The following formulas might be useful:

FUNDAMENTAL CONSTANTS

ϵ_0	$= 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$	(permittivity of free space)
μ_0	$= 4\pi \times 10^{-7}\,\mathrm{N/A^2}$	(permeability of free space)
c	$= 3.00 \times 10^{8}\,\mathrm{m}/s$	(speed of light)
e	$= 1.60 \times 10^{-19}\,\mathrm{C}$	(charge of the electron)
m	$= 9.11 \times 10^{-31}\,\mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}(\sqrt{x^2+y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi \quad = r^2 dr\, d\cos\theta\, d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

The Dirac Delta Function:

In one dimension: $\delta(x-x') = \begin{cases} 0 \text{ for } x \neq x' \\ \infty \text{ for } x = x' \end{cases}$

$$\int_a^b f(x)\delta(x-x_o)\,dx = \begin{cases} 0 \text{ for } x_o \notin [a,b] \\ f(x_o) \text{ for } x_o \in [a,b] \end{cases}$$

Maxwell's Equations

Translation of equations in SI system to those using Gauß system:

$$q,\rho,I,\ldots \text{ [Gauß]} \quad = \quad \frac{1}{\sqrt{4\pi\varepsilon_0}} q,\rho,I,\ldots \text{ [SI]}$$

$$\vec{E} \text{ [Gauß]} \quad = \quad \sqrt{4\pi\varepsilon_0}\vec{E} \text{ [SI]}; \quad \vec{B} \text{ [Gauß]} \quad = \quad \sqrt{4\pi\varepsilon_0}c\vec{B} \text{ [SI]}$$

Maxwell's Equations in free space:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \text{ [SI]}, \vec{\nabla}\cdot\vec{E} = 4\pi\rho \text{ [Gauß]}; \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \text{ [SI]}, \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \text{ [Gauß]}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad ; \quad \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) \text{ [SI]}, \vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t} \text{ [Gauß]}$$

Lorentz Force Law: $\vec{F} = Q\left(\vec{E} + \vec{v}\times\vec{B}\right)$ [SI], $\vec{F} = Q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$ [Gauß];

Continuity Equation: $\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$

AUXILLARY FIELDS

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

In linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0\chi_e\mathbf{E}, & \mathbf{D} = \epsilon\mathbf{E} \\ \mathbf{M} = \chi_m\mathbf{H}, & \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

P = polarization of medium = density of electric dipole moments

M = magnetization of medium = density of magnetic dipole moments

ENERGY, MOMENTUM, AND POWER

Energy: $$W = \frac{1}{2}\int\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)d\tau$$

Momentum: $$\mathbf{P} = \epsilon_0 \int (\mathbf{E}\times\mathbf{B})\,d\tau$$

Poynting vector: $$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$$

Larmor formula: $$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2a^2}{c^3}$$

Electrostatics (NO moving charges)

Coulomb's law (Fundamental Form – single point charge q at position $\vec{r}_q$):

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}_q}{\left|\vec{r}-\vec{r}_q\right|^3}$$

Gauss' law (integral version):

$$\oiint_{\text{Closed Surface}} \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}Q_{\text{enclosed}}$$

Field of a spherically symmetric charge distribution in free space:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\frac{Q(\text{encl within radius } r)}{r^2}\hat{r}$$

Field of a cylindrically symmetric charge distribution in free space:

$$\vec{E}(\vec{r}) = \frac{1}{2\pi\varepsilon_o}\frac{Q/L(\text{encl within radius } s)}{s}\hat{s}$$

Electric potential:

$$V(\vec{r}) = -\int_{\text{Some path from } \vec{r}_0}^{\vec{r}} \vec{E}(\vec{r}\,')\cdot d\vec{l}(r') \quad ; \quad \vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r})$$; $\mathbf{r}_0$ is the reference point where $V = 0$.

Potential of a single point charge q at position $\vec{r}_q$:

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\vec{r} - \vec{r}_q|}$$

Work done on a charge Q moved from point $\mathbf{r}_1$ to $\mathbf{r}_2$:

$$W = Q\,(\,V(\mathbf{r}_2) - V(\mathbf{r}_1)\,)$$

Potential energy: $U_{pot}(Q \text{ at } \mathbf{r}) = Q \cdot V(\mathbf{r})$

Energy of a point charge distribution:

$$W = \frac{1}{2} \cdot \sum_{i=1}^{n} q_i \left[\sum_{j \neq i} \frac{1}{4\pi\varepsilon_0} \frac{q_j}{|\vec{r}_i - \vec{r}_j|} \right]$$

Electric Dipole moment

$$m_1(\hat{r}) = \iiint \rho(\vec{r}\,')r'(\hat{r}'\cdot\hat{r})\,d^3r' = \hat{r}\cdot\vec{p}\,; \quad \vec{p} = \iiint \vec{r}\,'\rho(\vec{r}\,')\,d^3r'$$

Potential of a (pure) dipole:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{1}{\vec{r}^{\,2}} \hat{r}\cdot\vec{p}\ ;\ \text{field: } E(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r^3}\left[3\,(\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}\right]$$

In spherical coordinates (dipole pointing along z-axis):

$$V(r,\theta,\varphi) = \frac{p\cos\theta}{4\pi\varepsilon_o r^2}\,; \quad \text{field}: \vec{E}(r,\theta,\varphi) = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

Force on a dipole: $\vec{F} = (\vec{p}\cdot\vec{\nabla})\,\vec{E}$; Torque: $\vec{N} = \vec{p} \times \vec{E}$.

Energy of a dipole in an electric field: $W = -\vec{p}\cdot\vec{E}$

Magnetostatics

Cyclotron motion

$|p_{perp}| = |QBR|$ (momentum perpendicular to the magnetic field B for a particle with charge Q, orbiting on a circular or helical trajectory with radius R);

$\omega = \frac{|QB|}{m}$ (angular velocity of a particle with charge Q, mass m in a field B).

Work on moving charges done by magnetic forces: None.

Current densities produced by some charge distribution, moving with velocity v:

Volume current density: $\mathbf{J} = \rho \mathbf{v}$. Force on this current: $\mathbf{F} = \iiint \mathbf{J}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}')\, d^3r'$

Surface current density: $\mathbf{K} = \sigma \mathbf{v}$. Force on this current: $\mathbf{F} = \iint \mathbf{K}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}')\, da(r')$ (take *average* of **B**).

Line current: $\mathbf{I} = \lambda \mathbf{v}$. Force on this current: $\mathbf{F} = \int \mathbf{I}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}')\, d\ell(r')$.

Total current flowing through some area A: Volume current density => $I = \iint \mathbf{J} \cdot d\mathbf{a}$

Surface current density => $I = \int K_{perp}\, dl$

(= Line integral along the intersection of the area A and the surface on which **K** is confined, of the component of **K** perpendicular to that line)

Continuity equation for Magneto- and Electrostatics: $\vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t} = 0.$

Biot-Savart law for a steady volume current density:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_{\text{All Space}} \vec{J}(\vec{r}\,') \times \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3} d^3 r'$$

For the two other kinds of current distributions, the integral goes over a surface or a path.

Ampere's law in integral form

$$\oint_{\text{Closed Loop}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl}$$

, where I_{encl} is the current going through any surface spanned by the loop, in the direction of the normal on that surface (which is related to the direction of the path around the loop via the right-hand rule).

Magnetic Dipole

The field is $\vec{B}_{dipole}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m} \right]$. For a flat wire loop surrounding area a with current I, the dipole moment is $\vec{m} = Ia\hat{n}$ ($\hat{n}$ is the normal on the area).

Torque and force on a magnetic dipole: $\vec{N} = \vec{m} \times \vec{B}$; $\vec{F} = (\vec{m} \cdot \vec{\nabla})\, \vec{B}$

Electromagnetic waves

Prototype – plane wave:

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos\left(\vec{k}\cdot\vec{r} - \omega t\right); \quad \vec{B}(\vec{r},t) = \vec{B}_0 \cos\left(\vec{k}\cdot\vec{r} - \omega t\right)$$

with

$$k = \frac{2\pi}{\lambda}; \omega = 2\pi f = \frac{2\pi}{T} = k\cdot v_{phase} = k\cdot c/n$$

$$\vec{E}_0 \perp \hat{k}; \vec{B}_0 = \frac{\hat{k}}{v_{phase}} \times \vec{E}_0 = \frac{\vec{k}}{\omega} \times \vec{E}_0$$

where λ is the wave length, *f* is the frequency, $\vec{k}$ is the wave vector, v_{phase} is the phase velocity (which is equal to $c = 3 \cdot 10^8$ m/s in vacuum and equal to *c*/*n* in a medium with refractive index *n*).

Complex form: $\vec{E}(\vec{r},t) = \mathrm{Re}\left\{\vec{E}_c(\vec{r},t)\right\}; \vec{E}_c(\vec{r},t) = \vec{E}_{0c} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

Energy density (energy per unit volume, J/m^3):

$$\frac{\Delta E}{\Delta Vol} = \frac{\varepsilon_0}{2}\vec{E}^2(\vec{r},t) + \frac{1}{2\mu_0}\vec{B}^2(\vec{r},t) = \varepsilon_0 \vec{E}_0^2 \cos^2\left(\vec{k}\cdot\vec{r} - \omega t\right) \Rightarrow \left\langle \frac{\Delta E}{\Delta Vol} \right\rangle = \frac{\varepsilon_0}{2}\vec{E}_0^2 \text{ (average)}$$

Average energy current density (Intensity, “brightness”; W/m^2):

$$\frac{\Delta E}{\Delta Area \Delta t} = c\frac{\Delta E}{\Delta Vol} = \frac{c\varepsilon_0}{2}\vec{E}_0^2 = \frac{1}{2\mu_0 c}\vec{E}_0^2 = \left|\left\langle \vec{S} \right\rangle\right|$$

with the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t) = \frac{\hat{k}}{\mu_0 c}\vec{E}_0^2 \cos^2\left(\vec{k}\cdot\vec{r} - \omega t\right)$$

Momentum flux density (amount of momentum in $\vec{k}$ –direection carried through a unit surface perpendicular to $\vec{k}$ per unit time):

$$\left\langle S \right\rangle / c = \frac{\varepsilon_0}{2}\vec{E}_0^2$$

Radiation pressure *P* on a surface of area *A*:

$$P = 2\frac{\left\langle S \right\rangle}{c} A \cos^2\theta \text{ (reflection at incident angle θ relative to normal)}$$

$$P = \frac{\left\langle S \right\rangle}{c} A \cos\theta \text{ (absorption)}$$