PHYSICS 313 - Winter/Spring Semester 2017 - ODU

Astrophysics - Problem Set 4 – Solution

Problem 1

Please answer the following questions with "Y" or "N":

1a) The radiation pressure in an isotropic "photon gas" is 1/3 of its energy density. True? **Y**

1b) The (ordinary) pressure in an ideal gas is also 1/3 of its energy density. True? **N** [It's actually 2/3 of its energy density]

1c) The larger the opacity, the longer the mean free path for photons. True? **N** [It's exactly the opposite – larger opacity means shorter mean free path.]

1d) An optical depth of $\tau=1$ along some direction means that a photon going in that direction has a $1/e$ chance to escape unscathed. True? **Y**

1e) Sun glasses with strong UV protection tend to have a larger optical depth for $\lambda < 400$ nm than for $\lambda = 600$ nm. True? **Y**

1f) If a photon reaches the surface of the sun from an optical depth of $\tau=100$ in the sun's outer layers, its total travel path length will be equal to the straight-line distance from the depth it started at to the surface. True? **N** [It's more like 100 times that much, due to the random walk]

1g) Does pressure always have to increase as you go deeper into a star (i.e. with decreasing distance from its center)? **Y**

1h) Is there any other mechanism of energy transport in a star than radiation? **Y** [Convection]

1i) The optical depth of some layer of material **only** depends on its opacity. True? **N** [It also depends on its density.]

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Problem 2

Answer the following questions with (brief) derivations and final numerical results:
Note: Use the fact that one Hydrogen atom has an atomic weight of roughly 1, meaning 1 g per mol or 1 kg per 1000 mol = $6 \cdot 10^{26}$ atoms. Using the mass of the sun, I find that it contains roughly $1.2 \cdot 10^{57}$ H atoms.

2a) Assume sun was powered by chemical instead of nuclear reactions. To simplify the calculations, assume that sun is made entirely of hydrogen atoms, and each atom undergoes a chemical reaction liberating 1 eV per atom. How long would sun be able to shine at its present luminosity under these assumptions? **15,844 years**

Ans.: Multiply $1.2 \cdot 10^{57}$ with 1 eV/atom and with $1.6 \cdot 10^{-19}$ J/eV gives $1.9 \cdot 10^{38}$ J. This would be radiated in 15,844 years

2b) How would you answer change if the reaction is instead fusion into helium, with a total energy released pro hydrogen atom of 6.7 MeV? (Still assume ALL of the sun starts out as hydrogen and all of it gets converted to helium). **Ans.**: of course 6.7 million times as long - **100 Billion years.**

2c) Using the luminosity, mass and surface temperature of stars at either end of the main sequence, calculate their radii and their average densities as multiples of those of the sun. [Low-mass end: $M = 0.07\ M_{sun}$, $L = 5 \cdot 10^{-4}\ L_{sun}$, $T = 1700$ K : $\boldsymbol{R = 0.26\ R_{sun}, \rho = 4\ \rho_{sun}}$
High-mass end: $M = 100\ M_{sun}$, $L = 10^6\ L_{sun}$, $T = 53{,}000$ K; $T_{sun} = 5800$ K]
$\boldsymbol{R = 12\ R_{sun}, \rho = 0.06\ \rho_{sun}}$

Note: We use $L = \sigma T^4 * 4\pi R^2$ to get the radius and $\rho = M/(4\pi R^3/3)$ for the density.

Problem 3

Assume a star is surrounded by a thin, very low-density (dilute) spherical shell of gas (with a radius significantly larger than the star's), which it heats up to some high temperature. Under certain conditions, observers on Earth see this sphere as a shining ring surrounding the star (see photo). Using what you have learned about optical depth, opacity and radiation transport, how can you explain this?
(3-5 sentences)

Answ.: Since the gas has low density and the shell is thin, it will have a small optical depth, $\tau << 1$. This means that all light produced in the shell (due to its high temperature) eventually reaches the outside, with minimal absoprtion. However, because there is not much material in the shell, there simply is not much light emitted perpendicular to its surface – not enough for us to see the part of the surface closest to us (in the middle of the "ring"). On the other hand, at the edges, we are looking through the shell at a shallow angle (or even at 90 degrees relative to the normal), which means we are looking at a much larger "physical depth" and therefore much more material, all of which we can see because of the small optical depth of the shell even in that direction.