



PHYSICS 102N

Spring 2010

Week 14
Relativity



What is Relativity?



- Well known concept since Galilei's times:
 - Motion only makes sense **relative** to something (e.g., relative to an inertial coordinate system)
 - (Inertial) coordinate systems can move relative to each other
 - Laws of Physics should be the same in different inertial systems, even if they move relative to each other
 - E.g.: $\mathbf{F} = m\mathbf{a}$ is true in **all** inertial systems
- But you have to make some assumptions. Galileo:
 - Time passes uniformly, with same “pace”, in all inertial systems (IS)
 - Different observers agree on time elapsed between 2 events
 - In particular, they agree whether 2 events happened simultaneously or not
 - Length of an object is the same if measured from different IS

Here are some consequences of these assumptions



- Distance between 2 events is NOT universal (bouncing ball on train)
- relative velocities add



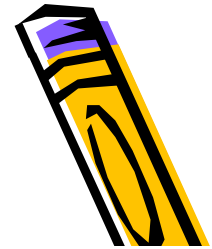
What does Einstein say?



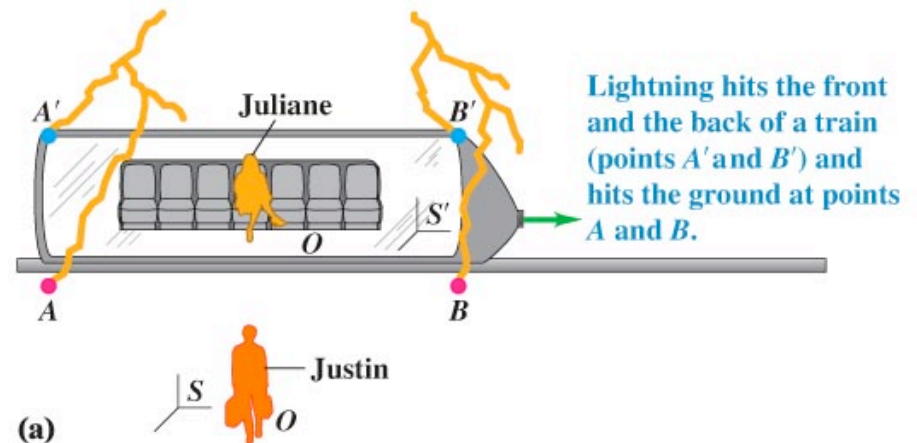
- Experimental fact: relative velocities do NOT add once you get close to the velocity of light ($c = 299\,792\,458$ m/s)
 - Light emitted from an object moving with 1000 m/s relative to “ground” travels with velocity c BOTH relative to ground AND relative to object!
 - Think about this one for 1 minute - or a few months (like Einstein did)...
- This has severe consequences:
 - Time does NOT pass with equal pace in all IS
 - Time difference between two events will come out different as measured from different IS
 - remember equivalent result for distance between 2 events even for Galileo
 - Shortest time difference is measured by system where both events happen at the same position (all other IS measure longer times = “time dilation”)
 - Two events might be simultaneous in one IS and non-simultaneous in another! They may even occur in opposite order in different IS
 - Length of an object is measured differently from different IS
 - Longest distance is measured by IS where object is at rest; all other IS measure shorter length = “length contraction”



Examples



- Non-simultaneity
 - Justin sees both at once
 - Juliane sees B' before A'
 - BOTH are correct!
answer depends on IS



- Twin Paradox
 - One twin travels with spaceship at 98% of c
 - other twin stays back on Earth
 - upon return, 1st one has aged 6 years but second one 30 years!
 - Note: 1st twin is NOT in an IS, so his description is NOT equivalent

- Length Contraction

- 1st twin from above appears only 1/5 as long as 2nd twin when measured by 2nd twin
- Total travel distance of 1st twin = 30 light years according to 2nd one, but only = 6 light year according to 1st one.



Consequences for Mass, Energy and Momentum



- Q: Why can't we accelerate something beyond the speed of light ($F = ma$)?
- A: Because inertia ("m") becomes infinitely big!
 - Kinetic energy contributes to inertia of an object: Inertia = $m + K.E./c^2$
 - ALL kinds of energy contribute to inertia of an object!
 - Mass is just one special contribution to inertia => can define total energy $E = m \cdot c^2 + \text{other types of energy} \Rightarrow \text{inertia} = E / c^2$
 - Correct form of energy due to mass and kinetic energy alone:

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

- Correct reformulation of momentum: inertia times velocity =>

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\vec{v}$$



Note: $(E/c^2)^2 - (p/c)^2 = m^2$; m is also called "rest mass"

Examples



- Fast particles are “heavier” (have more inertia)
 - Protons at the Large Hadron Collider in Geneva (CH) need a humongous magnetic field to be bent around a humongous circle, although they “only” travel with 99.999999% of the speed of light
- Fast particles live longer (time dilation)
 - Cosmic ray muons can travel through 30 km of atmosphere (0.1 ms at speed of light) even though their lifetime is only 0.0022 ms)
- Energy can be converted into mass (pair creation)
 - “Atom smashers”: We routinely produce new particles (e.g., pions with mass 1/7 of proton mass) by smashing high-energy electrons (1/1836 of proton mass) into a target: $m(\text{pion}) < \text{K.E.}(\text{electron})/c^2$
- Mass can be converted into energy (annihilation, nuclear decay)
 - Antimatter bomb in “Angels and Demons”: $e^+ + e^- \rightarrow \text{light (photons)}$
 $E(\text{light}) = \{m(e^+) + m(e^-)\} * c^2$
 - Nuclear Fission Energy: Mass of all fission products combined < initial mass of fissioning nucleus



Not weird enough yet? => General Relativity

- Special Relativity:
 - Time and space are related - time elapsed and distances measured depend on frame of reference
 - Introduce “4-dimensional Space-Time Continuum”
 - Every event has 4 coordinates: ct and \mathbf{r}
- General Relativity: Describe gravity as a DISTORTION of this space-time continuum
 - distances are altered near masses
 - time elapsed depends on nearby masses
 - straight lines get bent
 - parallel lines can converge
- Basic idea: distortion (“local curvature”) of space-time fabric is proportional to mass (energy) density

Mathematical formulation crazy complicated - involves differential geometry, non-Euclidean metric, tensors, and Einstein's field equations



Guidance: Equivalence Principle



- Remember: All objects fall with the SAME acceleration independent of their mass!
 - $\mathbf{a} = \mathbf{F}/m$, but $\mathbf{F} = -m\mathbf{g} \Rightarrow \mathbf{a} = -\mathbf{g}$
 - THIS is the reason why we can describe gravity as a property of space and time itself (which are the only things entering acceleration)
 - ANY freely falling system is (locally) INDISTINGUISHABLE from an IS!
 - Objects inside a falling elevator or a circling space station follow all of Newton's laws - in particular Newton's FIRST law!
 - Vice versa, the laws inside a stationary system WITH gravity are the same as those governing motion in an accelerated system with NO external force
 - "Force free" objects accelerate \rightarrow drop to the floor or follow curved trajectories
 - Consequences:
 - even light must follow a curved path around a massive object
 - time dilation (remember twin paradox) \Rightarrow clocks go slower close to massive objects
 - "it isn't the falling that hurts, but the sudden stop at the end!"



Examples



- Ordinary gravity: Things fall to the ground, planets orbit the sun...
 - “Old” explanation: Force of gravity pulls them
 - “New” explanation: They follow curved paths in space-time, which itself is curved because of the presence of masses (earth, sun,...)
 - By the same token, light follows curved paths near massive objects
 - observable during total eclipse of the sun: stars seem to be “moved”
- Clocks run slow deep inside a gravitational field
 - Clock on surface of Earth runs more slowly than clock circling at high altitude
 - Compare with clock at center and rim of rotating disk (twin paradox!) - “centrifugal acceleration” is equivalent to gravity
 - Important practical effect you have to account for to make your GPS work!
 - Gravitational “redshift” - photons lose energy ($E = hf$)
 - Most extreme: near a “black hole” (photons from INSIDE would lose more energy than they have -> they never come out!)



Large-scale structure of the Universe



- Is the Universe as a whole curved?
 - If there is “too much” mass in the Universe, ALL “straight” paths lead back to themselves (the curvature is positive, like on a sphere)
 - You could see (in principle) the back of your head
 - The present expansion would ultimately come to a halt, reverse and then end in a “big crunch”
 - If there is “too little” mass in the Universe, all “straight” paths diverge from each other (the curvature is “negative”, like a saddle)
 - The present expansion continues forever, only slightly slowed down
 - If there is exactly the right amount of mass (“critical density”), the Universe is “flat” (ordinary geometry applies, parallel lines never meet...)
 - Expansion goes on forever, but becomes slower and slower and *nearly* stops
 - Not enough visible mass for that. Dark Matter?
- The truth is even weirder!
 - The universe SEEMS to be flat (“critical density”)
 - BUT: the expansion seems to be accelerating
 - Reason: Dark Energy (Huh?)

