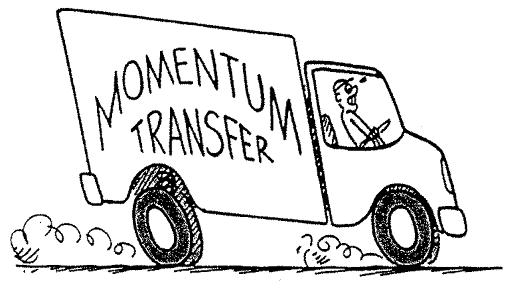
PHYSICS101 - Fall Semester 2010 - ODU

Midterm Exam (2^{nd} test) Solution

Part I: Multiple Choice/Numeric – Self-paced Clicker Test

Problem 1

The brakes are applied on a speeding truck, exerting a constant braking force, and it comes to a smooth stop. If the truck were heavily loaded so it had twice the total mass, but only 1/2 the velocity (and the same braking force would be applied), the skidding distance would be



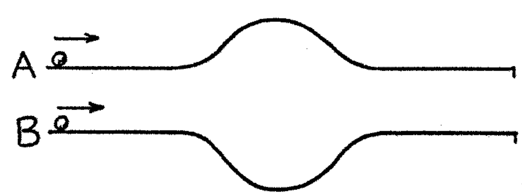
1. the same
2. 2 times as far
3. ½ times as far
4. ¼ times as far

Solution: As already shown in class, the answer is **3**: Since K.E. = ½ mv^2, doubling the mass doubles the kinetic energy, but then halving the speed brings it down to ¼ yielding an overall factor of ½. You need to do ½ as much work to stop a car with ½ the kinetic energy. Since work = force x distance, and the braking force is the same, the distance needed is only ½.

Problem 2

Two smooth tracks of equal length have "bumps"—A up, and B down, both of the same curvature. If the initial speed = 2 m/s, and the speed of the ball at the bottom of the curve on Track B is 3 m/s, then the speed of the ball at the top of the curve on Track A is

1. 1 m/s
2. 3 m/s
3. It won't even make it to the top of curve A



Solution: **3**: In both cases, the initial K.E. = ½ mv^2 = m/2 x 4 m^2/s^2. In case B, at the bottom of the bump, the kinetic energy has increased to m/2 x 9 m^2/s^2, which is higher by m/2 x 5 m^2/s^2. Energy conservation tells us that therefore the potential energy must be lower by an amount mgh = m/2 x 5 m^2/s^2 (which implies gh = -2.5 m^2/s^2 or h = -0.25 m). In case A, the ball

would have to **gain** the same amount of potential energy to make it to the top of the bump, meaning that it would have to lose $m/2 \times 5\ m^2/s^2$ in kinetic energy (because of total energy conservation). That is impossible since it is starting out with less than that **total** – therefore it will never make it up the bump.
Because this problem is a bit tricky, it was treated as Extra Credit for your total score.

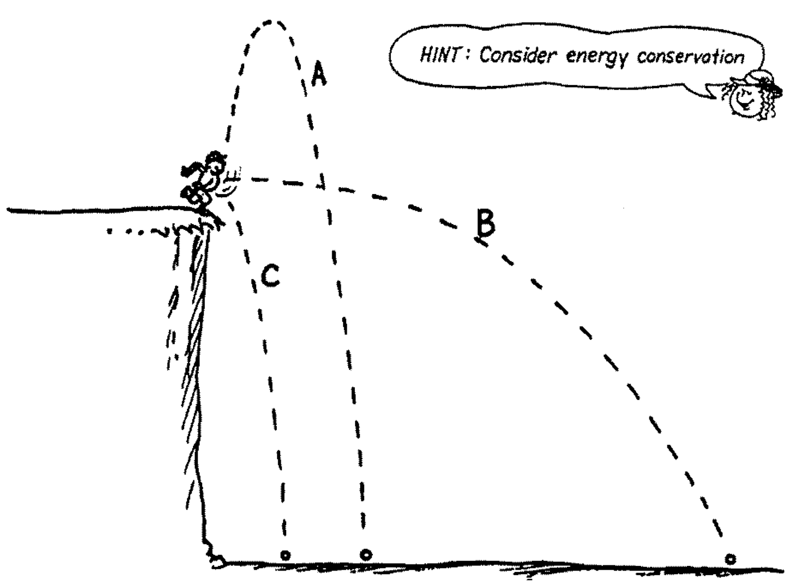
Problem 3

Would it "stub" your (bare) toes harder to kick a heavy bowling ball than to kick a small golf ball?

1. Yes, because when your toes "collide" with the bowling ball, a lot more momentum is transferred to them than when kicking the golf ball
2. Yes, because the golf ball is softer than the bowling ball
3. No, your toes will be "stubbed" harder by the golf ball since it requires more energy to kick
4. Neither; the force exerted on your toes is the same in both cases

(pick the best choice of the 4 possible answers given)
Solution: **1**: When your toes hit the bowling ball, they come nearly to a complete stop due to the great mass of the ball. In the case of the golf ball, they only slow down a little because its mass is much smaller. Since the momentum transferred to your toes is equal to their change in momentum, which is equal to their change in velocity times their mass, the bowling ball definitely imposes a much larger transfer of momentum to your toes. In turn, this requires a larger force, and therefore "stubs" your toes more noticeably. (Don't) try it out!

Problem 4



Three baseballs are thrown from the top of the cliff along paths A, B, and C. If their initial speeds are the same and air resistance is negligible, the ball that strikes the ground below with the greatest speed will follow path

1. A (first up)
2. B (initially horizontal)
3. C (intially downwards)
4. Either A or C
5. All three strike with the same speed

Solution: **5**: This follows simply from total energy conservation: The three baseballs all start out from the same height (hence same potential energy *mgh*) in the same kinetic energy (since their initial speed is the same and K.E. = ½ mv^2). The total energy for all 3 is therefore the same and, due to energy conservation, remains the same no matter whether they go first up, sideways or down. Whenever each one of them hits the ground, they once again have the same potential energy (*mgh* = 0) and therefore the same K.E. (which must now equal their total, conserved energy) and therefore the same speed.

Problem 5

I am throwing a tennis ball horizontally away while standing on ice skates, on slippery ice. Which of the following statements is correct?

1. I move backwards with the same speed as the tennis ball has after my throw.
2. I transfer a lot more impulse to the tennis ball than it transfers to me.
3. The magnitude of the impulse I impart on the tennis ball is **solely** (only) determined by the force I exert during the throw.
4. The tennis ball and I have the same kinetic energy after the throw.
5. The sum of my momentum plus that of the tennis ball is the same before and after the throw.

Solution: **5**: Since no external forces are acting (slippery ice = no friction), the law of conservation of momentum applies. This law is simply stated in the last of the 5 options. Since the initial total momentum is zero (everything at rest), it must be true that my own momentum after throwing the ball must be equal in magnitude but opposite in sign to that of the ball. But momentum = *mv*, and since MY mass is much larger than the ball's mass, same momentum must mean much smaller speed! #2 is just plain wrong – it contradicts Newton's 3rd Law. #3 is wrong because the impulse is the product of force times time, so it depends not **only** on force. #4 is also not correct – my speed is much smaller than that of the ball, and since K.E. goes like speed squared, mine is much smaller than that of the ball.

Problem 6

A car of mass 1200 kg driving with initial velocity 15 m/s crashes into a loaded minivan at rest, with mass 2800 kg. After the collision, both cars stick together. What is the common velocity of the both of them? (Enter all digits, including possibly a period, into your clicker, than press the "enter" button).

Solution: This can be answered by applying the equation

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f$$

from the formula sheet. m_1 = 1200 kg, $\mathbf{v}_{1i}$ = 15 m/s, $\mathbf{v}_{2i} = 0$ => the right hand side is equal to 1200 kg x 15 m/s = 18,000 kg m/s. After the collision, both cars stick together, so both must have the same final velocity $\mathbf{v}_f$. Momentum conservation says $(m_1 + m_2)\,\mathbf{v}_f$ = 4000 kg $\mathbf{v}_f$ must be equal to 18,000 kg m/s. Dividing by 4000 kg on both sides gives $\mathbf{v}_f$ = **4.5** m/s.

Problem 7

For the case described above, calculate the average force (in Newton) between the car and the minivan if the collision takes place over a time span of 0.1 seconds. *Hint*: Calculate the impulse first! (Enter all digits, including possibly a period, into your clicker, than press the "enter" button)

Solution: Since we know that impulse = force x time, we can figure out the necessary force if we calculate the change in momentum for either one of the 2 cars (= impulse; both cars must receive the same magnitude of impulse because of Newton's 3rd Law). It's easier to calculate the impulse for the 2nd car (the loaded minivan): Initially, its momentum is zero. After the collision, it's 2800 kg x 4.5 m/s (see above) = 12,600 kg m/s which is equal to the impulse. Dividing by the time gives the force: F = 12,600 / 0.1 kg m/s^2 = 126,000 N.

Since this problem is also somewhat involved, it was again treated as Extra Credit.

Problem 8

I can throw a ball upwards at 10 m/s. If you can throw the same ball upwards with twice that velocity (20 m/s), how much higher will the ball go? Ignore air resistance

1. Same height
2. Twice the height
3. Four times the height
4. 1.4 times (√2) times as high

Solution: **3**: This is once again a straightforward application of energy conservation – even easier than the 1st problem, and very similar to examples I did in class. Please make sure you understand the answer!

Part II: Word Problems
– answer with complete sentences and step-by-step calculations

Problem 8

Consider a system made of two identical balls of mass m that are brought into a collision. Here are 2 different possible scenarios for this collision:
a) The two balls, made of highly elastic rubber, roll frictionless on a flat, horizontal track with velocities of equal magnitude but opposite direction, $\pm v$, until they bounce off each other, and recoil. Each ball has now the exact negative of its initial velocity.
b) The two balls, made out of "sticky" putty, roll frictionless on the flat, horizontal track until they crash into each other, and stick together afterwards. Again, they start out with equal (in magnitude) but opposite (in direction) velocities $\pm v$, and in the end they both have velocity zero.
For each of these two scenarios, describe in detail the initial and final momenta of each ball and the total initial and final momentum, as well as their initial and final kinetic energies. Is total momentum conserved in either scenario? How about total kinetic energy? If one of these quantities is NOT conserved, why not? (Where does it "go"?)
Solution:
a) The initial momentum of the first ball is $+mv$ and that of the second ball $-mv$, by definition. The total initial momentum is the sum of the two, which gives zero. The balls roll with constant velocity until they hit each other, when each reverses its velocity in an instant. Now the first ball has momentum $-mv$ and the second ball $+mv$, and the sum (total final momentum) is again zero. Momentum is conserved as it must, since no other (frictional) forces are present. Both balls have the same kinetic energy, K.E. = $\frac{1}{2} mv^2$, both before and after the collision. The total kinetic energy is therefore mv^2, and is the same before and after the collision (meaning it's conserved). This the hallmark of an elastic collision (actually the **definition**!)
b) Initially, everything is exactly the same as in case a). Again, the total momentum is zero and the total kinetic energy is mv^2. After the collision, since the balls stick together and stop moving, each has momentum zero individually, and the total momentum is

again zero and thus conserved. But the kinetic energy is now also zero in the end, meaning it was **not** conserved. Since energy cannot be destroyed, it must have been transformed into other forms, like heat and deformation energy. This is the hallmark of an **inelastic** collision.
Note that all of these answers are totally straightforward – all you had to do is read the problem carefully, as well as the formula sheet.

Problem 9

A roller coaster car (5000 kg) starts (at rest) at a height of 50 m above ground and rolls (without braking, friction or air resistance) through a complicated sequence of bumps and valleys, loop-de-loops and other pieces of track until it finally arrives at ground level. What will be its speed at that moment (while it is at ground level)? Explain which fundamental concepts of Physics you would use to arrive at the answer, and explain each step of your solution carefully. Why is the answer independent of the detailed structure of the whole roller coaster track?
Solution: This problem can be best solved using the concept of total mechanical energy, which is conserved in this case: No frictional, air resistance or braking forces are present, and therefore no work is done on the car. This also explains why the exact structure of the track is immaterial – everytime the car goes up or down, some kinetic energy gets converted into potential energy and back, but the total will never change and therefore will be the same once the car reaches ground level.
Initially, the total energy is all due to potential energy, since it starts at rest and therefore its kinetic energy is zero:
$E_{tot} = U_{pot} = mgh$ = 5000 kg x 10 m/s^2 x 50 m = 2,500,000 J (2.5 MJ).
At the end of the track, at ground level, $h = 0$ and therefore the potential energy is zero. Therefore, all of the total energy must now be in the form of kinetic energy, and we have
2,500,000 J = E_{tot} = $K.E.$ = ½ mv^2 = 2500 kg v^2
Dividing both sides by 2500 kg yields v^2 = 1000 (m/s)^2. Taking the square root on both sides gives us the desired answer: v = √1000 m/s = 31.6 m/s (71 miles/hour!)