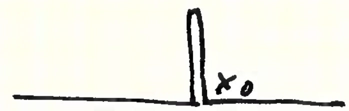
09/30/2014 PHY-323

* X eigen state $|\psi_{x_0}\rangle \longrightarrow \psi_{x_0}(x) = \delta(x - x_0)$



Very narrow spike

* P, $|\phi_p\rangle \longrightarrow \phi_p(x) = e^{i\frac{p}{\hbar}x}$

eigen state

→ $(X\psi)_{(x)} = x\,\psi(x)$

→ $(P\psi)(x) = \frac{\hbar}{i}\frac{d}{dx}\psi(x)$

→ $X|\psi_{x_0}\rangle = x_0|\psi_{x_0}\rangle$

→ $P|\phi_p\rangle = p|\phi_p\rangle$

→ $i\hbar\frac{d}{dt}|\psi\rangle = \underline{H}|\psi\rangle$ → Hamiltonian operator

quantimechanical entity on how to update a ~~quantity~~ state vector

for comparison:

Newton sez $\left[\dot{p}_x = F_x = \frac{-d\,V(x)}{dx}\right]$

where,

$V_{(x)}$ = potential energy at x

Remember: $\vec{E}(x,t) = Re\ e^{ikx}\ e^{-i\omega t}$ $\quad \omega = \frac{E}{\hbar}$

$$\frac{d}{dt}\bar{E}(x,t) = -i\frac{E}{\hbar}E(x,t) \longrightarrow i\hbar\frac{d}{d\theta}|\psi\rangle$$

= energy

H = operator, that represents Energy

→ it has to be a function of P, X: $H(P,X) = T_{kin} + V_{pot}$

$$= \frac{p^2}{2m} + V(x)$$

that is why H must represent energy

$\frac{1}{2}mv^2 = T_{kin}$

$P = mV \Rightarrow V = \frac{P}{m}$

$\frac{1}{2}m\frac{p^2}{m^2} = \frac{p^2}{2m}$

$$\rightarrow H(\psi)(x) = -\frac{\hbar^2}{2m}\cdot\frac{d^2}{dx^2}[\psi(x)] + V(x)\psi(x)$$

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

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if $\quad \Theta|\psi_0\rangle = \Theta|\psi_0\rangle \longrightarrow |\psi_0\rangle$ represents a system with value $[\Theta]$ for observable.

Same

if $\Omega|\psi_\omega\rangle = \omega|\psi_\omega\rangle$ $|\psi_\omega\rangle$ represent a system with value ω for observable Θ

where,

Ω is an operator representing an observable Θ.

if All Eigen values of Ω are contained in a set $\{\omega\}$, Then only a member (ω_i) of that ~~state~~ set can result from a measurement on any $|\psi\rangle$.

→ Afterwards

The stat vector is given by $|\psi_{\omega_i}\rangle$

"Collapse" of WF

→ The probability to mesure ω_j given $|\psi\rangle$ is $P(\omega_j) = |\langle \psi_{\omega_j} | \psi \rangle|^2$

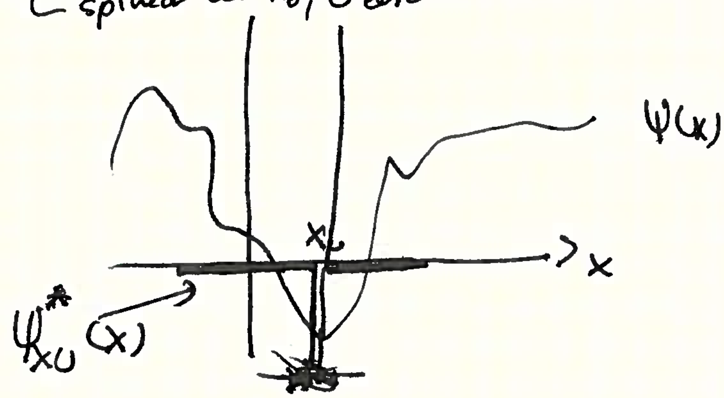
$$\cancel{\sqrt{|\langle \psi_{\omega_j} | \psi_{\omega_j}\rangle|} \sqrt{\langle \psi | \psi \rangle}}$$

– assume ψ_{ω_j} and ψ are already normalized

Ex $|\psi\rangle = \psi(x)$ what is the probability of mesuring x_0?

$$\langle \psi_{x_0} | \psi \rangle = \int_{-\infty}^{\infty} \psi^*_{x_0}(x)\, \psi(x)\, dx = \psi(x_0)$$

↑ spiked at x_0, 0 else



$$\underset{\uparrow\, x_0+\Delta x}{\mathrm{Prob}(x_0, x_0+\Delta x)} = \psi^*(x_0)\,\psi(x_0)\Delta x = \underbrace{|\psi(x_0)|^2}_{P(x)} \cdot \Delta x$$

↓ Probability Density

Δ prob. $(p_0, p_0+\Delta p)$

$$\left| \int_{-\infty}^{\infty} \phi_{p_0}^{*}(x) \cdot \psi(x) \cdot dx \right|^2 \Delta p$$

$$= \left| \int e^{-\frac{i p_0}{\hbar} x} \cdot \psi(x) \cdot dx \right|^2 \Delta p$$

inverse fourier transform of $\psi(x)$.

Expectation value of x.

$\rightarrow$ $\langle x \rangle = \int_{-\infty}^{\infty} p(x) \cdot dx = \int_{-\infty}^{\infty} x \cdot \psi^*(x) \cdot \psi(x) \cdot dx$

$$= \int \psi^*(x) (X \psi)(x) \cdot dx$$

$$= \langle \psi | X | \psi \rangle$$

$\rightarrow$ The expectation value $\langle \omega \rangle$ of Ω is given by $\langle \psi | \Omega | \psi \rangle$ in general.

Note: $\langle \omega \rangle$ is the average value of the observable if you keep repeating the measurement of Ω on <u>different</u> systems <u>all</u> represented by the <u>same</u> state vector $|\psi\rangle$.

vs. If you repeat the measurement on the <u>same</u> system over and over (in rapid succession), you simply get the initial value ω_1 again and again, not a distribution with $\langle \omega \rangle$ as centroid.