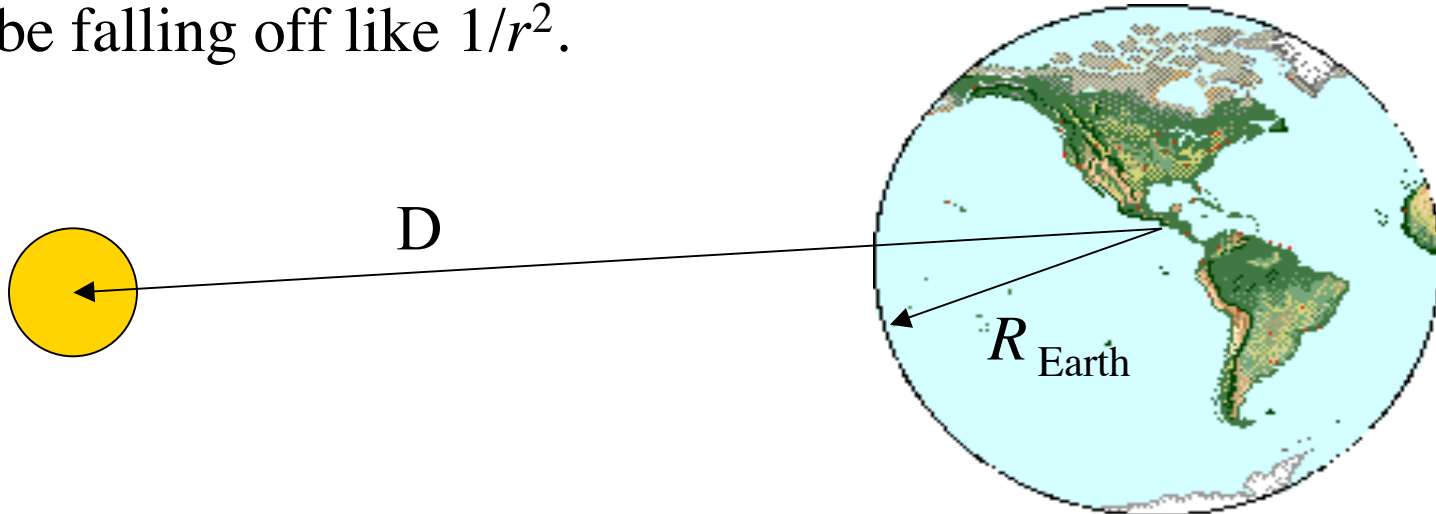


Gravitation

- Objects on the surface of Earth fall down with acceleration $a_{\text{rad}} = g = 9.81 \text{ m/s}^2$; Earth's radius is $R_{\text{Earth}} = 6380 \text{ km}$.
- Moon circles the Earth once every $27.3 \text{ d} = 2.36 \cdot 10^6 \text{ s} \Rightarrow \omega = 2.66 \cdot 10^{-6} / \text{s}$. Moon is $D = 384,000 \text{ km}$ away $\Rightarrow a_{\text{rad}} = D\omega^2 = 0.00272 \text{ m/s}^2$ (3600 times smaller). D is 60 times bigger than R_{Earth} ! \Rightarrow Gravitational force must be falling off like $1/r^2$.



Newton's Law of Gravitation

- All masses are accelerated with the same acceleration at the same distance from Earth $\Rightarrow F \propto m$ (since $a = F/m$)
- Newton's 3rd law \Rightarrow Earth is attracted to any mass with force proportional to $M \Rightarrow F \propto m M$
- The force is proportional to the distance squared: $F \propto 1/r^2$
- Need proportionality constant: $G \Rightarrow$
 $F = G m M / r^2$
- Universal constant \Rightarrow Universal force law for any two bodies with masses m, M at distance r !
- Measure G using torsion balance $\Rightarrow G = 6.7 \cdot 10^{-11} \text{N m}^2/\text{kg}^2$
- Plug in numbers: on the surface of earth,
 $g = F/m = G M / R_{\text{Earth}}^2 \Rightarrow M = 5.97 \cdot 10^{24} \text{ kg}$

Important points

$$\vec{F}(\text{on } m_2 \text{ at } \vec{r}_2 \text{ due to } m_1 \text{ at } \vec{r}_1) = -G \frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^2} \text{ pointing from } m_2 \text{ to } m_1$$

- Universal $1/r^2$ force law - describes not only gravity, but also electromagnetism ...
- Valid not only for point masses, but for spherical extended masses as well (measure r from the center)
- G can be measured with torsion balance (but it's hard because it is so small) -> Value: $6.673 \cdot 10^{-11} \text{N m}^2/\text{kg}^2$
- Force always acts **on** center of gravity of an object (roughly equal to center of mass)
- Force always points **to** center of gravity of attracting mass (along distance vector)
- Mass m exerts equal and opposite force on mass M as that exerted by M on m .
- Superposition: Gravitational forces add

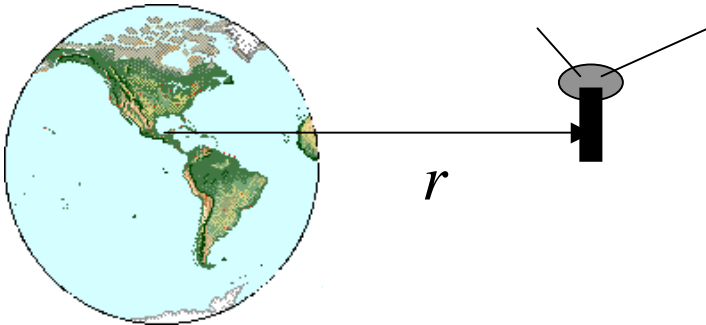
Example I

- Gravity on the surface of moon
 - Radius of Moon: 1740 km = 27.3% of Earth's radius (roughly 1/4)
 - IF density were the same: Mass of Moon \approx 1/64 Mass of Earth (really 1/81 = 1.23%)
 - Gravitational acceleration on Moon $\approx (1/80)/(1/4)^2 = 1/5 = 2 \text{ m/s}^2$ (really 1.62 m/s²)



Example II

- Project “Lagrange Point”
Put a satellite between Sun and Earth where the net gravitational force is zero
 - Satellite can be stationary (looking at Earth)
- Where to put?
 - $M_{\text{Earth}} = 6 \cdot 10^{24} \text{ kg}$, $M_{\text{Sun}} = 2 \cdot 10^{30} \text{ kg}$, $D_{\text{E-S}} = 1.5 \cdot 10^{11} \text{ m}$.
 - Require $GmM_{\text{Earth}}/r^2 = GmM_{\text{Sun}}/(D_{\text{E-S}} - r)^2$
 $\Rightarrow (D_{\text{E-S}} - r) = 577 r \Rightarrow r = D_{\text{E-S}}/578 = 259,000 \text{ km}$.



Example III

- Two steel balls floating in space (initially at rest). Masses $M = 10$ kg, $m = 5$ kg, $d = 0.1$ m apart.
 - Initial gravitational attraction:
 $F = G m M / d^2 = 3.34 \cdot 10^{-7}$ N (on each)
 - Initial acceleration: $3.34 \cdot 10^{-8}$ m/s² for M ,
 $6.68 \cdot 10^{-8}$ m/s² for m .
 - After 60 s, first one has moved 0.06 mm. Second has moved 0.12 mm.
 - Center of mass remains at same point (1/3 of the way from first to second mass).

Tidal forces

- Because gravitational forces vary like $1/r^2$, they can be different on different parts of the same object.
- Example: Gravitational force exerted by moon on Earth: Largest at point closest to moon, less large on center of Earth, least on opposite side.
- Variation of force over an object of diameter D :
$$\Delta F/m = D \cdot 2GM/r^3$$
- Tend to elongate objects along connecting line
- Examples: Oceans on Earth (and atmosphere, Earth itself); Moon; black holes