

Homework Problem Set 8 - Solution

Problem 1

Setting the centripetal acceleration, $a = v^2/r$, equal to the gravitational acceleration due to a central mass M yields

$$\frac{v^2}{r} = \frac{GM}{r^2} \Rightarrow M = \frac{rv^2}{G} = \frac{(50,000 \times 3 \cdot 10^{16} \text{ m})(200,000 \text{ m/s})^2}{6.67 \cdot 10^{-11}} = 9 \cdot 10^{41} \text{ kg} = 4.5 \cdot 10^{11} M_{\text{sun}}$$

So, compared to the sun, the whole Milky Way appears to be more than 20 times as massive per unit of luminosity (making it likely that it contains “dark matter”).

Problem 2

Using the result from my previous lectures, $r_s = 2GM/c^2 = 3 \text{ km } M/M_{\text{sun}}$, we get a Schwarzschild radius of $1.2 \cdot 10^{10} \text{ m} = 0.08 \text{ AU}$, well inside the orbit of Mercury. In comparison,

- 1) A star with mass $4 \cdot 10^6 M_{\text{sun}}$ and density 10^{-5} kg/m^3 would have a volume of $8 \cdot 10^{41} \text{ m}^3$ and therefore (assuming a sphere) a radius of $r = 5.75 \cdot 10^{13} \text{ m} = 384 \text{ AU}$, which is much larger than the observed upper limits (below).
- 2) Light travels 300,000 km in 1 second, which comes out to 7.2 AU in one hour. This would be the diameter of an object that fluctuates coherently within 1 hour, so the radius would be even smaller, 3.6 AU.
- 3) This distance of closest approach for S0-16 would be $(1-\epsilon)900 \text{ AU} = 45 \text{ AU}$, which is larger than the upper limit in 2) and definitely can accommodate the calculated Schwarzschild radius, but is incompatible with a “Betelgeuse”-type giant star.
- 4) The resolution of such an interferometer would be 8000 pc times 10^{-3} arcsec. By the very definition of a parsec, 1 pc times a resolution of 1 arcsec equals 1 AU. Therefore, 8000 pc times 10^{-3} arcsec must equal 8 AU, again in agreement with 2) and other limits as well as the Schwarzschild radius, but inconsistent with a stellar object. So the best bet is indeed a Black Hole.

Problem 3

$$L = \frac{m_{\text{sun}} c^2}{30,000 \times 3.15 \cdot 10^7 \text{ s}} = 1.9 \cdot 10^{35} \text{ W} = 500 \text{ Mio} \times L_{\text{sun}}. \text{ So the luminosity of the central}$$

black hole exceeds that of the sun by an even larger factor than its mass!