Work and Energy

- Newton showed momentum **p** = m**v** is an important concept
  - Relevant for collisions - momentum is conserved
  - Can tell you about the change in motional state during collision
  - Change in momentum = Impulse = F x Δt
- Leipnitz proposed ($^1/_2$) mv^2 as relevant quantity
  - Compare distance needed to stop moving object - 2x the velocity => 4x the distance! (average v **X** time to get v to zero)
  - Nowadays called "kinetic energy"
  - Change in energy = WORK = F · Δs (force time distance)
  - Many other forms of energy - the sum of all of them is conserved
- Momentum tells you "oomph", energy tells you "ouch"

Work and Energy

- Work: The total **effect** a force **F** has **on** an object (mass point, closed system,…), based on the change in its position
  - Given by Force times Displacement (see later)
  - Changes motional (kinetic) state (or internal state, mechanical state,…)
  - Can be positive or negative
  - Can be transferred from one object to another, but no object can "have" work
  - Analogy: Transfer of cash
- Energy: The **ability** to **do** work
  - Property **of** the object (mass point, closed system,…)
  - Changed by work done **on** object
  - Can be exchanged between objects/systems, but **can not** be created or destroyed (energy conservation)
  - Analogy: Net Worth

Work and Kinetic Energy I

- Prototype example - free fall starting at rest:
  - Force: mg ; Acceleration g ; Velocity $v(t) = g \cdot t$; Distance fallen $\Delta s = {}^1/_2\, gt^2$
  - Work done after time t: $\Delta W = F\Delta s$ / $= mg \cdot ({}^1/_2\, gt^2) = {}^1/_2 m \cdot (gt)^2 = {}^1/_2 m \cdot (v(t))^2$
  - = K.E. (t)
- In general:

CHANGE of kinetic energy of some object: $\Delta\ K.E. = \Delta {}^1/_2 m\mathrm{v}^2$

EQUALS

Work done on that object: $\Delta W = F\Delta s$

Work and Kinetic Energy II

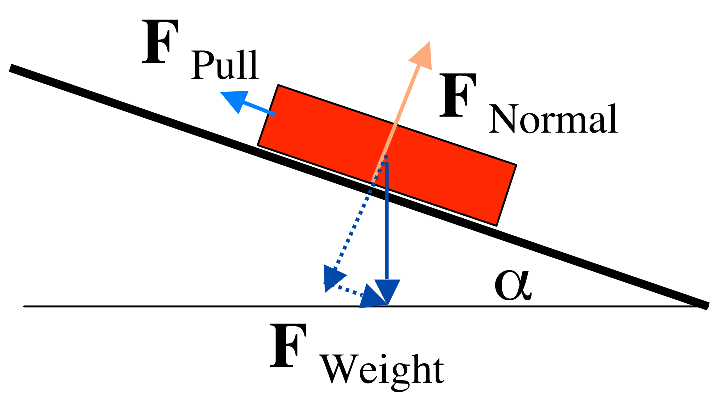
- Dimension: Displacement times Force
  Unit: Nm = J (Joule)

1) Specify a force acting **on** an object
2) Multiply displacement in the direction of the force with that force:
   $\Delta W = F \, \Delta s \cos\phi$
   - If **F** is in the direction of Δs, then positive work is done on object
   - If **F** is in opposite direction of Δs, then negative work is done on object
   - If **F** is perpendicular to direction of Δs, then no work is done at all.
3) Add work done due to all (external) forces acting on object:
   $\Delta W = \Sigma \mathbf{F}_i \cdot \Delta \mathbf{s} = \Sigma \Delta W_i$
4) Set equal to change in kinetic energy:
   $\Delta K.E. = {}^{m}/_{2}\, v_f^2 - {}^{m}/_{2}\, v_i^2 = \Delta W$

- Examples: Pushing car up incline, car rolling down (demo); Catching a ball; ball bouncing off wall, ball getting stuck in wall

Work and Kinetic Energy - Example

- Car rolling down ramp: Gravity does work, normal force doesn't => only displacement in vertical direction counts
- Final velocity same as for free fall! $^1/_2\, mv^2 = mg\Delta h$
- Car moving up ramp: Weight does negative work $\Delta W = -mg\Delta h$
- Slows down car or I have to supply additional work



Important Notes

- Even if $\Sigma \mathbf{F}_i \neq 0$, Work done can be zero:
  - No displacement: Holding a book, pushing against a wall
  - Direction of displacement perpendicular to Force: Moving sideways (constant height) in gravity field, circular motion (constant speed)
  - Normal forces and static friction **never** do work, but tension can and kinetic friction can, too (always negative).
- Kinetic energy is a scalar quantity (unlike momentum!): K.E. = $^m/_2\, v^2 = W\,(0 \to v)$, always positive
  - Independent of direction (circular motion at constant speed doesn't change kinetic energy)
  - Depends on system of reference
- **Everything** I said **only** valid in Inertial System of Reference

Power

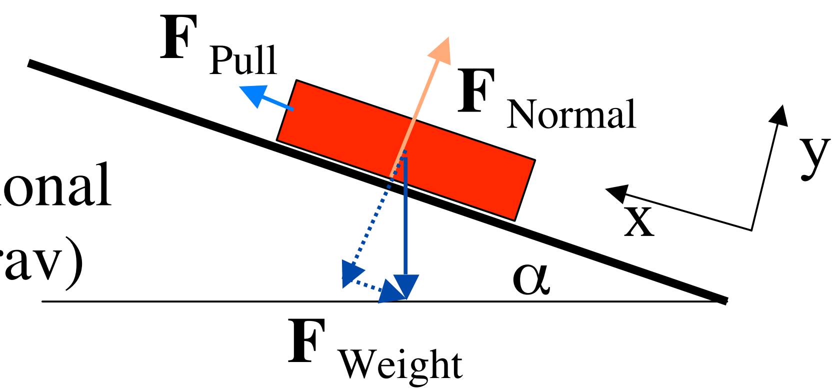
- Work done per unit time:
  $P = \Delta W / \Delta t$ (average)
- Unit: Watt = Joule/s, kW (kiloWatt) ->
  new unit for energy&work: Ws, kWh…
- $\Delta W = \mathbf{F} \cdot \Delta \mathbf{s} \Rightarrow P = \mathbf{F} \cdot \Delta \mathbf{s} / \Delta t = \mathbf{F} \cdot \mathbf{v}$
- Example:
  1000 kg Car accelerating from 0 to 20m/s in 5 s => $F = 4000$ N,
  $v_{ave} = 10$ m/s => $P_{ave} = 40$ kW = 53 hp .
- Different way to calculate:
  K.E. (final) = 200,000 J = ΔW in 5 s

Potential Energy

- So far: Considered all forces equal, calculate work done by net force only => Change in kinetic energy.
  - Analogy: Pure Cash Economy
- But: Some forces seem to be able to "store" the work for you (when they do negative work) and "give back" the **same** amount (when they do positive work).
  - Analogy: Bank Account. You pay money in (ending up with less cash) - the money is stored for you - you can withdraw it again (get cash back)
- These forces are called "conservative" (they conserve your work/money for you)

Potential Energy - Example

- Car moving up ramp: Weight does negative work
  $\Delta W(\text{grav}) = -mg\Delta h$
- Depends only on initial and final position
- Can be retrieved as positive work on the way back down
- Two ways to describe it:
  1) No net work done on car on way up
  2) Pulling force does positive work that is stored as gravitational potential energy $\Delta U = -\Delta W(\text{grav})$



Total Mechanical Energy

- Dimension: Same as Work
  Unit: Nm = J (Joule)   Symbol: $E = K.E. + U$

1) Specify all **external** [*)] forces acting **on** a system
2) Multiply displacement in the direction of the net external force with that force:
   $\Delta W_{ext} = F \, \Delta s \cos\phi$
3) Set equal to change in total energy:
   $\Delta E = {}^{m}/_{2} v_f^2 - {}^{m}/_{2} v_i^2 + \Delta U = \Delta W_{ext}$
   $\Delta U = -W_{int}$

[*)] We consider all non-conservative forces as external, plus all forces that we don't want to include in the system.

Example: Gravitational Potential Energy *)

- I. Motion in vertical (y-) direction only:

  $$\Delta U = -W_{grav} = mg\Delta y$$

- External force: Lift mass m from height y_i to height y_f (without increasing velocity) => Work gets stored as gravitational potential energy $\Delta U = mg\,(y_f - y_i) = mg\,\Delta y$
- Free fall (no external force): Total energy conserved, change in kinetic energy compensated by change in potential energy $\Delta K.E. = {}^m/_2\, v^2 = -mg\,\Delta y$
- Example: Throw baseball up with 20 m/s (accelerate over 0.5m). Maximum height? Force needed?

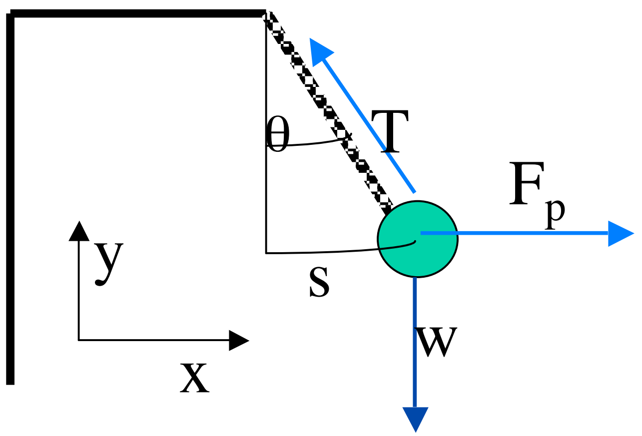
*) Here the system consists of Earth plus object!

Gravitational Potential Energy II

- Important point: Potential energy has **no** absolute zero (like kinetic energy does - in a given reference frame!).
- Depends on choice of **reference point**: **You decide** where you want U to be 0.
- Call that point $h = 0$.
- Define potential energy as $U = mgh$.
- => Total energy $E = {}^{m}/_{2}\, v^2 + mgh$.
- Choice arbitrary - other choice means constant offset in definition of U and E .
- No **observable** depends on that choice! All that counts are differences ΔU. BUT: you must specify reference point when quoting E and U !

Work on a pendulum - an Example

- Slowly pushing a pendulum bob sideways:



1) Tension does no work (perpendicular to motion)
2) Pushing force does work - equal to potential energy stored by gravitational force
3) After letting go, gravitational potential energy gets converted to kinetic energy at the bottom

Pendulum cont'd.

- On the way up: Net work done by external force (pushing) increases total energy ($K.E. + U_{grav}$) stored in the system
- On the way down: No other (non-grav.) force => Total Energy conserved ($\Delta E = 0$) $\Delta E = \Delta K.E. + \Delta U = 0 \Rightarrow \Delta K.E. = {}^{m}/_{2}\, v^2 = -\Delta U = -mg\Delta h$
- Important points:
  - Minus sign: **negative** work done by conservative force **increases** the potential energy due to that force (putting cash into account)
  - Potential energy is stored in **system** (pendulum and gravitation, bank account)
  - Total energy (cash+account balance) changes through work done by forces **external** to system (push, cash influx)
  - System must be "leakproof" (conserve work): return to initial condition -> same potential energy (conservative forces)

Elastic Potential Energy

- So far: Considered system object - gravitational field;
  Potential energy = - gravitational internal work (inside system)
- Now: Consider system -> mass attached to spring;
  Potential energy = - internal work done by spring
  - Call x = 0 unstretched position of spring
  - Force exerted by spring $F_x = -kx$
  - Work done by spring $\Delta W = -k/2\,(x_f^2 - x_i^2)$
  - Potential energy stored in spring-cart system:
    $U = -\Delta W = k/2\,(x_f^2 - x_0^2)$
    where x_0 is the point where we declare U to be $U = 0$.
  - $x_0 = 0 \Rightarrow U = k/2\, x^2$ (convenient, **not** unique)
  - Note: $U > 0$ stretched **and** compressed

Elastic Potential Energy cont'd.

- No other (non-elastic) force =>
  Total Energy conserved ($\Delta E = 0$)
  $\Delta E = \Delta \text{K.E.} + \Delta U = 0 \Rightarrow \Delta {}^{m}/_{2}\, v^2 = -\Delta U$
  Example: Oscillation
- Non-elastic force present =>
  $\Delta E = \Delta \text{K.E.} + \Delta U = \Delta W \Rightarrow$
  $\Delta {}^{m}/_{2}\, v^2 = -\Delta U + \Delta W$
- Elastic and gravitational force present =>
  $E = \text{K.E.} + U_{el} + U_{grav} =$
  ${}^{m}/_{2}\, v^2 + k/2\, x^2 + mgh$.
  (several bank accounts)
- Note: Elastic forces are conservative because work done only depends on initial and final position.

Total Mechanical Energy - Final Version

1) Specify all forces acting on an object
2) Separate out all **conservative** forces (Work done depends only on initial and final position). Incorporate them into the system of the object as potential energy U
3) Add all **external** forces acting **on** the system (= all other forces), call the result "net (external) force".
4) Multiply displacement in the direction of the net force with that force: $\Delta W_{ext} = \mathbf{F} \cdot \Delta\mathbf{s} = F\, \Delta s \cos\phi$
5) Set equal to change in total energy: $\Delta E = {}^{m}/_{2}\, v_f^2 - {}^{m}/_{2}\, v_i^2 + \Delta U = \Delta W_{ext}$
6) $\Delta U = -\Delta W_{int}$; $\Delta\, {}^{m}/_{2}\, v^2 = -\Delta U + \Delta W_{ext}$

Examples: Pumpkin falling on spring-loaded platform (without and with air resistance); bungee jump

Other types of Energy

1) Electromagnetic energy (see later in the semester).
   Examples: Charged capacitors (electrostatic energy), current-carrying coils (magnetic energy), …
2) Chemical energy (really a special kind of electromagnetic energy).
   Examples: Batteries, fuel, …
3) Sound, light, nuclear,…
4) Internal energy (heat, pressure,…) -see next semester (PHYS102)

Important Points

- Energy concept is useful:
  - Calculate change in velocity without knowing Force as F(t)
  - Understand levers, hydraulic systems, mechanical advantage…
- Energy concept is fundamental: Energy is conserved!
  - Kinetic energy (always positive - "cash")
  - Gravitational energy (can be + or -; "checking account with overdraft")
  - Elastic potential energy (always positive - "savings")
  - Chemical, electrical, sound, light, …. Energy
  - Heat energy (less useful - "cash flushed down the drain")
- Efficiency = fraction of useful work/total energy transferred