PHYSICS313 - Spring Semester 2011 - ODU

Section 2 - 2nd Homework Problem Set - Solution

Problem 1

Given the total mass *M* and radius *R*, the density is $\rho = \dfrac{M}{\frac{4\pi}{3}R^3}$. Now let's assume we already have "assembled" a sphere of radius $r < R$ and we ask "how much additional gravitational potential energy do we get from putting another layer of thickness d*r* on top of this sphere?". The answer of course is

$$dU_{pot} = -\frac{Gm(r)}{r}dm = -\frac{G\frac{4\pi r^3\rho}{3}}{r}4\pi r^2\rho dr = -\left(4\pi\rho\right)^2\frac{G}{3}r^4 dr$$

. After integrating over r, we get $U_{pot} = -\left(4\pi\rho\right)^2\frac{G}{3}\frac{R^5}{5} = -\left(\frac{4\pi\rho}{3}R^3\right)^2\frac{3}{5}\frac{G}{R} = -\frac{3}{5}\frac{GM^2}{R}$

Now, for either type of core collapse, the potential energy in the final state will be so much more negative than the initial one (because R decreases by at least a factor 100) that we can ignore the latter. So we just plug in the numbers for mass (2.8×10^{30} kg) and radius in above formula and find the following:

White Dwarf – $U_{\mathrm{pot}} = -6.28 \cdot 10^{43}$ J

Neutron Star – $U_{\mathrm{pot}} = -3.14 \cdot 10^{46}$ J

Total rest mass energy - $Mc^2 = 2.52 \cdot 10^{47}$ J , much more than the White Dwarf but only 10x more than the Neutron Star gravitational potential energy.

Problem 2

The linear distance for a step $\Delta\theta$ in θ direction (increasing latitude) is simply $R\Delta\theta$ (*R* is Earth's radius) and, given that the distance from Earth's axis of a circle of constant latitude θ is $r_\perp = R\cos\theta$ (or $R\sin\theta$ if you use ordinary spherical coordinates), the linear distance due to a step in longitude $\Delta\phi$ is $r_\perp\,\Delta\phi = R\cos\theta\,\Delta\phi$. These two directions are always orthogonal, so the total distance travelled, Δs, is given by $\Delta s^2 = R^2\Delta\theta^2 + R^2\sin^2\theta\Delta\phi^2$

Following the definition of a metric, this implies for our 2-dimensional metric on the surface of Earth $g = \begin{pmatrix} R^2 & 0 \\ 0 & R^2\sin^2\theta \end{pmatrix}$ if we identify $\theta = x_1$ and $\phi = x_2$.

Problem 3

We know that the universe must be filled with an unknown substance called "dark matter" – in fact, it makes up the vast majority of all mass. One of the leading contenders for this dark matter are neutral, weakly interacting particles ("Wimps"). In turn, many physicists believe that these wimps could be the so-called "supersymmetric partners" of existing neutral particles (neutralinos), e.g. the partner of the Z boson ("Zino"). Therefore, many experiments are looking actively for these particles – which is difficult, since they interact only weakly. Most "dark matter detectors" are based in deep underground labs (to shield more ordinary cosmic ray background) and try to detect the recoil atoms from interactions between the detector material and the (putative) wimps. There are also attempts do use space-based detectors to look for possible decay products of Wimps or their interactions with ordinary matter. Finally, there is now a big program underway at the European Accelerator Center "CERN" to produce and detect supersymmetric particles. If confirmed, supersymmetry is an extension of the standard model of particle physics which will double the number of fundamental particles.

There are other, even more speculative particles that may exist but haven't been found yet – so-called axion, or heavy partners of the photon, etc. People are looking for them, but no guarantee that they will ever be found.

On the other hand, there is no doubt that gravity waves must exist – this follows from General Relativity and also has been indirectly inferred from the motion of double-pulsar systems. These waves are basically ripples in space-time which lead to elongation and compression (by tiny amounts) of all matter in their way. Again, there are several gravity wave detectors in operation on Earth (e.g. the twin LIGO detectors) which use interferometry with km long arms and high precision optics to detect these compressions down to the level of a fraction of a nuclear size. So far, no signals have been found, because the existing detectors require some rather unusual cosmic events to generate a big-enough signal: coalescence of two black holes, for instance. Future versions will be more and more sensitive and a discovery is all but guaranteed – gravity can be considered part of the standard model (although it doesn't mesh well with quantum mechanics yet). Gravity waves can be considered manifestations of “gravitons” – particles analog to photons in light waves.