

## Section 2 – 1<sup>st</sup> Homework Problem Set

### Problem 1

Look up the total energy released in the pp chain for each  ${}^4\text{He}$  nucleus produced. Compare with the total energy released in the “triple-alpha” reaction. For both reactions, calculate the energy released **per nucleon** (i.e., divide by the total number of nucleons involved). Assuming the same power (energy production rate), how much longer would you expect “hydrogen burning” to last than “helium burning”? (Obviously, there are other factors to consider, e.g. the overall luminosity, so this is only a very rough estimate).

### Problem 2

Re-derive the expressions for non-relativistic and ultra-relativistic pressure for an ideal (non-interacting) Fermi gas. Start with the definition of pressure as the force per unit surface area. Assume that each particle crashing into a surface of area  $A$  transfers a momentum  $2p_x$  (x-component of its momentum). Find the number of particles that crash into this surface per unit time in terms of their velocity components  $v_x$  and their density  $n$ . What follows for the pressure on  $A$ ? Assume that the average of  $\langle v_x p_x \rangle = 1/3 \langle \mathbf{v} \mathbf{p} \rangle$  (equipartition in all 3 dimensions), but count only particles moving in **one** direction (e.g., to the right), not the opposite direction (which don't crash into the same side of  $A$ ). Express  $\langle \mathbf{v} \mathbf{p} \rangle$  in terms of the kinetic energy of the particle for the two cases we're after, and find the pressure in terms of the density and average kinetic energy per particle,  $E_{\text{tot}}/N_{\text{tot}}$ . Plug in our results for the latter from the lecture for a non-relativistic and from a ultra-relativistic Fermi gas to get the final results.

### Problem 3

Using all the numbers and equations from the lecture for Sirius B (or from above), calculate the ultra-relativistic electron Fermi gas pressure and the pressure at the center derived assuming a constant density. (This is of course a very rough estimate).

What would happen to both pressures (central pressure and ultra-relativistic electron Fermi gas pressure) if you increased the density 8-fold? Discuss the consequences for the stability of a relativistic Fermi gas.