PHYSICS 313 - Winter/Spring Semester 2017 - ODU

Astrophysics – LAST Problem Set 12 – Solution

Problem 1

Mark each of the following statements with “Y” or “T” if they are correct, and with “F” or “N” if they are incorrect:

1a) To the best of our knowledge, the Universe has a significant positive radius of curvature. **F**

1b) Unless something changes in the composition of the universe, it will eventually grow exponentially with time. **T**

1c) The redshift of the light from any far-away source embodies the change in the size of the universe since emission of that light. **T**

1d) Some of the objects from which we receive light today are nearly 50 Billion light years away from us right now. **T**

1e) Since emission of the CMB radiation, the universe has increased nearly 1100 times in size. **T**

1f) If 1e) is correct, then it follows that the emission of the CMB radiation must have happened at a time when the universe was exactly 1/1100 of its present age (time since the big bang). **F**

Problem 2

The following are NOT multiple choice questions! Instead, answer each question with **five ordered** single digits (i.e., arrange the digits 1-5 in the correct order):

2a) The following ingredients are all affecting the evolution of the universe. Please write down **all of the 5 numbers in the correct order** indicating the relative importance of each ingredient for the dynamic evolution (i.e., its energy density) **today**. Begin with the most important and end with the least important. **4, 2, 5, 1, 3** [We don’t know the last one for sure, but it’s plausible]

1 – Radiation
2 – Dark matter (nonrelativistic “dust”)
3 – Curvature
4 – Dark energy
5 – Baryonic matter (the stuff of galaxies and intergalactic gas – also nonrelativistic “dust”).

2b) How would this ordering change if you look back to a time where the universe was only 1000 years old? **1, 2, 5, 4, 3**

Problem 3 – Give written responses (in complete sentences) and show the mathematical derivation as well as your final result for each question

Before the modern cosmological standard model including dark matter and dark energy, people thought that the universe was practically empty (which it would be without these two ingredients – baryonic matter makes up only 4-5% of the critical density ρ_0 today). Yet they knew it was expanding (by observing the light from that puny amount of matter that is in stars and galaxies).

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For the following, assume a Hubble constant H_0(today) = 70 km/s / Mpc = 1/14Byr. Also let's use for the present age of the Universe t_0 = 14 Byr.
a) Let's begin assuming $\rho_{tot}(t) = 0$ exactly (no matter, no radiation, no dark energy; of course, even without dark matter this is only an approximation and only valid for more recent times). What can you conclude about the curvature of the Universe?

Answ.: Since by definition, the sum of all fractional densities is 1,
$\Omega_M^0 + \Omega_R^0 + \Omega_\Lambda^0 + \Omega_K^0 = 0 + 0 + 0 + \Omega_K^0 = 1$ and $\Omega_K^0 = -\dfrac{Kc^2}{H_0^2 a_0^2}$, the coefficient K must be negative and by convention K = -1. The universe has negative curvature, which means it is open.

b) Under the same assumption above, what will be the time-dependence of the scale parameter a(t)?

Answ.: Using $\dot{a}(t) = a(t) H_0 \sqrt{\left(\Omega_M(t) + \Omega_R(t) + \Omega_\Lambda(t) + \Omega_K(t)\right)}$ and $\Omega_K(t) = \Omega_K^0 \dfrac{a_0^2}{a^2}$, we find $\dot{a}(t) = aH_0 \sqrt{\Omega_K^0 \dfrac{a_0^2}{a^2}} = H_0 a_0 =$ const. $\Rightarrow a(t) = a_0 + H_0 a_0 \left(t - t_0\right)$. The Universe grows linearly with time. Obviously, it started out with zero size when $t_s = t_0 - \dfrac{1}{H_0}$, so we can also write this result more simply as $a(t) = H_0 a_0 \left(t - t_s\right)$. Finally, we are free to choose the origin of time (i.e., what moment we call $t = 0$), so we might as well put that moment at time t_s (in other words, $t_s = 0$ which also yields $t_0 = 1/H_0$ = 14 Byr). This gives us the simplified equation $a(t) = H_0 a_0 t$

c) Still using the same assumptions, calculate how far away from us a quasar was when it emitted a lightray 12 Byr ago that we observe today. What will be the redshift of this light? How far is that same quasar away today?

Answ.: We use the the equation $\dfrac{dr_c}{dt} = \dfrac{c}{a(t)}$ to describe the motion of the light in the co-moving coordinate system. Plugging in our result from above yields $dr_c = -\dfrac{c}{H_0 a_0 t} dt$ (the minus sign reflects the fact that the light is moving **towards** us). This gives for the distance of the quasar in co-moving coordinates $\int_{r_c(emit)}^{0} dr_c = -r_c(emit) = -\dfrac{c}{H_0 a_0} \int_{t(emit)}^{t_0} \dfrac{dt}{t} = -\dfrac{c}{H_0 a_0} \ln\left(\dfrac{t_0}{t(emit)}\right)$.
Here, $t_0 = 1/H_0$ = 14 Byr and t(emit) = 14 Byr – 12 Byr = 2 Byr, so the logarithm is ln(7) = 1.946.
It follows that $D(emit) = a(t(emit)) \cdot r_c(emit) = H_0 a_0 \dfrac{ct(emit)}{H_0 a_0} \ln\left(7\right) = 1.946 \cdot 2Blyr$ which is 3.892 Billion light years. Since the Universe has increased in size 7-fold since the emission, the redshift is z = 6 and today the same quasar is 7 times further away, namely 27.2 Blyr.

d) Now let's add the 5% of matter. Without **detailed** calculation, explain why (and roughly at what value of the scale factor, $a(\mathrm{t})/a_0$) the importance of the curvature term would become smaller than the importance of the cold matter ("dust") term in the evolution of $a(\mathrm{t})$.

Answ.: The evolution equation for the scale factor in this case would read like $\dot{a}(t) = a(t)H_0\sqrt{\left(\Omega_M(t)+\Omega_K(t)\right)}$ where $\Omega_K(t) = \Omega_K^0 \frac{a_0^2}{a^2} = 0.95\frac{a_0^2}{a^2}$ and $\Omega_M(t) = \Omega_M^0 \frac{a_0^3}{a^3} = 0.05\frac{a_0^3}{a^3}$. As a increases (in the future), the matter term becomes less and less important. Vice versa, in the past, when a was much smaller, this term was more important, and in fact at a time when a was 1/19 of its present value, both the matter and the curvature term would have been equal in size. Even further back in the past, the matter term would have dominated. This shows why a non-zero curvature is so problematic – even if it starts out being only a small contribution to the overall density in the past, it grows to dominate everything and becomes maximal (as long as there is no dark energy around).

e) Ultimately, in any universe that contains any radiation at all, the radiation must have dominated the evolution of $a(\mathrm{t})$ at some (very early) time. Explain why!

Answ.: The contribution due to radiation, $\Omega_R(t) = \Omega_R^0 \frac{a_0^4}{a^4}$, grows even more quickly if we go backwards in time (towards smaller and smaller a) than the matter contribution, $\Omega_M(t) = \Omega_M^0 \frac{a_0^3}{a^3}$.

If the Universe was arbitrarily small at some point in the past, we can always find a size a such that the radiation term dominates all others. Another point worth making is that, at early enough times, all **types** of matter start behaving like radiation, since the temperature will be so high that the average energy per degree of freedom is much larger than the rest mass energy mc^2.