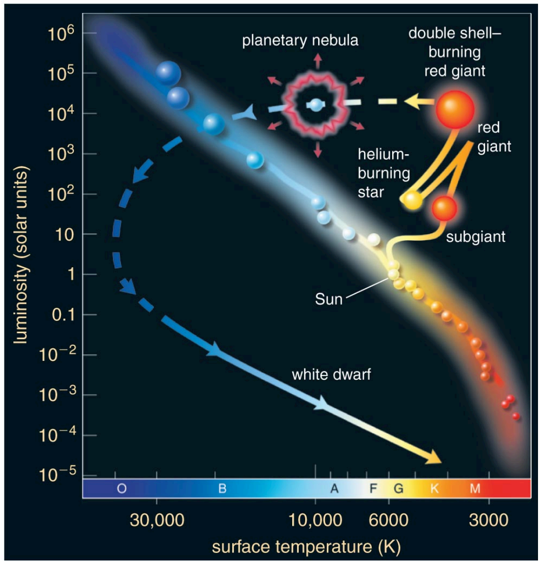
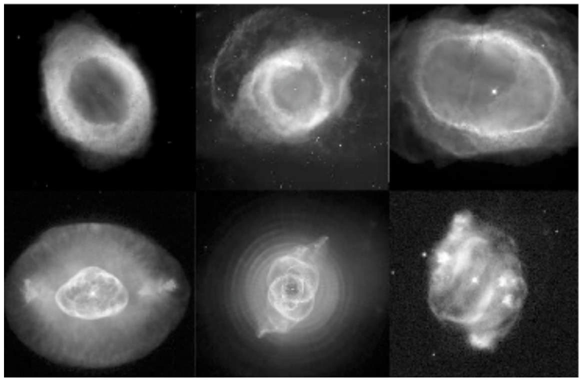
# White Dwarfs




- Reminder: Last stages of sun and similar-sized stars

  Last stage: Helium burning stops, core collapses and significant fraction of mass gets ejected as planetary nebula



- What happens with the core after the final collapse? => White Dwarf! (Example: Sirius B)
  - Core contracts until "Fermi pressure" of electrons balances gravitational attraction
  - Final size typically <1% of present solar radius => Density $10^6$ times larger than that of the sun! Temperature $10^7$ K at center

# Example: Sirius B

- Visual companion of Sirius A, 50 yr orbit
  - $M = M_{sun}$
  - $T$ = 27,000 K, Lumi = 3% of sun => $R = 0.008\ R_{sun}$ = 5500 km
  - => density = $2 \cdot 10^6$ x density(sun) = $3 \cdot 10^9$ kg/m$^3$; $10^{57}$ nucleons $2 \cdot 10^{36}$ nucleons/m$^3$ , $10^{36}$ e$^-$/m$^3$; Atoms <1/20 of radius apart
- Pressure at center:

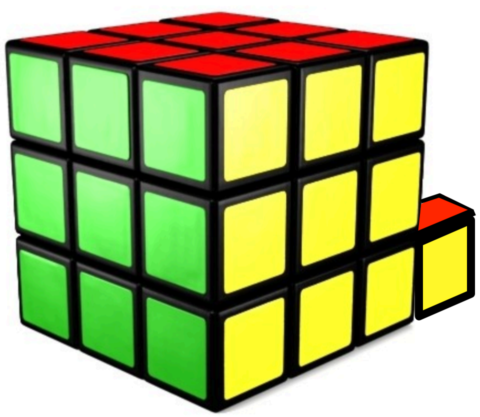
$$dP = -\frac{GM}{r^2}\rho\,dr \approx -\frac{G}{r^2}\frac{4\pi r^3}{3}\rho^2 dr = -\frac{4\pi G\rho^2}{3} r\,dr \Rightarrow$$

$$P(R) - P(0) = -\frac{4\pi G\rho^2}{3}\int_0^R r\,dr = -\frac{4\pi G\rho^2}{3}\frac{R^2}{2} \Rightarrow \quad P(0) \approx \frac{2\pi G\rho^2 R^2}{3} \approx 3.9 \cdot 10^{22}\ \text{N/m}^2$$

  - Ideal Gas: $P = nRT/V \approx 1.4 \cdot 10^{13}$ N/m$^2 \cdot T$/K => several orders of magnitude missing. Solution? => Degenerate Fermi-Gas

# Interlude: Fermi Gas

- Pauli exclusion principle: No two fermions (spin 1/2 particles) can be in the same quantum state
- Heisenberg uncertainty principle: $\Delta p \cdot \Delta x \approx \hbar$ => two states are indistinguishable if they occupy the same "cell" $dV \cdot d^3p = h^3$ in "phase space" (except for factor 2 because of spin degree of freedom)
  - "Heuristic" explanation: standing waves!
  - Assume cubic box of side length $L$, volume $V = L^3$.
  - Only possible states have wave length $\lambda = L, L/2, L/3\ldots$
  - Corresponding momenta: $p = h/\lambda = h/L, 2h/L, 3h/L\ldots$ in all 3 dimensions =>
  - New states require an addition of $(h/L)^3$ in volume to momentum space



# Interlude: Fermi Gas

- Pauli exclusion principle: No two fermions (spin 1/2 particles) can be in the same quantum state
- Heisenberg uncertainty principle: $\Delta p \cdot \Delta x \approx \hbar$ => two states are indistinguishable if they occupy the same "cell" $dV \cdot d^3p = h^3$ in "phase space" (except for factor 2 because of spin degree of freedom) => for volume $V$ and "momentum volume" $d^3p = 4\pi p^2 dp$ we find for the Number of states between $p \ldots p+dp$:

$$dN = 2\frac{V}{h^3}4\pi p^2 dp = \frac{V}{\pi^2\hbar^3}p^2 dp \quad \Rightarrow \quad N_{tot} = \frac{V}{\pi^2\hbar^3}\frac{p_f^3}{3} \quad \Rightarrow \quad p_f = \hbar\left(3\pi^2\right)^{1/3} n^{1/3}; \quad n = \frac{N_{tot}}{V}; \quad N_{tot} = \frac{M_{star}}{0.001\text{ kg}}\frac{N_A}{2}$$

$$h = 2\pi\hbar$$

- Sirius B: $p_f$ = 670 keV/c for electrons (semi-relativistic - $m_e$ = 511 keV/$c^2$!)

Sun: $6 \cdot 10^{56}$ $e^-$

– total kinetic energy:

$$E_{tot}^{kin} = \int_0^{p_f} E(p)\frac{V}{\pi^2\hbar^3}p^2\,dp = \begin{cases} \int_0^{p_f}\frac{p^2}{2m}\frac{V}{\pi^2\hbar^3}p^2\,dp = \frac{1}{2m}\frac{V}{\pi^2\hbar^3}\frac{p_f^5}{5} = \frac{3}{5}N_{tot}\frac{p_f^2}{2m} = \frac{3\hbar^2}{10m}N_{tot}\left(3\pi^2\right)^{2/3}\left(\frac{N_{tot}}{V}\right)^{2/3} = \frac{3\hbar^2\left(\frac{9\pi}{4}\right)^{2/3}}{10m}\frac{N_{tot}^{5/3}}{R^2}\text{ ; non-rel.} \\ \int_0^{p_f} pc\frac{V}{\pi^2\hbar^3}p^2\,dp = \frac{Vc}{\pi^2\hbar^3}\frac{p_f^4}{4} = \frac{3}{4}N_{tot}cp_f = \frac{3}{4}\hbar c N_{tot}\left(3\pi^2\right)^{1/3}\left(\frac{N_{tot}}{V}\right)^{1/3} = \frac{3\hbar c\left(\frac{9\pi}{4}\right)^{1/3}}{4}\frac{N_{tot}^{4/3}}{R}\text{ ; ultra-relativistic} \end{cases}$$

# White Dwarf Stability

- If *R* **de**creases, gravitational energy more negative:

$$\frac{dV_{pot}^{grav}}{d(-R)} = -\frac{d}{dR}\left(-\frac{3GM^2}{5R}\right) = -\frac{3GM^2}{5R^2}$$

- …while kinetic energy goes up:

$$\frac{dE_{tot}^{kin}}{d(-R)} = -\frac{d}{dR}\left(\frac{3\hbar^2}{10m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^2}\right) = \frac{3\hbar^2}{5m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^3}\text{; non-rel.}$$

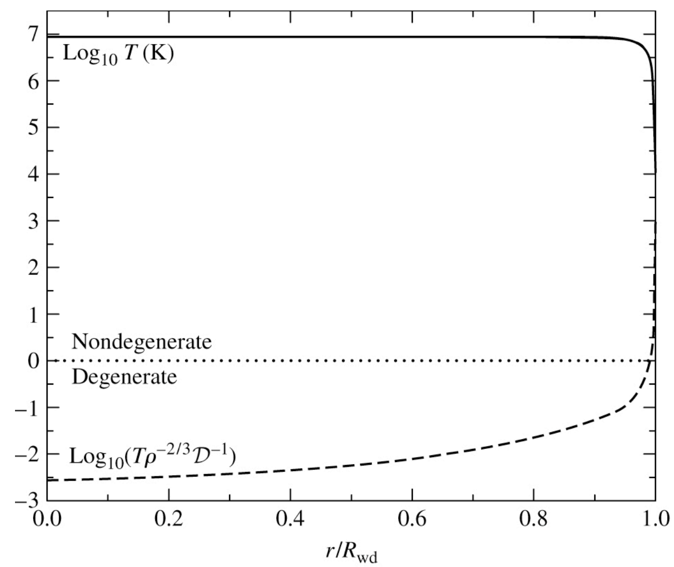
- Compare: Equilibrium if sum of derivatives = 0

$$-\frac{3GM^2}{5R^2} + \frac{3\hbar^2}{5m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^3} = 0 \Rightarrow \quad R = \frac{\hbar^2 N_{tot}^{5/3}}{m_e G M^2}\left(\frac{9\pi}{4}\right)^{2/3} \propto \frac{M^{5/3}}{M^{6/3}}$$

= 7280 (really: 5500) km / $(M/M_{sun})^{1/3}$

# White Dwarf Structure

- Center (most of volume):
  - High density, degenerate Fermi gas
  - Uniform temperature (high heat conductance)
    - initially $10^9$ K (from collapse), quickly cools to a few $10^6$ - $10^7$ K
  - mostly C, O
- Shell (thin layer, 1% in R):
  - hydrogen, helium
  - insulates star, much lower T -> much reduced radiation ($\propto T_{core}^{7/2}$)
  - further slowdown due to crystallization
  - Oldest white dwarfs have cooled to about 3500K -> can estimate age of galaxy to $10^{10}$ yr



# White Dwarf **INStability**

- If $R$ **de**creases, gravitational energy more negative:

$$\frac{dV_{pot}^{grav}}{d(-R)} = -\frac{d}{dR}\left(-\frac{3GM^2}{5R}\right) = -\frac{3GM^2}{5R^2}$$

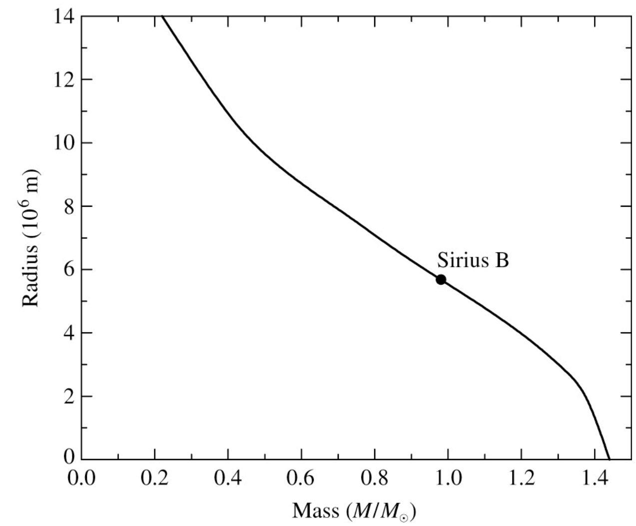
- …while kinetic energy goes up more slowly:

$$\frac{dE_{tot}^{kin}}{d(-R)} = -\frac{d}{dR}\left(\frac{3\hbar c}{4}\left(\frac{9\pi}{4}\right)^{1/3}\frac{N_{tot}^{4/3}}{R}\right) = \frac{3\hbar c}{4}\left(\frac{9\pi}{4}\right)^{1/3}\frac{N_{tot}^{4/3}}{R^2}\text{ ;fully rel.}$$

- Compare: Once first term is greater than 2nd term (depends only on $M$ and $N$), no amount of shrinking can stabilize system -> collapse

# => Chandrasekhar Limit

- For less massive, larger white dwarfs:
  - $R \approx 5600$ km $(M/M_{sun})^{-1/3}$ => $V \propto 1/M$; $\rho \propto M^2$
  - $p_f = 670$ keV/c x $(n/n_{SiriusB})^{1/3} = 670$ keV/c x $(M/M_{sun})^{2/3}$




- as mass increases, gas becomes more and more relativistic and radius becomes even smaller => runaway collapse ($R \propto M^{-\infty}$)

- Mass limit $M_{ch} = 1.4\ M_{sun}$

- Above that mass (for a stellar remnant after blowing off outer hull) electron Fermi gas pressure not sufficient for stability -> neutron Fermi gas (see later)