Lecture 9/22/2016

The **expectation value :** <X> is the mean, or average value for a given probability distribution. It is also denoted by the Greek letter μ.

For a discrete set of probabilities, $<X>= \sum x_i Prob(x_i)$

For a continuous set of probabilities, $<X>=\int xp(x)dx$ and

$<f(x)>=\int f(x)p(x)dx$

We require that $\int p(x)dx=1$

"Variance" is denoted by

$\sigma^2=\int(x-\mu)^2p(x)dx = \int x^2p(x)dx - 2\mu\int xp(x)dx + \mu^2\int p(x)dx =$

$<X^2> - 2\mu^2+ \mu^2 =<X^2> - \mu =<X^2> - <X>^2$, with the limits of integration ranging from some X_{min} to X_{max}. This relationship expresses the difference between $<X^2>$ and $<X>^2$. They are **NOT** interchangeable.

Standard deviation σ = square root of variance

Example of Problems where probability is required:

Position of and electron: Find the probability that the electron may be found between x=100nm--200nm: Prob(100nm--200nm) ≈ p(150nm)x100 nm. Calculate its expectation value. Example: <X> = 500nm with σ= 100nm. which means 500 nm is the best prediction with an uncertainty of ±100 nm.

Units for Subatomic Physics:

$1eV=1.602x10^{-19}J$

$m = E/c^2$ -> new unit for mass: eV/c^2, e.g. $m_{electron}= 511{,}000eV/c^2$

New unit for Momenta: eV/c

$h=6.66x10^{-34}J \cdot s$ and $\hbar=h/2\pi =197.33\ nm \cdot eV/c$

The upper limit to how precise a system's position and momentum may be specified (predicted or measured) is given as $h/4\pi$. This can also be expressed as follows:

$\sigma_p\sigma_x \geq \hbar/2$ which is known as the **Heisenberg Uncertainty Principle.**

Classical Mechanics

The defining properties include position (r), momentum (p) and mass (m).

$p = m(dr/dt) \rightarrow$ can predict $r(t+\Delta t)$ from $r(t)$ and p

r -> $F = dp/dt$ -> can predict $p(t+\Delta t)$ from $p(t)$ and $F(r)$.

Quantum Mechanics

The defining property is the state vector, $|\Psi>$. Properties of this "state vector"?

1) Contains ALL information that one CAN have about a particle/system
2) Can be used to predict probability for any measurement outcome
3) Describes how a system evolves in the future: $|\Psi>(t) => |\Psi>(t+\Delta t)$

ONE example for a state vector: Functions mapping from the Real Numbers to the complex numbers, $\Psi(x)$.

In order to determine a probability for something contained in the state vector, you must multiply by its complex conjugate in order to calculate a non-complex probability. For quantum mechanics to apply the techniques used in probability, we require that $\int\Psi^*(x)\Psi(x)dx=1$.

Here, the probability density of finding the particle near x is given by

$p(x) = \Psi^*(x)\Psi(x)$. Because the state vector is complex-valued, it can simultaneously also encode probabilities for other observables (like a flagpole which can have many shadows depending on where the light comes from).