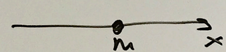
$|\psi\rangle(t)$

1-dim motion 

$|\psi\rangle(t) \rightarrow \quad \psi(x,t)$

Normalization $\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$

$$\boxed{i\hbar \frac{\partial \psi(\vec{x},t)}{\partial t} = H \psi(\vec{x},t)}$$

Prob.: $[x \ldots x+\Delta x] = \int_x^{x+\Delta x} |\psi(x,t)|^2 dx \simeq |\psi(x,t)|^2 \cdot \Delta x$

$$\underbrace{H = \frac{P^2}{2m} + V(x)} \xrightarrow{3D} \frac{P_x^2 + P_y^2 + P_z^2}{2m} + V(x,y,z)$$

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\cdot\psi(x)$$

"Stationary" solutions:

$$\psi(x,t) = \varphi_E(x) \cdot e^{-i\frac{E}{\hbar}t}$$

$$H|\varphi_E\rangle = E|\varphi_E\rangle$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_E(x) + V(x)\varphi_E(x) = E\cdot\varphi_E(x)$$

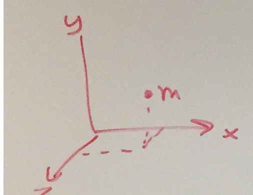
Observable Ω; $\rightarrow$ operator w/ eigenvalues ω_i; eigenvectors $|\varphi_{\omega_i}\rangle$

$\rightarrow$ Prob.$(\omega_i) = |\langle\varphi_{\omega_i}|\psi\rangle|^2$

$\rightarrow$ collapse $\Rightarrow$ $|\varphi_{\omega_i}\rangle$

$$\langle\Omega\rangle_\psi = \langle\psi|O|\psi\rangle = \int_{-\infty}^{\infty}\psi^*(x)\, O\, \psi(x)\, dx$$

$\longrightarrow$ 1D $\to$ 3D



$|\psi\rangle(t) \to \psi(\vec r, t)$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy\,dz\, |\psi(x,y,z,t)|^2 = 1$$

$$\iiint d\tau(\vec r)\, |\psi(\vec r,t)|^2 = 1$$

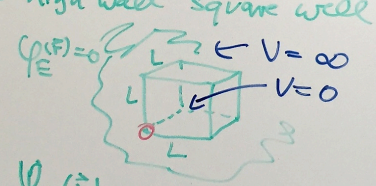
$$\Delta\,\text{Prob}(\vec r, \Delta\tau) = \iiint_{\Delta\tau} d\tau' |\psi(\vec r,t)|^2 = |\psi(\vec r,t)|^2 \cdot \Delta\tau$$

$$i\hbar \frac{\partial}{\partial t}\psi(\vec r,t) = \underbrace{-\frac{\hbar^2}{2m}\left(\underbrace{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}}_{\Delta = \vec\nabla^2}\right)\psi(\vec r,t) + V(\vec r)\,\psi(\vec r,t)}$$

2 steps: 1) $\to$ $\varphi_E(\vec r)$, $\hat H \varphi_E(\vec r) = E\varphi_E(\vec r)$

$$\psi_E(\vec r,t) = \varphi_E(\vec r)\cdot e^{-\frac{i}{\hbar}Et}$$

∞ high wall square well



$$\varphi_E(\vec r) = \varphi_{E1}(x)\,\varphi_{E2}(y)\,\varphi_{E3}(z)$$

$$P_x \to \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$P_y \to \frac{\hbar}{i}\frac{\partial}{\partial y}$$

$$P_z \to \frac{\hbar}{i}\frac{\partial}{\partial z}$$

$$\boxed{(X\psi)(x,y,z) = x\cdot\psi(x,y,z)}$$

$$[Y\psi](\vec r) = y\,\psi(\vec r)$$

$$[Z\psi](\vec r) = z\,\psi(\vec r)$$

Inside:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2 \varphi_{E1}(x)}{\partial x^2}\varphi_{E2}(y)\varphi_{E3}(z) + \varphi_{E1}\varphi_{E3}\frac{\partial^2 \varphi_{E2}}{\partial y^2} + \varphi_{E1}\varphi_{E2}\frac{\partial^2 \varphi_{E3}}{\partial z^2}\right]$$

$$= E\,\varphi_{E1}\varphi_{E2}\varphi_{E3}$$

Find: 1) $-\frac{\hbar^2}{2m}\frac{\partial^2 \varphi_{E1}}{\partial x^2} = E_1 \varphi_{E1}$

2) $-\frac{\hbar^2}{2m}\frac{\partial^2 \varphi_{E2}}{\partial y^2} = E_2 \varphi_{E2}$

3) $-\frac{\hbar^2}{2m}\frac{\partial^2 \varphi_{E3}}{\partial z^2} = E_3 \varphi_{E3}$

$$\Rightarrow \quad E_1 \varphi_{E1}\varphi_{E2}\varphi_{E3} + E_2\varphi_{E1}\varphi_{E2}\varphi_{E3}$$

$$+ E_3\varphi_{E1}\varphi_{E2}\varphi_{E3} = E\,\varphi_{E1}\varphi_{E2}\varphi_{E3}$$

$$\varphi_{E1}(x) = \sqrt{\frac{2}{L}}\sin\frac{n_1\pi x}{L} \qquad E_1 = \frac{n_1^2\pi^2\hbar^2}{2mL^2}$$

$$\varphi_{E2}(y) = \sqrt{\frac{2}{L}}\sin\frac{n_2\pi y}{L} \qquad E_2 = \frac{n_2^2\pi^2\hbar^2}{2mL^2}$$

$$\varphi_{E3}(z) = \sqrt{\frac{2}{L}}\sin\frac{n_3\pi z}{L} \qquad E_3 = \frac{n_3^2\pi^2\hbar^2}{2mL^2}$$

$n_1, n_2, n_3 = 1, 2, 3 \cdots$

Ground state: $k, m, n = 1$

$E_1 = E_2 = E_3 \rightarrow E_{g.s} = 3\frac{\pi^2\hbar^2}{2mL^2}$

Next lowest state

$n_1 = 2$

$E_1 = \frac{4\pi^2\hbar^2}{2mL^2} \rightarrow E_{\varphi_{ex}} = 6\frac{\pi^2\hbar^2}{2mL^2}$

3x degenerate