$ / $m_s = -\frac{1}{2}$ H, He

• Fermions have <u>Pauli Exclusion Principle</u>

- two Fermions of the same type cannot have exact same quantum states (state vectors)

(e.g., only 2 electrons max. in ground state)

2) Highest number of protons in any atom,

$Z = 1, 2, \dots, 119$ $\quad \Rightarrow E_n = -R_y \frac{Z^2}{n^2}$

essentially, characteristic size of atom,

$a = \frac{a_0}{Z}$

e -54.4 eV

Helium: • $z=2$

e -54.4 eV

$+30$ eV for $e^- e^-$ repulsion

net binding energy $= -79$ eV

Refinements:

3) Reduced Mass

$$\mu = \frac{M_{neg} \cdot M_{Nucl}}{M_{neg} + M_{Nucl}}$$

So, $E_n = -\frac{\mu}{M_e} Ry \frac{Z^2}{n^2}$. Affects radius slightly; $a = \frac{a_0 M_e}{Z\mu}$

Note: muon, μ^- has mass $\approx 200 M_e$ $\Rightarrow$ E_n 200x larger, radius a 200x smaller

4) non-zero radius of the nucleus

Relativity

5) $$E = \sqrt{m^2c^4 + p^2c^2} \approx mc^2 + \frac{p^2}{2m} - \frac{p^4}{8m^3c^2}$$

Also, leads to spin-orbit interaction, lowering states with m_ℓ opposite in sign to m_s

Net effect of relativity (called "Fine structure"): states with different ℓ, same n are still degenerate, but different combinations of m_ℓ and m_s can have slightly lower or higher energies (no longer degenerate).

6) Quantum Electrodynamics (QED) breaks even the ℓ-degeneracy (a tiny bit) $\rightarrow$ "Lamb shift"

7) Nuclei have spins, too; can lead to slightly different energies for certain combinations of m_s (HyperFine Structure)