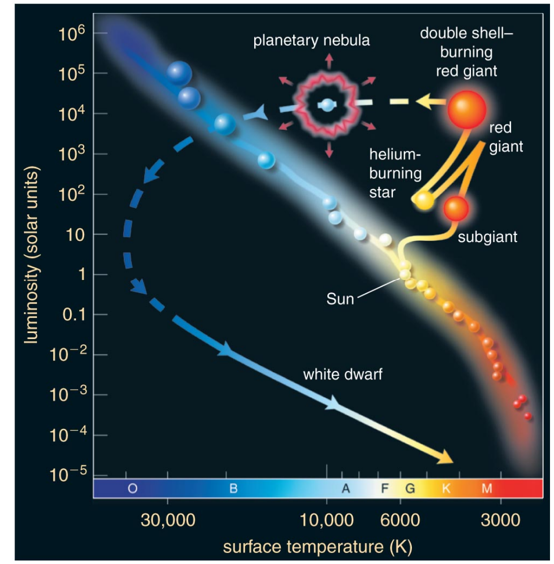
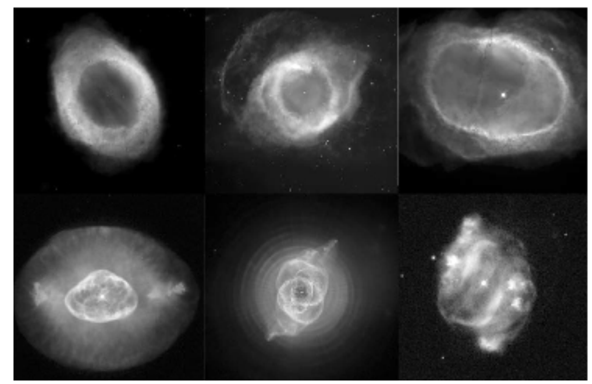
White Dwarfs



- Reminder: Last stages of sun and similar-sized stars

  Last stage: Helium burning stops, core collapses and significant fraction of mass gets ejected as planetary nebula



- What happens with the core after the final collapse? => White Dwarf! (Example: Sirius B)
  - Core contracts until "Fermi pressure" of electrons balances gravitational attraction
  - Final size typically 1% of present solar radius => Density 10^6 times larger than that of the sun! Temperature 10^6 K at center

Example: Sirius B

- Visual companion of Sirius A, 50 yr orbit
  - $M = M_{sun}$
  - T = 27,000 K, Lumi = 3% of sun => $R = 0.008\ R_{sun}$
  - => density = $2 \cdot 10^6$ x density(sun) = $3 \cdot 10^9$ kg/m^3 = $2 \cdot 10^{36}$ nucleons/m^3 , 10^{36} e$^-$/m^3
- Pressure at center:

$$dP = -\frac{GM}{r^2}\rho\, dr \approx -\frac{G}{r^2}\frac{4\pi r^3}{3}\rho^2 dr = -\frac{4\pi G\rho^2}{3} r\, dr \Rightarrow$$

$$P(R) - P(0) = -\frac{4\pi G\rho^2}{3}\int_0^R r\, dr = -\frac{4\pi G\rho^2}{3}\frac{R^2}{2} \Rightarrow \quad P(0) \approx \frac{2\pi G\rho^2 R^2}{3} \approx 3.9 \cdot 10^{22}\ \mathrm{N/m^2}$$

  - Ideal Gas: $P = nRT/V \approx 1.4 \cdot 10^{13}$ N/m^2 $\cdot$ T/K => several orders of magnitude missing. Solution? => Degenerate Fermi-Gas

Interlude: Fermi Gas

- Pauli exclusion principle: No two fermions (spin 1/2 particles) can be in the same quantum state
- Heisenberg uncertainty principle: $\Delta p \cdot \Delta x \approx h$ => two states are indistinguishable if they occupy the same "cell" $dV \cdot d^3p = h^3$ in "phase space" (except for factor 2 because of spin degree of freedom) =>
- Number of states between $p \ldots p+dp$:

$$dN = 2\frac{V}{h^3}4\pi p^2 dp = \frac{V}{\pi^2\hbar^3}p^2 dp \quad \Rightarrow \quad N_{tot} = \frac{V}{\pi^2\hbar^3}\frac{p_f^3}{3} \quad \Rightarrow \quad p_f = \left(3\pi^2\right)^{1/3} n^{1/3}\hbar$$

  - Sirius B: 670 keV/c for electrons (semi-relativistic - m_e = 511 keV/c^2!)

  – total kinetic energy:

$$E_{tot} = \int_0^{p_f} E(p)\frac{V}{\pi^2\hbar^3}p^2 dp = \begin{cases} \int_0^{p_f}\frac{p^2}{2m}\frac{V}{\pi^2\hbar^3}p^2 dp = \frac{1}{2m}\frac{V}{\pi^2\hbar^3}\frac{p_f^5}{5} = \frac{3}{5}N_{tot}\frac{p_f^2}{2m} = \frac{3}{10m}N_{tot}\left(3\pi^2\right)^{2/3}\left(\frac{N_{tot}}{V}\right)^{2/3}\hbar^2 & \text{;non - rel.} \\ \int_0^{p_f} pc\frac{V}{\pi^2\hbar^3}p^2 dp = \frac{Vc}{\pi^2\hbar^3}\frac{p_f^4}{4} = \frac{3}{4}N_{tot}cp_f = \frac{3}{4}N_{tot}\left(3\pi^2\right)^{1/3}\left(\frac{N_{tot}}{V}\right)^{1/3}\hbar c & \text{; ultra - relativistic} \end{cases}$$

White Dwarf Stability

- Pressure:

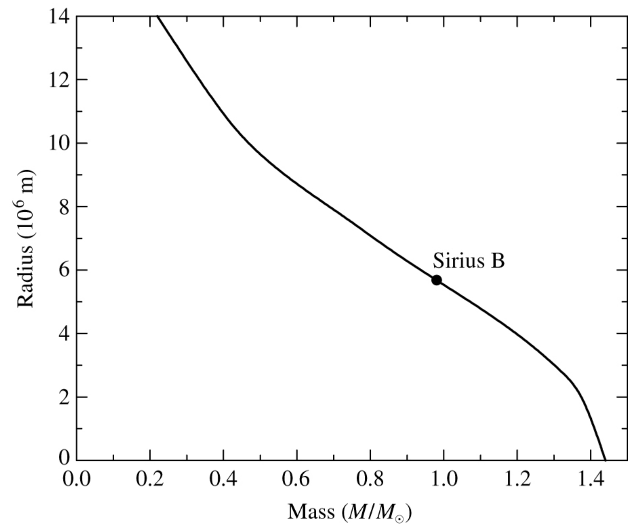
$$P = -\frac{dE_{tot}}{dV} = \begin{cases} -\frac{3\hbar^2}{10m} N_{tot}^{5/3} \left(3\pi^2\right)^{2/3} \frac{d}{dV} V^{-2/3} = \frac{2\hbar^2}{10m} N_{tot}^{5/3} \left(3\pi^2\right)^{2/3} V^{-5/3} = \frac{\left(3\pi^2\right)^{2/3} \hbar^2}{5m} n^{5/3} & \text{;non - rel.} \\ -\frac{3\hbar c}{4} N_{tot}^{4/3} \left(3\pi^2\right)^{1/3} \frac{d}{dV} V^{-1/3} = \frac{\hbar c}{4} N_{tot}^{4/3} \left(3\pi^2\right)^{1/3} V^{-4/3} = \frac{\left(3\pi^2\right)^{1/3} \hbar c}{4} n^{4/3} & \text{; ultra - rel.} \end{cases}$$

- Compare:

$$P(0) \approx \frac{2\pi G \rho^2 R^2}{3} = \left\{ \begin{array}{ll} \frac{\left(3\pi^2\right)^{2/3} \hbar^2}{5m_e} \left(\frac{\rho}{2m_N}\right)^{5/3} & \text{;non - rel.} \\ \frac{\left(3\pi^2\right)^{1/3} \hbar c}{4} \left(\frac{\rho}{2m_N}\right)^{4/3} & \text{;ultra - rel.} \end{array} \right\} \Rightarrow R^2 = \begin{cases} \frac{3\left(3\pi^2\right)^{2/3} \hbar^2}{5m_e 2\pi G \left(2m_N\right)^{5/3}} \rho^{-1/3} \propto \frac{R}{M^{1/3}} \Rightarrow & R^3 \propto \frac{1}{M} \\ \frac{3\left(3\pi^2\right)^{1/3} \hbar c}{8\pi G \left(2m_N\right)^{4/3}} \rho^{-2/3} \propto \frac{R^2}{M^{2/3}} \Rightarrow & M_{\max} \quad ! \end{cases}$$

=> Chandrasekhar Limit

- For less massive, larger white dwarfs:
  - $R \approx 5600$ km $(M/M_{sun})^{-1/3}$ => $V \propto 1/M$; $\rho \propto M^2$
  - $p_f = 670$ keV/c x $(n/n_{SiriusB})^{1/3} = 670$ keV/c x $(M/M_{sun})^{2/3}$
- as mass increases, gas becomes more and more relativistic and radius becomes even smaller => runaway collapse ($R \propto M^{-\infty}$)



- Mass limit $M_{ch} = 1.4\ M_{sun}$
- Above that mass (for a stellar remnant after blowing off outer hull) electron Fermi gas pressure not sufficient for stability -> neutron Fermi gas (see later)

White Dwarf Structure

- Center (most of volume):
  - High density, degenerate Fermi gas
  - Uniform temperature (high heat conductance)
    - initially 10^9 K (from collapse), quickly cools to a few 10^6 - 10^7 K
  - mostly C, O
- Shell (thin layer, 1% in R):
  - hydrogen, helium
  - insulates star, much lower T -> much reduced radiation ($\propto T_{core}^{7/2}$)
  - further slowdown due to crystallization
  - Oldest white dwarfs have cooled to about 3500K -> can estimate age of galaxy to 10^{10} yr