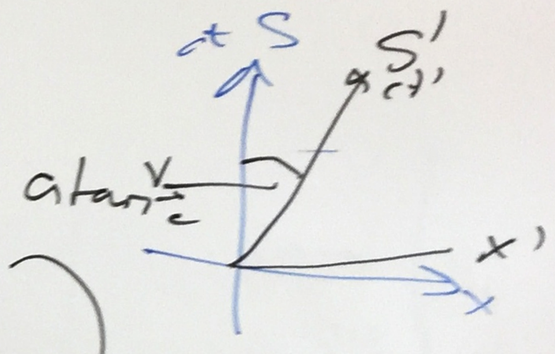
Inertial Systems { "Moving docks" — Time dilation; "Moving rulers" — Space contraction; Relativity of simultaneity



$$x = \frac{1}{\sqrt{1-v^2/c^2}}\left(x' + \frac{v}{c}ct'\right)$$

$$ct = \frac{1}{\sqrt{1-v^2/c^2}}\left(ct' + \frac{v}{c}x'\right)$$

$y = y' \qquad z = z'$

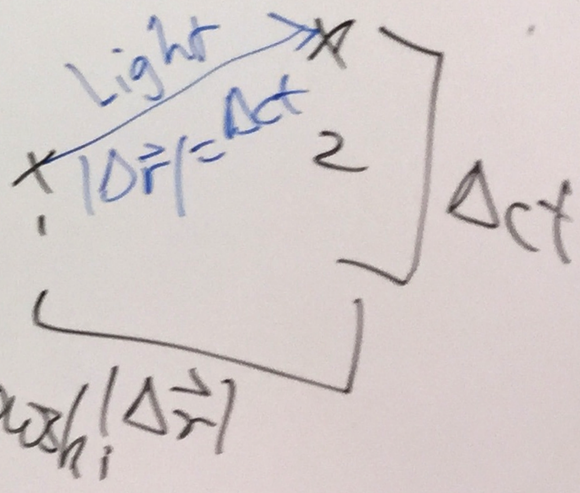
} Lorentz transformations

Invariant Interval between 2 events

$$\Delta s^2 = (\Delta ct)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

causally connected:
- $\Delta s^2 = 0 \iff$ light-like
- $\Delta s^2 > 0 \iff$ time-like
- $\Delta s^2 < 0 \iff$ space-like

} separation



$$T_{kin} - \frac{GMm}{R} > 0$$

$$\text{"} V_{pot}^{grav}$$

$$\Rightarrow \frac{m}{2}v^2 > \frac{GMm}{R}$$

What if $v = c$?

$$R > \frac{2GM}{c^2} = R_s$$

Metric

$\Delta x^\nu = 0, 1, 2, 3$

$$(\Delta ct, \Delta x, \Delta y, \Delta z)\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}\Delta ct\\ \Delta x\\ \Delta y\\ \Delta z\end{pmatrix} = \Delta s^2$$

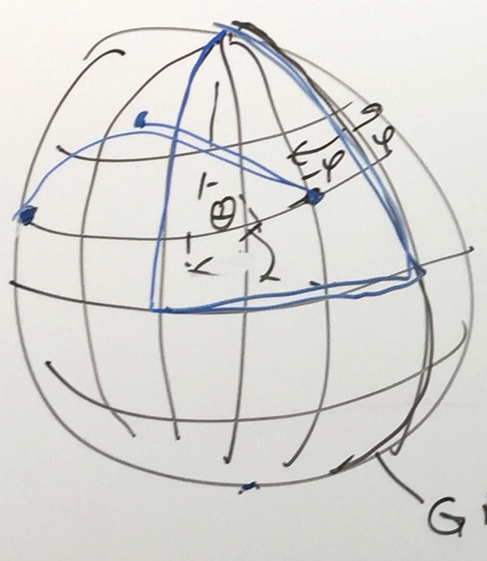
Δx^ν — Metric tensor g

$$= \sum_{\substack{\mu,\nu\\=0}}^{3} \Delta x^\mu g_{\mu\nu} \Delta x^\nu$$

All equal indices are summed over

Surface of Earth

φ, θ



Lat $= 90° - \theta$

Long $= \varphi$

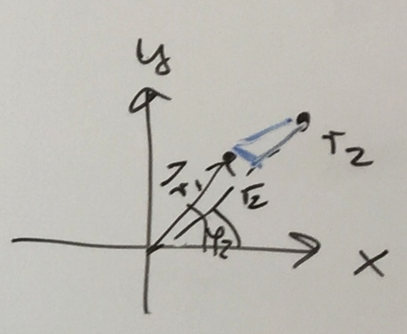
shortest distance between 2 points

$$\Delta s^2 = R^2\Delta\theta^2 + R^2\sin^2\theta\,\Delta\varphi^2$$

$$g = \begin{pmatrix}R^2 & 0\\ 0 & R^2\sin^2\theta\end{pmatrix}$$

= geodesic

circular coordinates



$|\vec{r}_2 - \vec{r}_1|^2$?

$\Delta x^2 + \Delta y^2$

$(r\Delta\varphi)^2 + \Delta r^2$

$$(\Delta r, \Delta\varphi)\begin{pmatrix}1&0\\0&r^2\end{pmatrix}\begin{pmatrix}\Delta r\\ \Delta\varphi\end{pmatrix}$$