

Homework Problem Set 8

Problem 1

Some halo stars and globular clusters circle our Milky Way at radii as large as 50 kpc. Assume they move with tangential velocity 200 km/s. Calculate the total mass required for the Milky Way to explain this motion in terms of Keplerian orbits (i.e., assuming a spherical mass M inside the orbit, calculate the magnitude of M in units of solar masses, 2×10^{30} kg). Compare with the visible luminosity, roughly 20 billion times that of the sun. *Note:* 1 pc = 3×10^{16} m.

Problem 2

Calculate the Schwarzschild radius of a black hole with mass $4 \cdot 10^6 M_{\text{sun}}$. Compare to the following sizes:

- 1) The radius of a star with the same mass and the density of a typical red giant like Betelgeuse (10^{-5} kg/m³; 1/100 millionth the density of water!)
- 2) The diameter of an object that varies on the time scale of 1 hour (assuming that for causality to hold, the object cannot be larger than the distance traveled by light in 1 hour)
- 3) The closest approach of the orbit of star S0-16, which has an orbit with major half axis of 900 AU and an eccentricity of $\epsilon = 0.95$.
- 4) The resolution of a long-baseline radio interferometer, 10^{-3} arcsec, over the distance from Earth to the galactic center (8 kpc).

Problem 3

What would be the luminosity of a black hole (only an apparent contradiction) that converts one solar mass completely into light every 30,000 years? Compare to the solar luminosity of 3.8×10^{26} W!