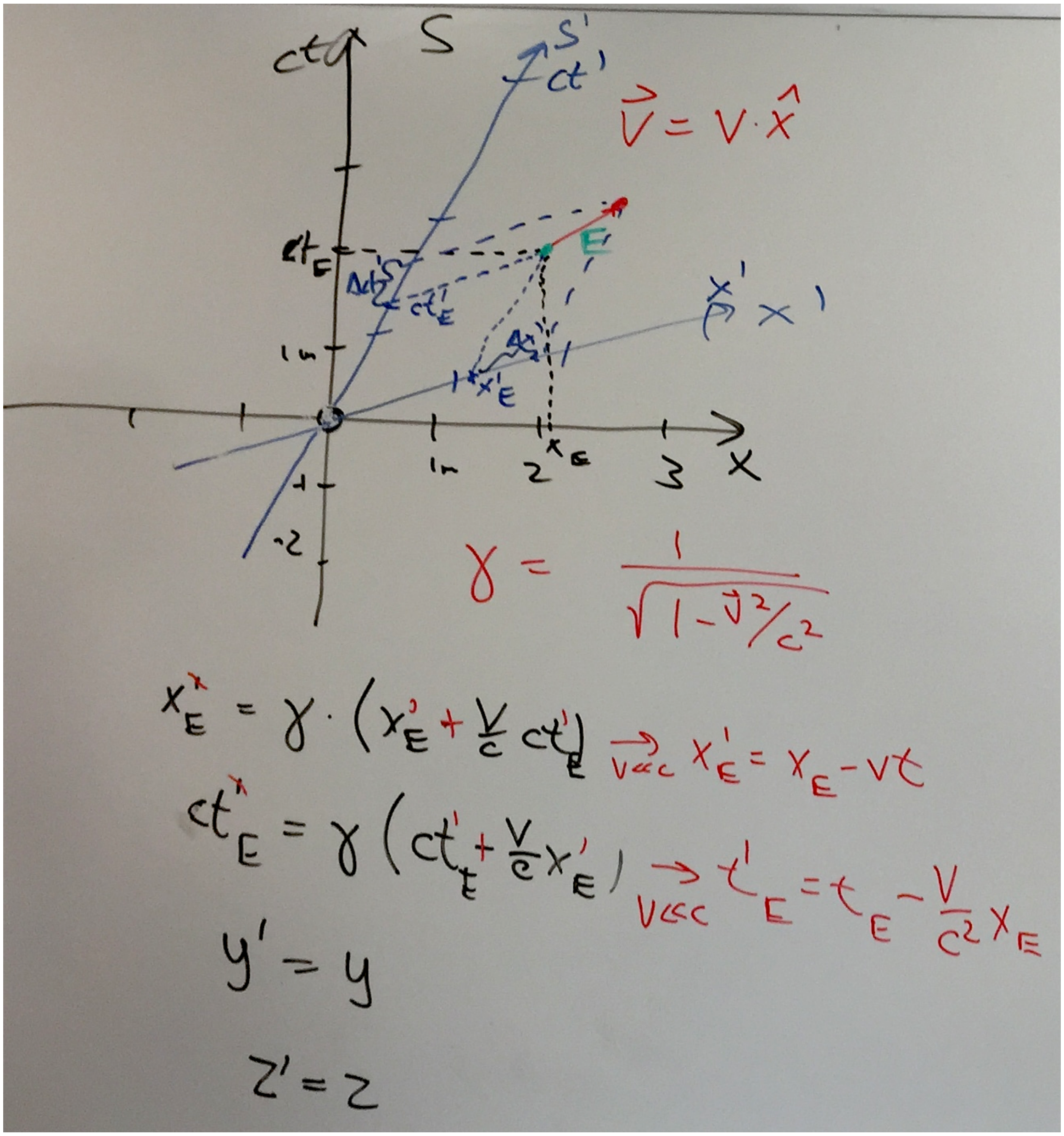
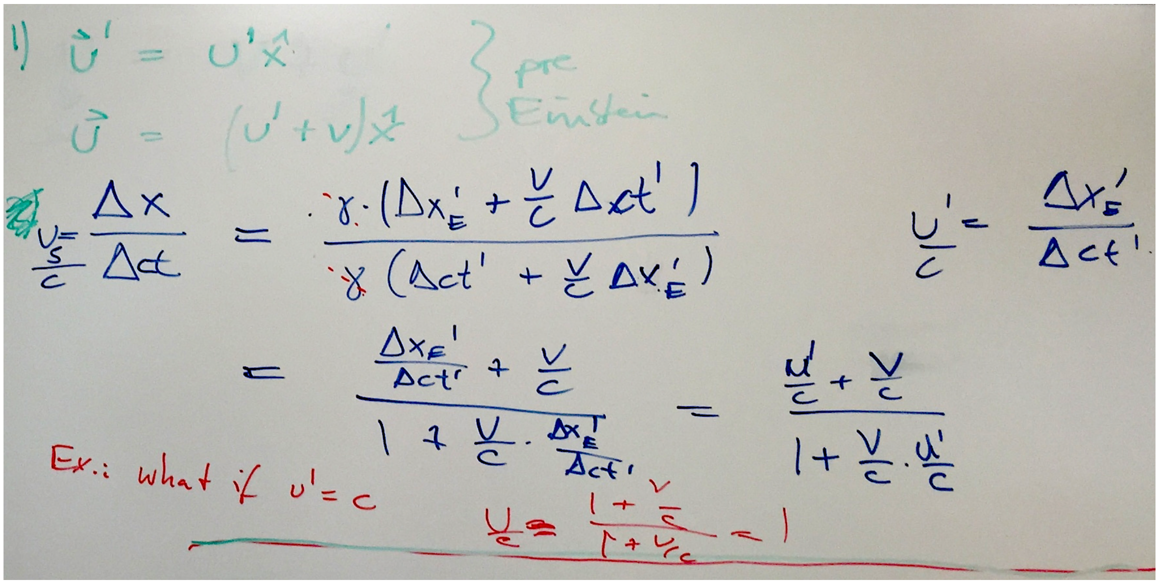
PHYS323 MODERN PHYSICS Lecture Notes 9/3/2015

Recap: Complete set of Lorentz transformations from one coordinate system S' to another, S (assuming S' is moving in positive x-direction with speed v relative to S, and their origins coincide):



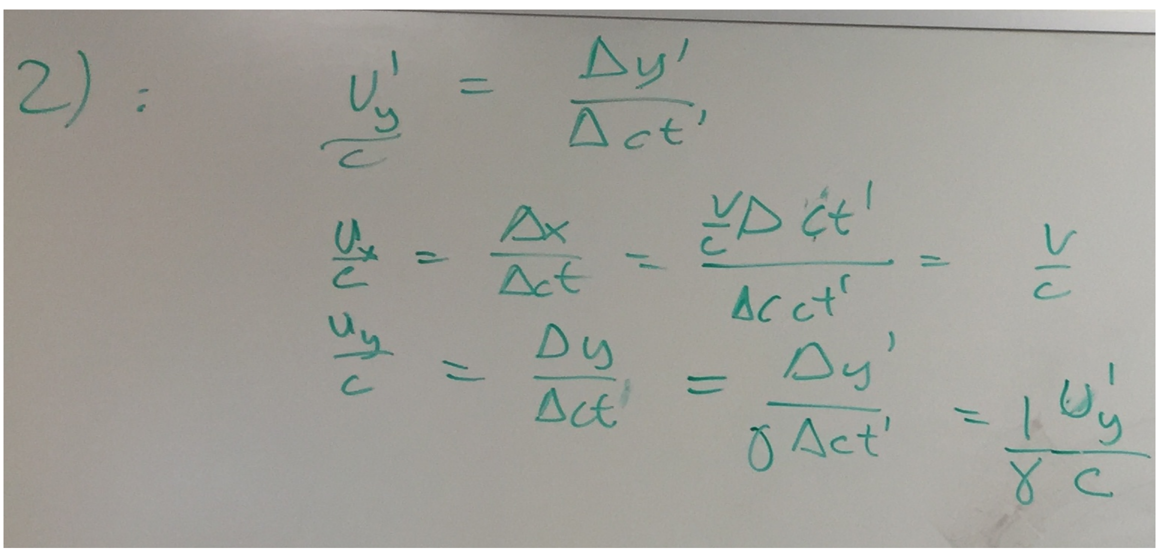
The inverse transformation, going from coordinates in S' to those in S, simply change the sign of v to –v.
(NOTE: The drawing also shows the definition of Δx, Δct etc. for two separate events indicated by the green dot labeled "E" and the red dot towards the upper right).

From these equations, we can derive the laws for the “addition of velocities”:



Here, we have a point the moves in +x’ direction in S’ with velocity u’ relative to S’. Then the velocity in S is also in the x-direction, and the magnitude is given by $\frac{u}{c} = \frac{v/c + u'/c}{1+(u'/c)(v/c)}$. In HW2 you will prove that this means u can never have a magnitude larger than c, no matter how large u’ and v are (as long as they are also below or at most equal to c).

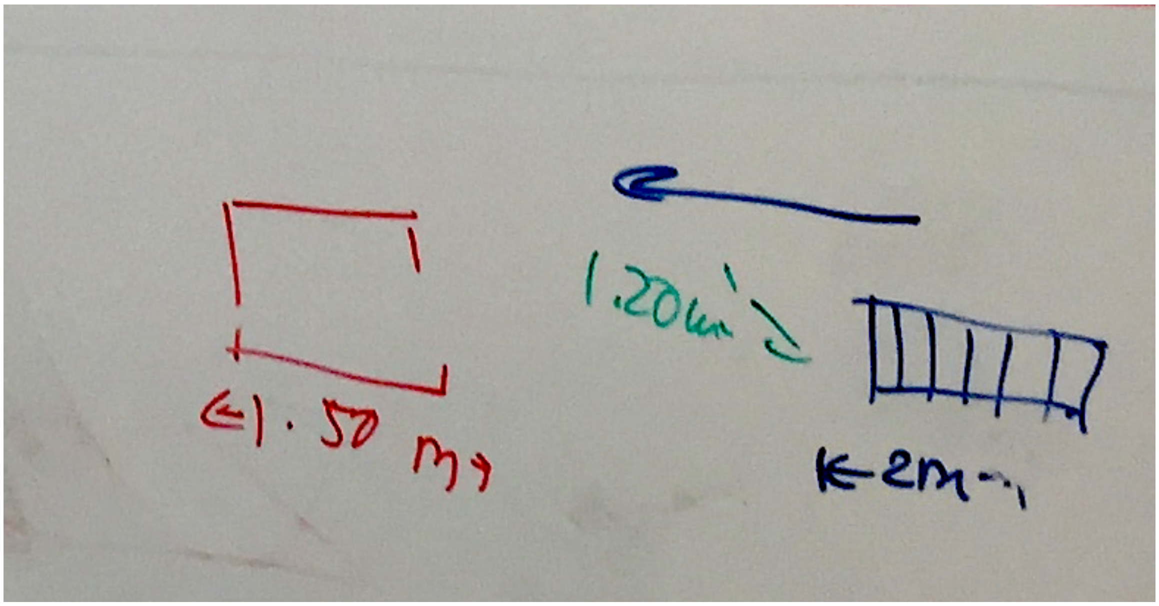
For an object moving sideways (with velocity u’ in y’-direction in S’), things are more complicated:



In general, all angles and directions change if you go from S to S’ or vice versa.

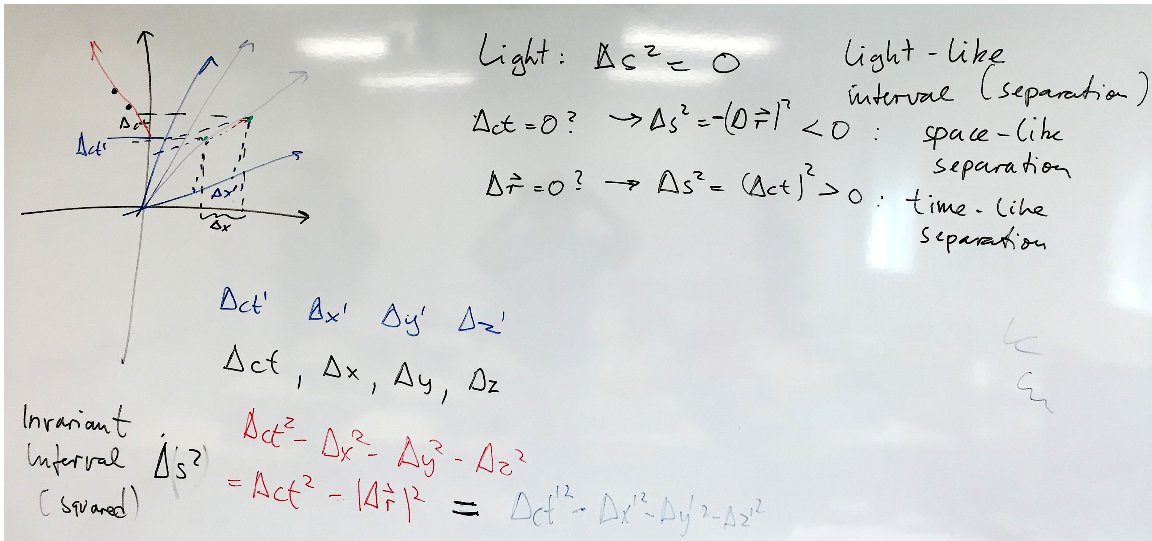
We discussed additional paradoxa:

1) Sandra Bullock's rescue by a space ship moving at 80% of c relative to her, by pulling her sideways through a 2 m long door (assuming she is also 2 m long).
From the spaceships point of view, she is length contracted to 1.2 m -> no problem
From HER perspective, the door is length-contracted to 1.2 m while she is 2 m long -> BIG problem. However, when the captain of the spaceship pulls both her head and her feet into the door sideways AT THE SAME TIME, to HER it looks like he pulls her head first and THEN her feet, so she slides in at an angle – NO problem. (SUGGESTION: DRAW A SPACE-TIME DIAGRAM OF THIS!)



2) Similarly : Barn and ladder paradox. Can the ladder be brought into the barn in one piece and then the door slammed shut?
From the barn's perspective, the ladder is length contracted to 1.2 m, so the end is inside the barn BEFORE the front hits the far wall -> YES.
From the ladder's perspective, its front hits the far wall long before the end is inside the barn – but the message "front hit the wall" takes a finite time to travel to the end of the ladder, so that one keeps moving undisturbed until after the barn door is closed. (AGAIN: Make a space-time diagram of this, including how the message travels – assume with the speed of light – it certainly can't go faster!)
CONSEQUENCE: There is no such thing as a "totally rigid object" in special relativity, since if you push on one end, the other will never move before a light signal could have reached it. (Causality – see below).

Finally, we discussed the invariant interval Δs^2 between two points (events) in space-time:



Although the individual components Δx, Δy, Δz and Δct in S can be all different from the primed counterparts in S', the results Δs^2 and $\Delta s'^2$ turn out to be exactly the same.
PROOF: Simply plug in the Lorentz Equations for Δx, Δy, Δz and Δct in S, in terms of the primed coordinates in S', and calculate Δs^2 and $\Delta s'^2$. (LITTLE EXERCISE IN ALGEBRA!)

In spite of the square, Δs^2 can be positive, zero or negative.

1) If Δs^2 is positive (see the two black dots in the screenshot above):
- it will be positive in ALL coordinate systems
- we say that "the separation between the two events (points in space-time) is time-like"
- All observers agree which of the two events occurred first and which second;
- the first is said to be "in the absolute past" of the second
- the second is said to be "in the absolute future" of the first
- this means that it is POSSIBLE that a signal COULD HAVE traveled from the 1st to the 2nd
- Therefore, the 1st COULD possibly have caused the 2nd (causality link).
- There exists a coordinate system S_0 where both events happen at the SAME position
- This would be the system of an object that moves from the 1st to the 2nd event (with $v \leq c$)
- The clock of this system would read off the elapsed time $\Delta ct_0 = \sqrt{\Delta s^2}$ between the 2 events
- We call Δct_0 the "eigentime difference" of the two events

2) If Δs^2 is zero, then the separation (interval) is called "light-like"
- This means a light ray COULD have traveled from one to the other
- Again, all coordinate systems agree on this (since c is the same in all!)
- Also, they all agree which one happened first and which one happened second (causality)

3) If Δs^2 is negative, the separation is called "space-like".
- This means there is NO possible causal connection between the two events.
- Event 2 is said to be "in the absolute elsewhere" relative to Event 1, and vice versa
- The ordering between the two events depends on the coordinate sytem:
- You can find a coordinate system where 1 is first and 2 is second, and another where it's the other way around.
- You can find a coordinate system where the two events are exactly simultaneous. In this particular coordinate system, their spatial distance is equal to $\Delta r = \sqrt{-\Delta s^2}$. (proper length)