Image Restoration

Reference

Gonzalez and Woods, Digital Image Processing, 2nd Ed, Prentice Hall, 2002.

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Image Restoration

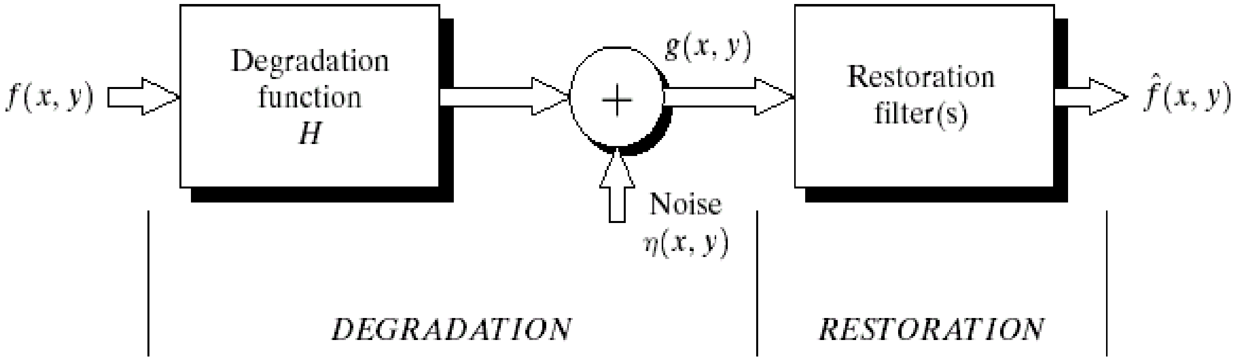
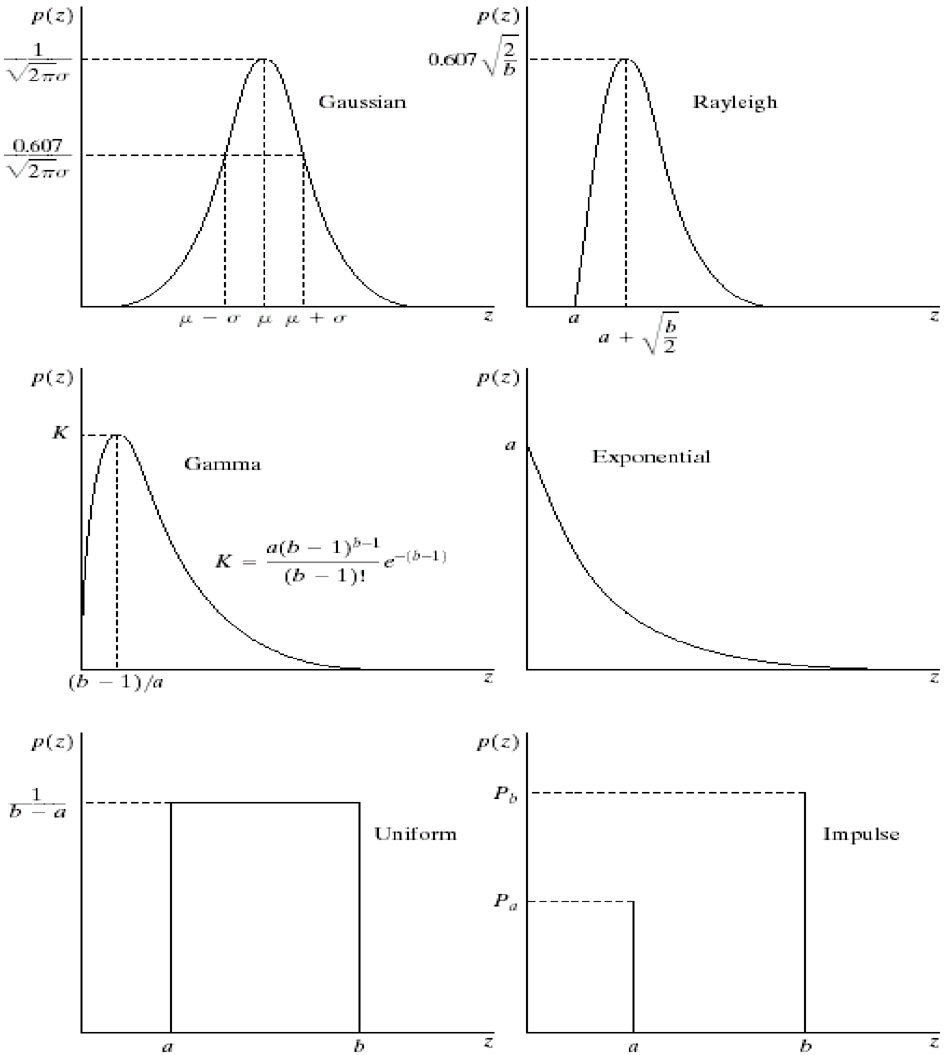


FIGURE 5.1 A model of the image degradation/ restoration process.

Noise Models



a b
c d
e f

FIGURE 5.2 Some important probability density functions.

•pdf of Gausssian random variable z:

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

70% of values are in [(μ-σ),(μ+σ)]

95% of values are in [(μ-2σ),(μ+2σ)]

•pdf of Rayleigh noise:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\,e^{-(z-a)^2/b} & , for \ z \ge a \\ 0 & , for \ z \le a \end{cases}$$

The mean and variance are :

$$\mu = a + \sqrt{\pi b/4} \quad and \quad \sigma^2 = \frac{b(4-\pi)}{4}$$

Noise Models

Erlang (Gamma) noise pdf: $$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az}, & for \quad z \ge 0 \\ 0, & for \quad z < 0 \end{cases} ; \mu = \frac{b}{a} \quad and \quad \sigma^2 = \frac{b}{a^2}$$

Uniform noise pdf: $$p(z) = \begin{cases} \dfrac{1}{b-a}, & if \quad a \le z \le b \\ 0, & otherwise \end{cases} ; \mu = a + b \big/ 2, \sigma^2 = \frac{(b-a)^2}{12}$$

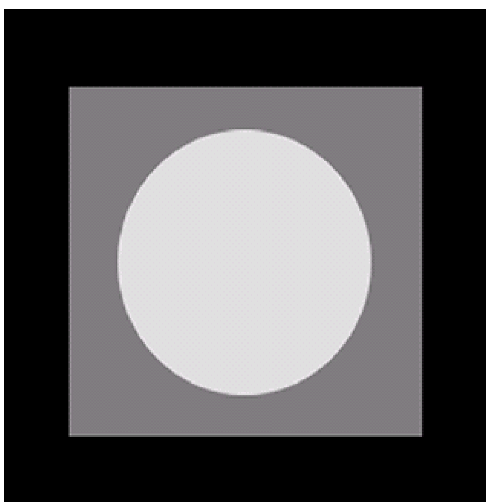


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Impulse (salt-and-pepper) noise pdf:

$$p(z) = \begin{cases} P_a, for\, z = a \\ P_b, for\, z = b; If\; b > a\; gray-level\; b\; will\; appear\; as\; a\; light\; dot. \\ 0, otherwise \end{cases}$$

Noisy Images

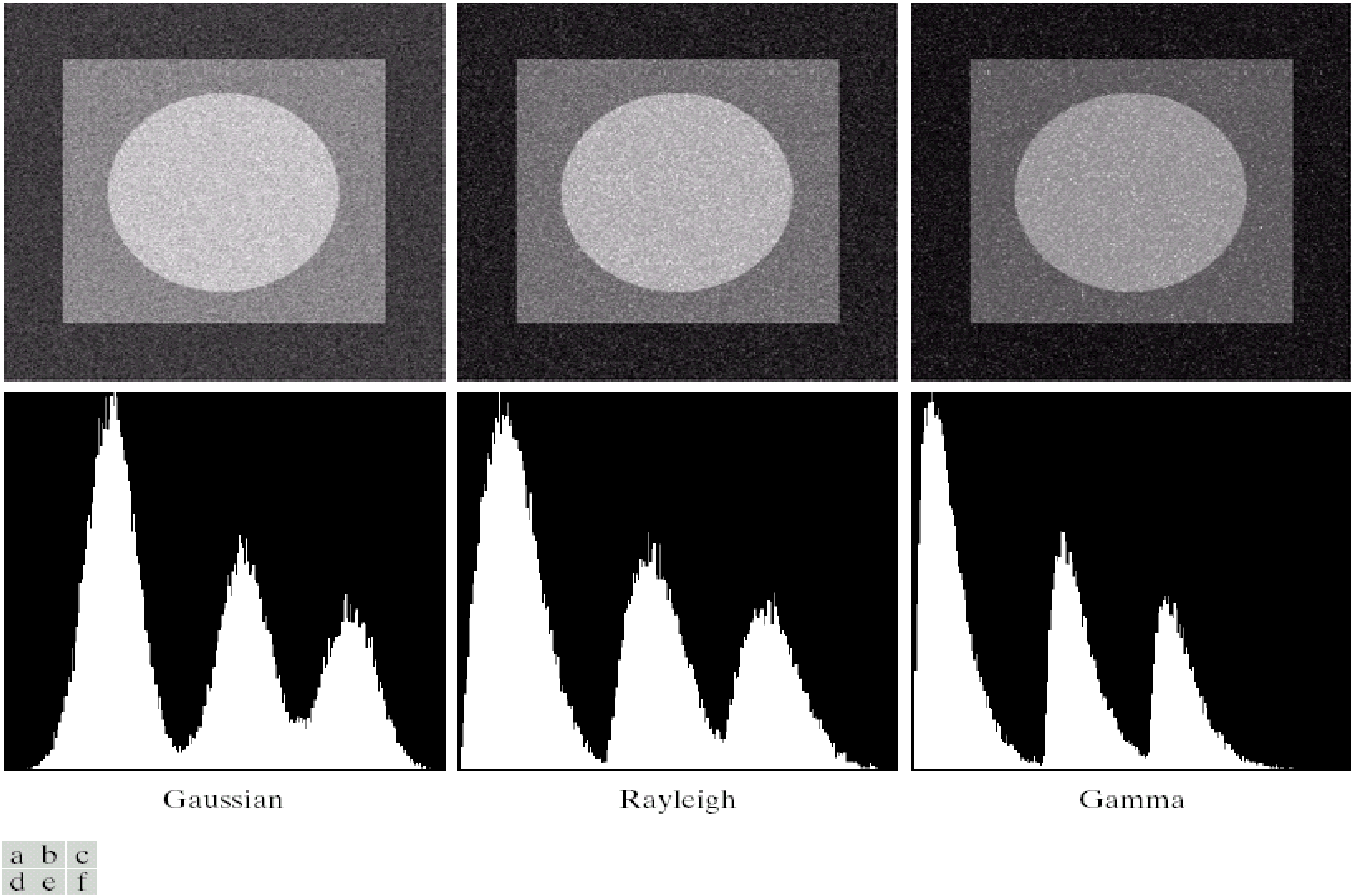


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Noisy Images

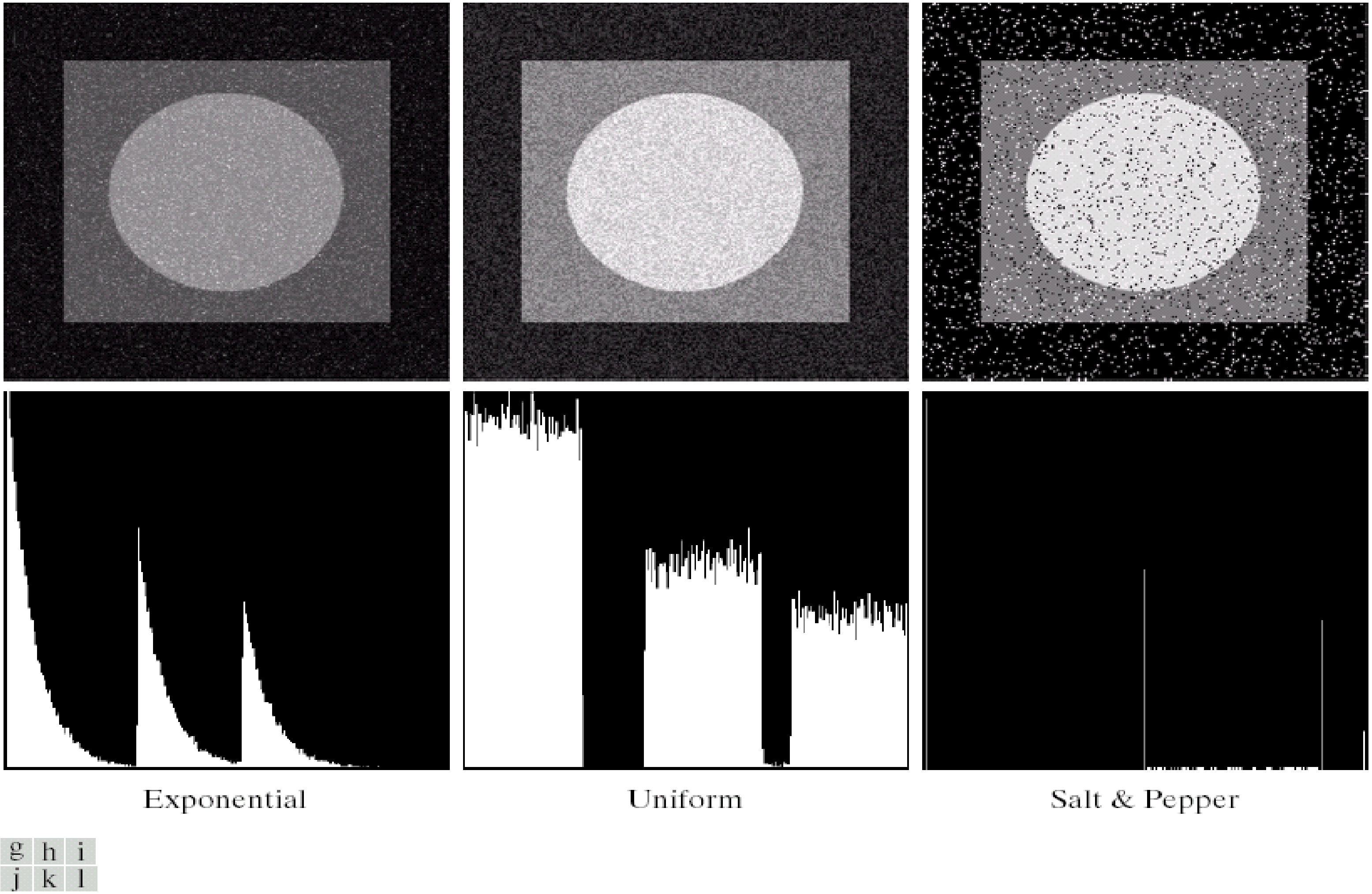
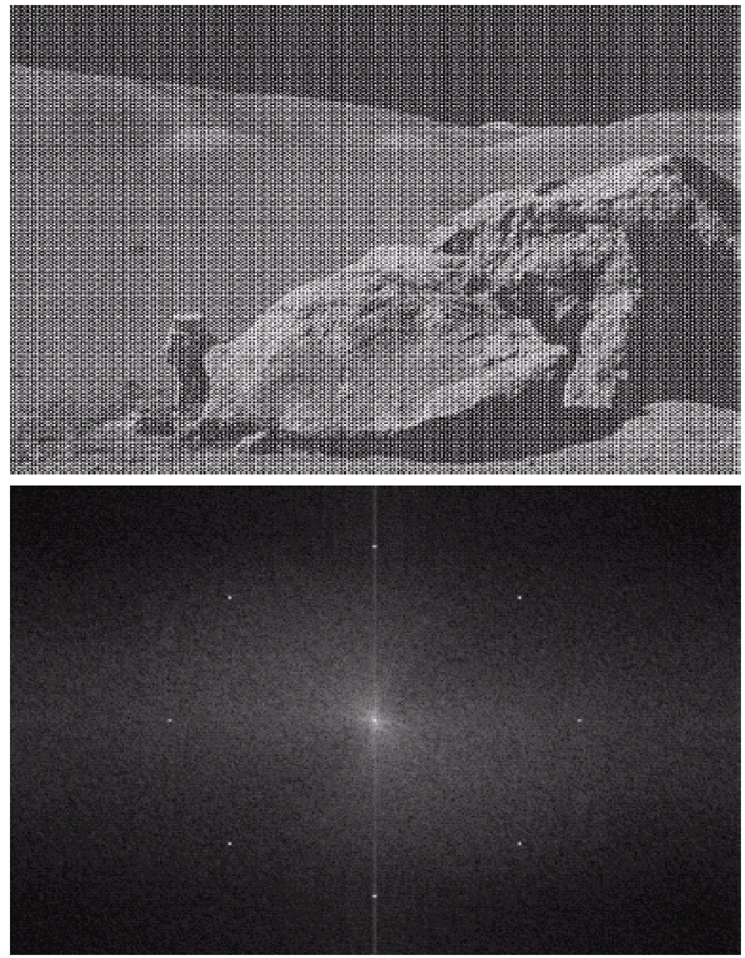


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Noisy Images



a
b

FIGURE 5.5
(a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Additive Noise Removal
Spatial Filtering

Arithmetic Mean filter

$$\bar{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Similar to convolution mask.

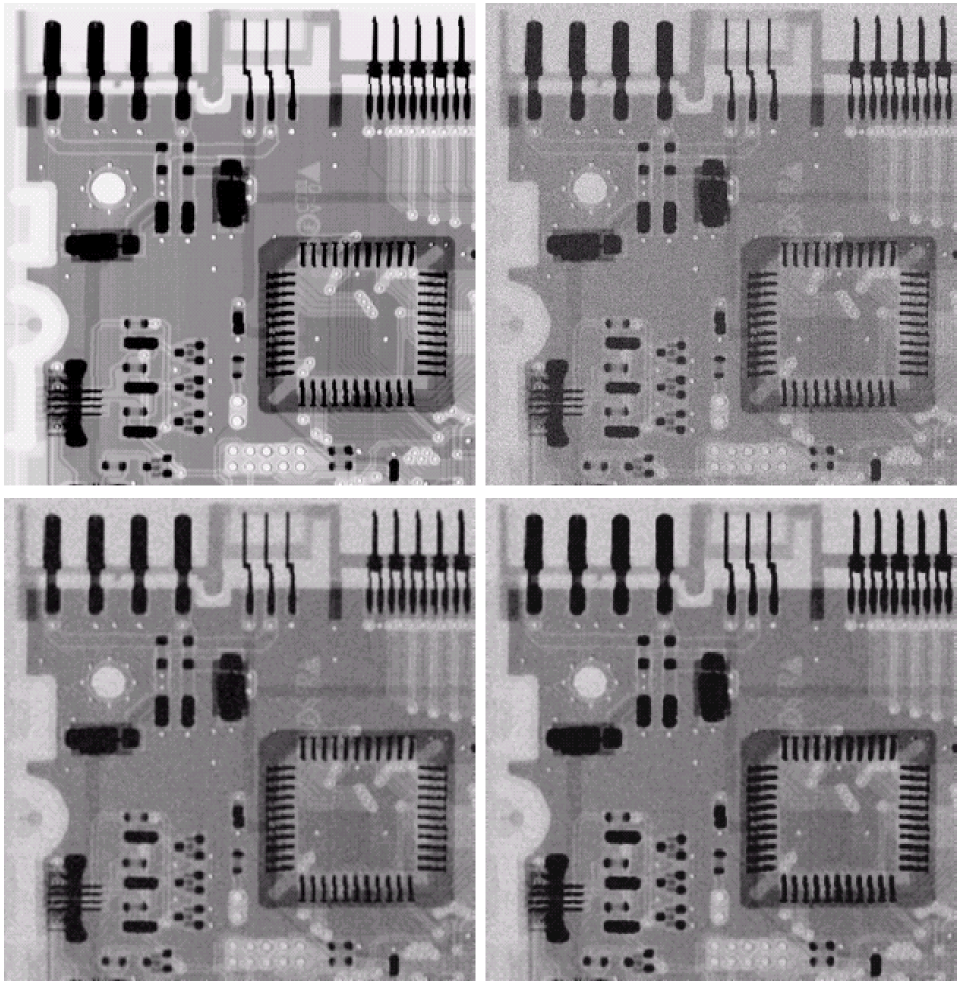
S_{xy} is the nxn filter

Geometric mean filter

$$\bar{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Harmonic filter

$$\bar{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Additive Noise Removal
Spatial Filtering

Contraharmonic mean filter

Q ->order of filter

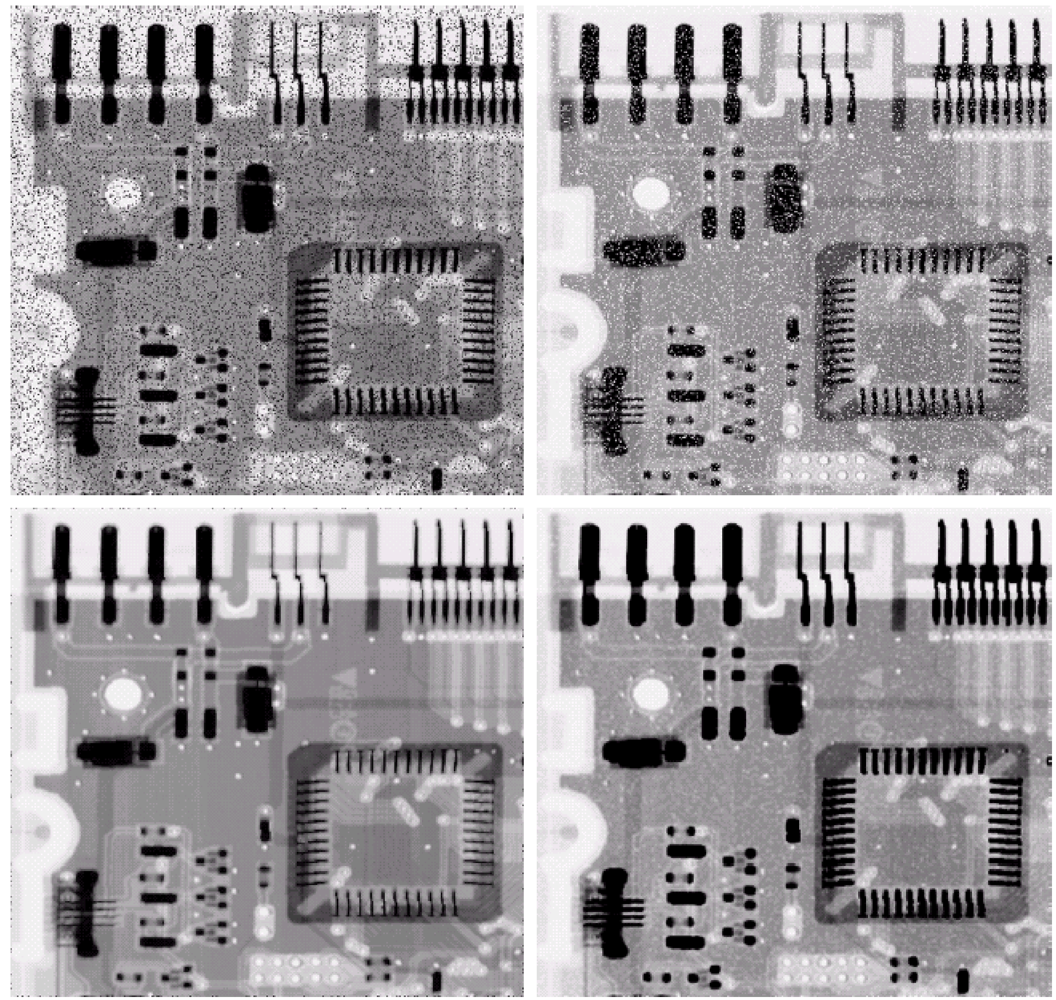
For +ve->removes pepper noise

For –ve->removes salt noise

For Q=0 ->arithmetic

For Q=-1 -> mean

$$\bar{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Spatial Filtering

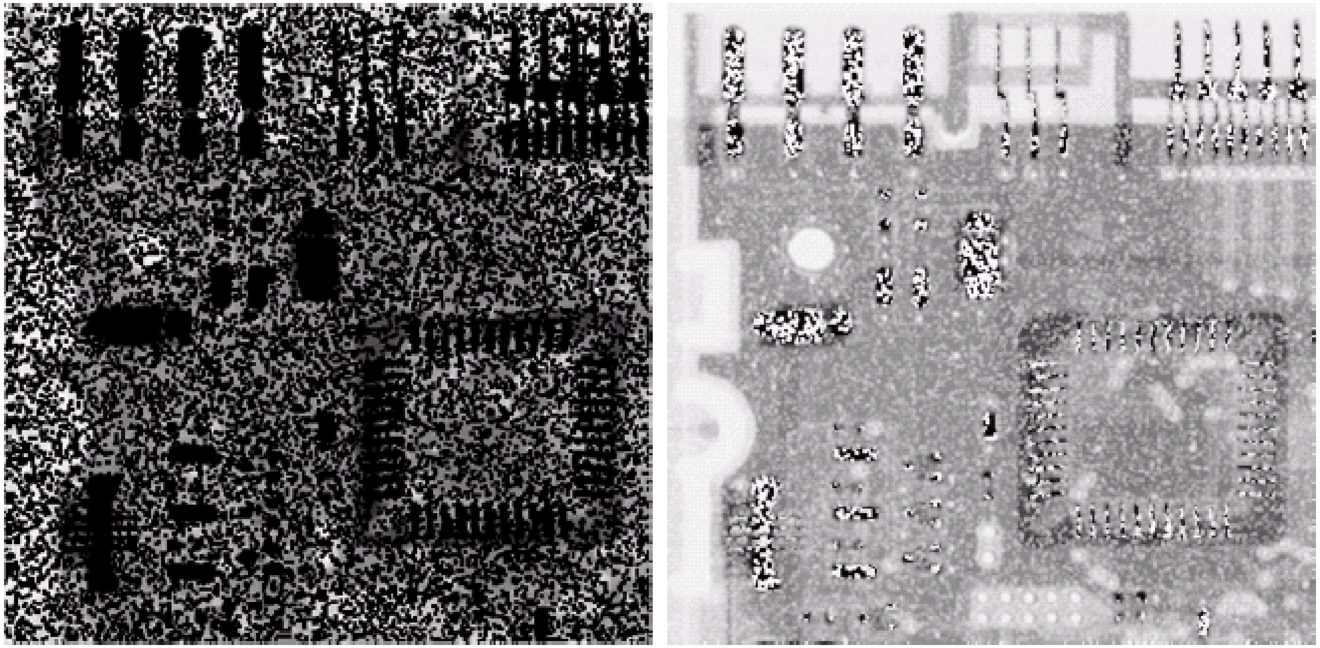


FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-Statistics Filters

Filters

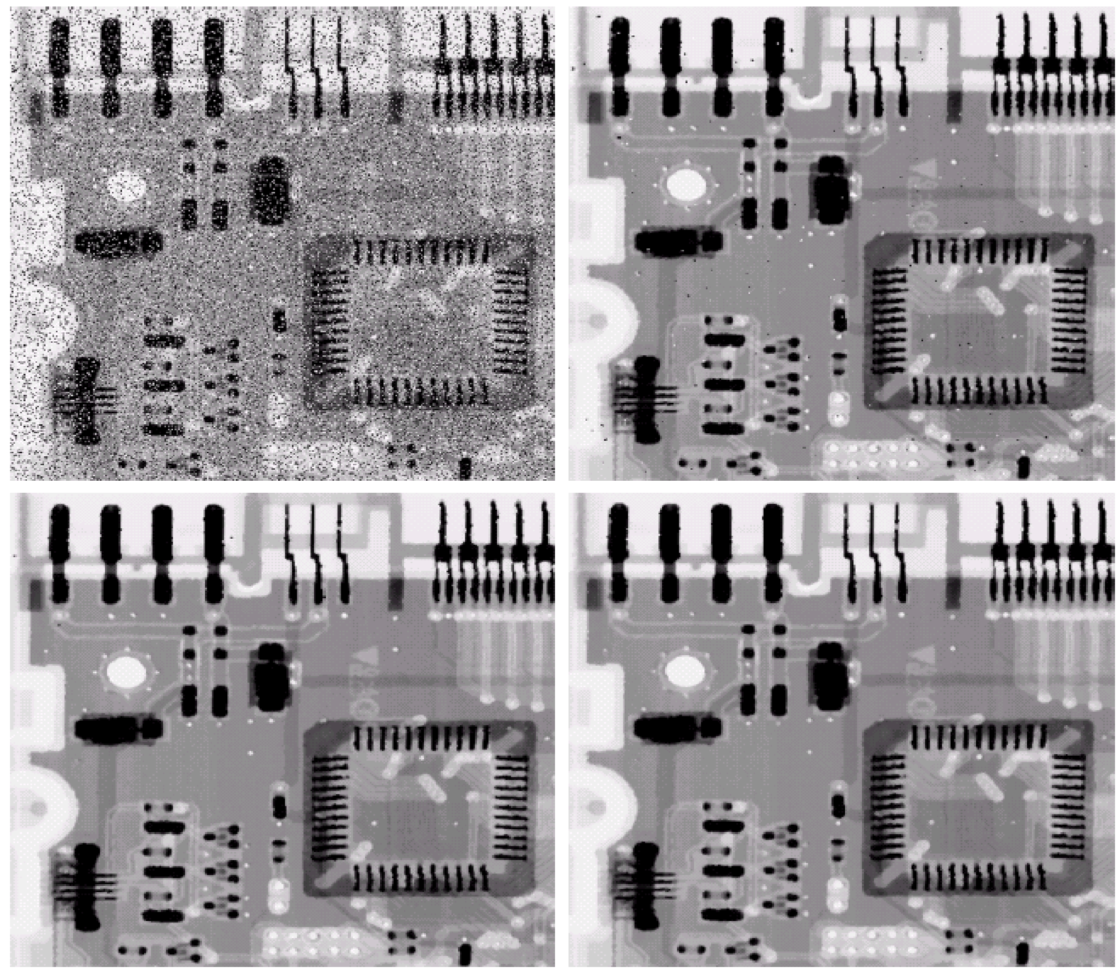
Median

Max and Min

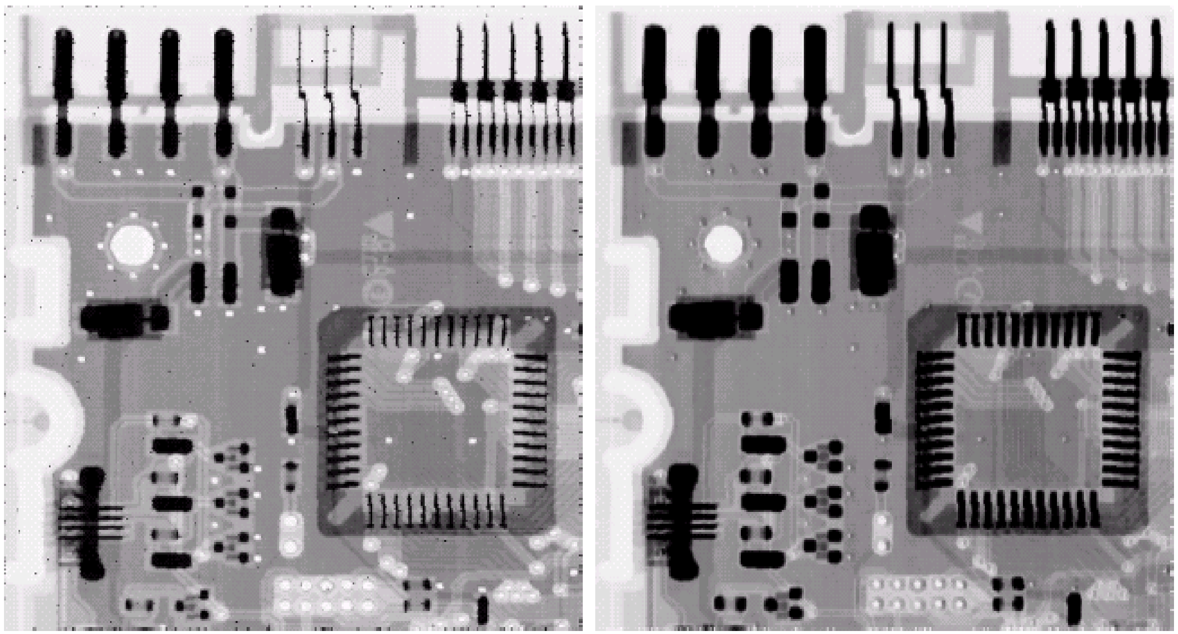
Midpoint

a	b
c	d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



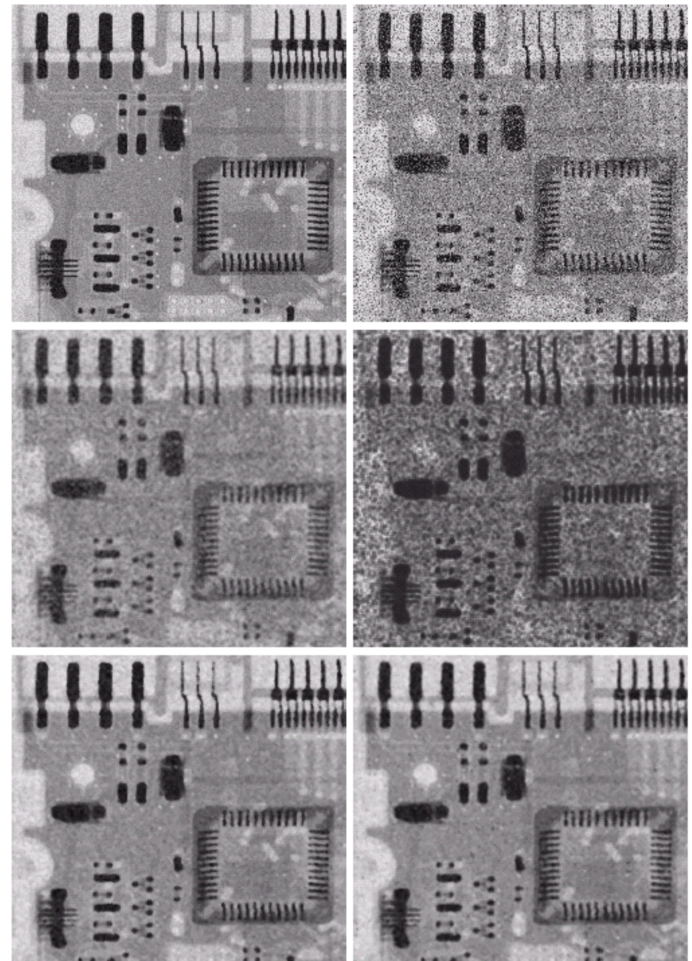
Spatial Filters



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Spatial Filters



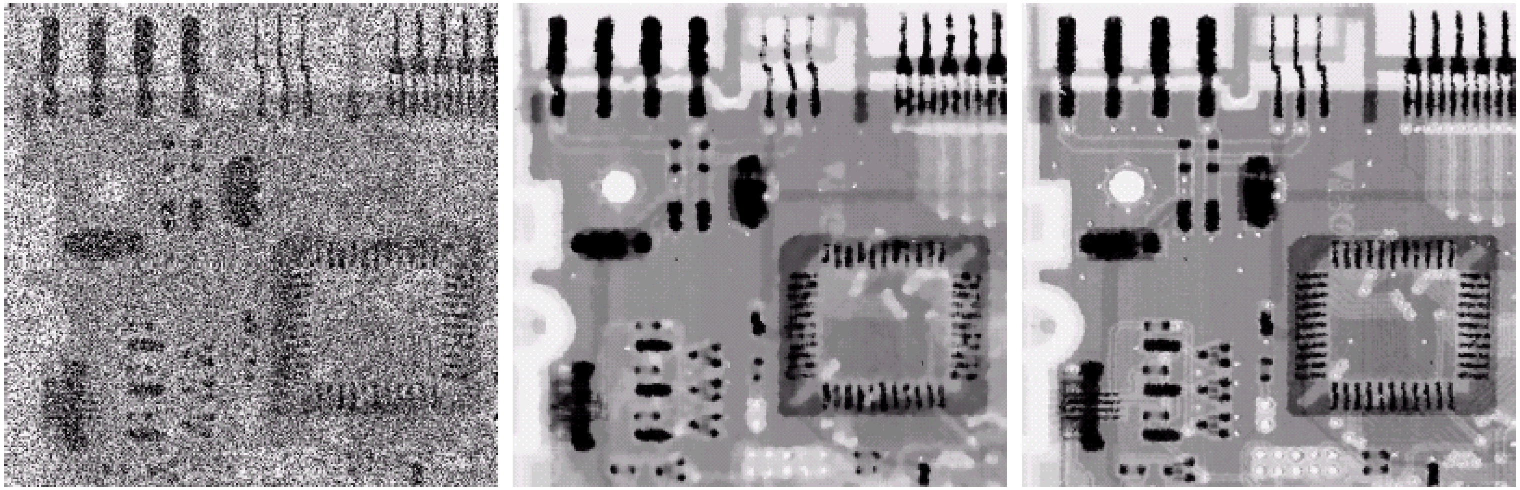
a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5 × 5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with *d* = 5.

Alpha trimmed mean filter

$$\bar{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t);\ d\ total\ gray\ levels$$

Adaptive Spatial Filtering



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Frequency filters

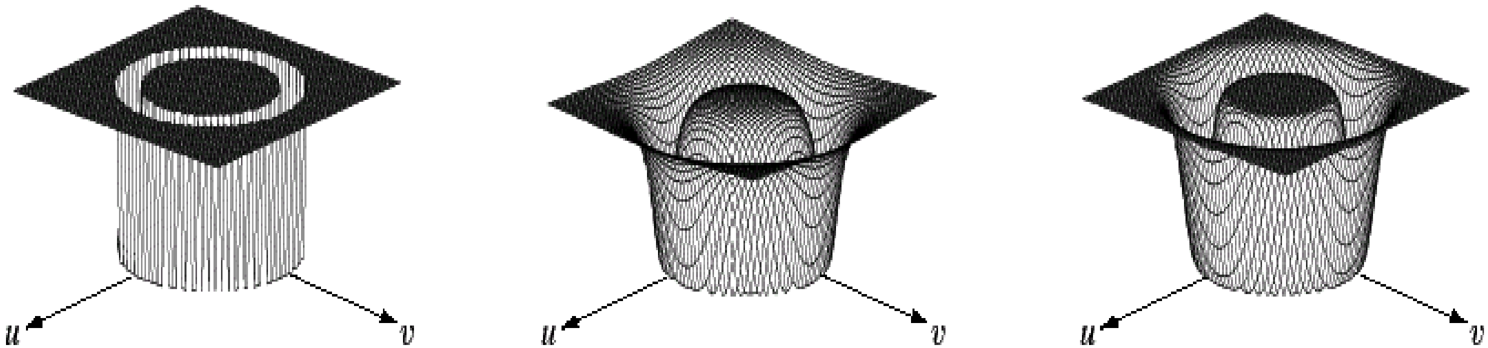
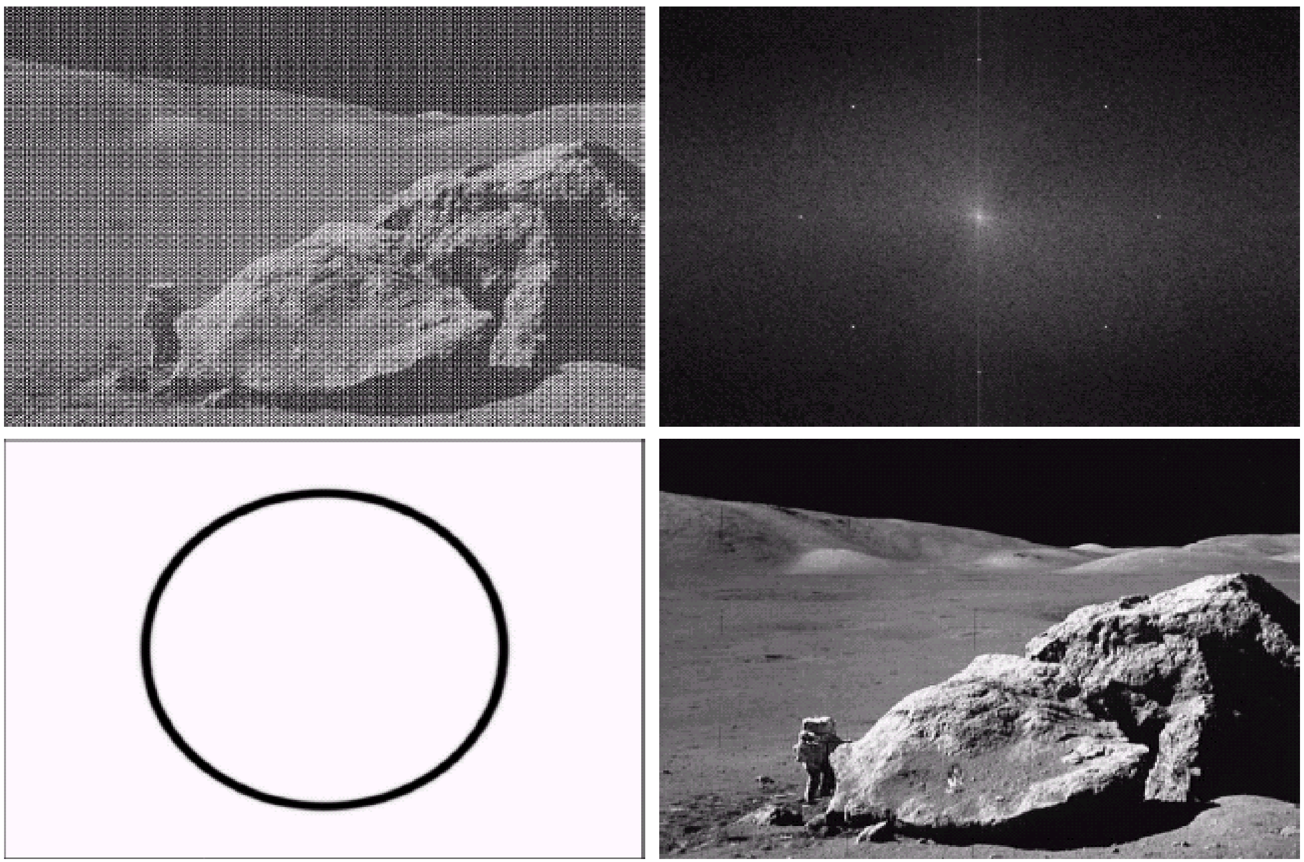


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Butterworth Filtering



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Frequency Filters

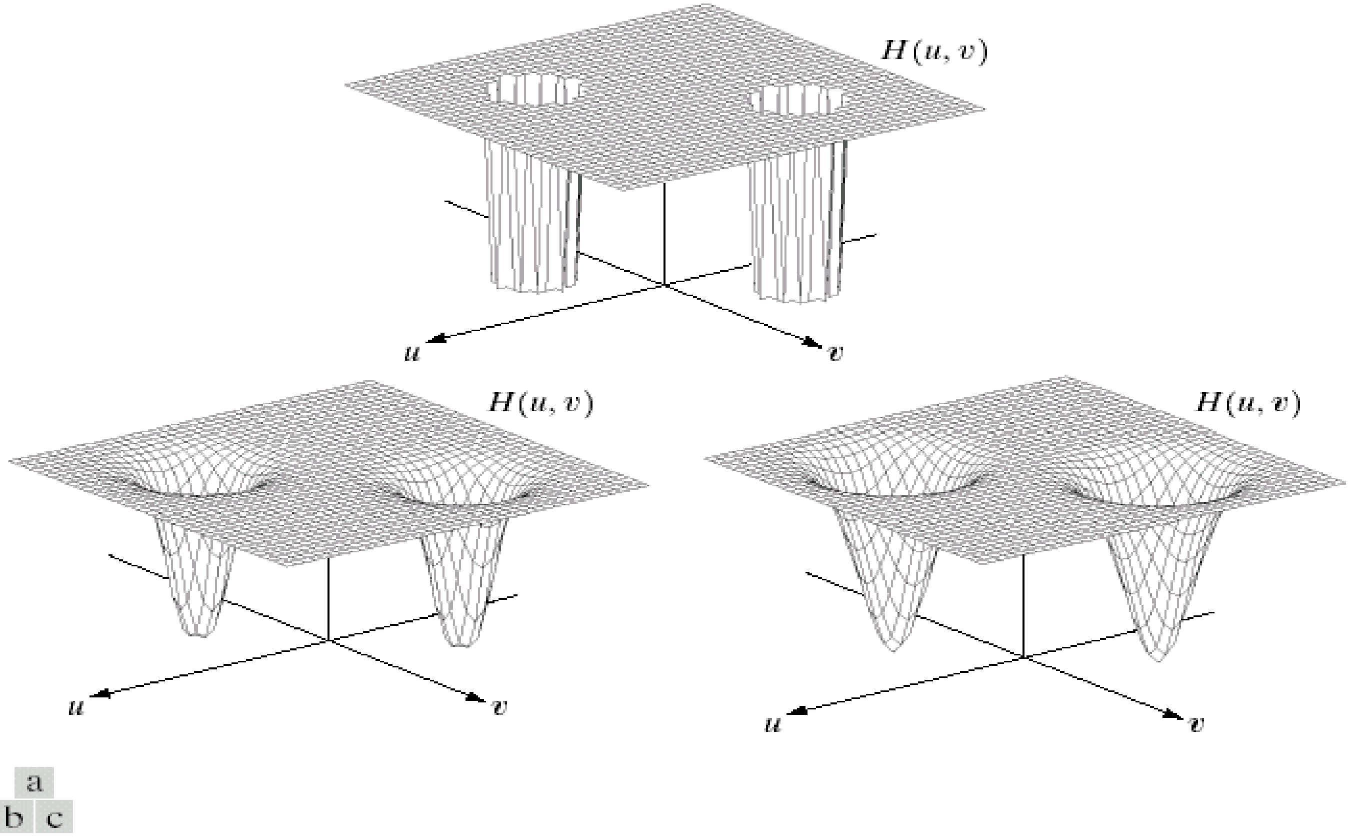
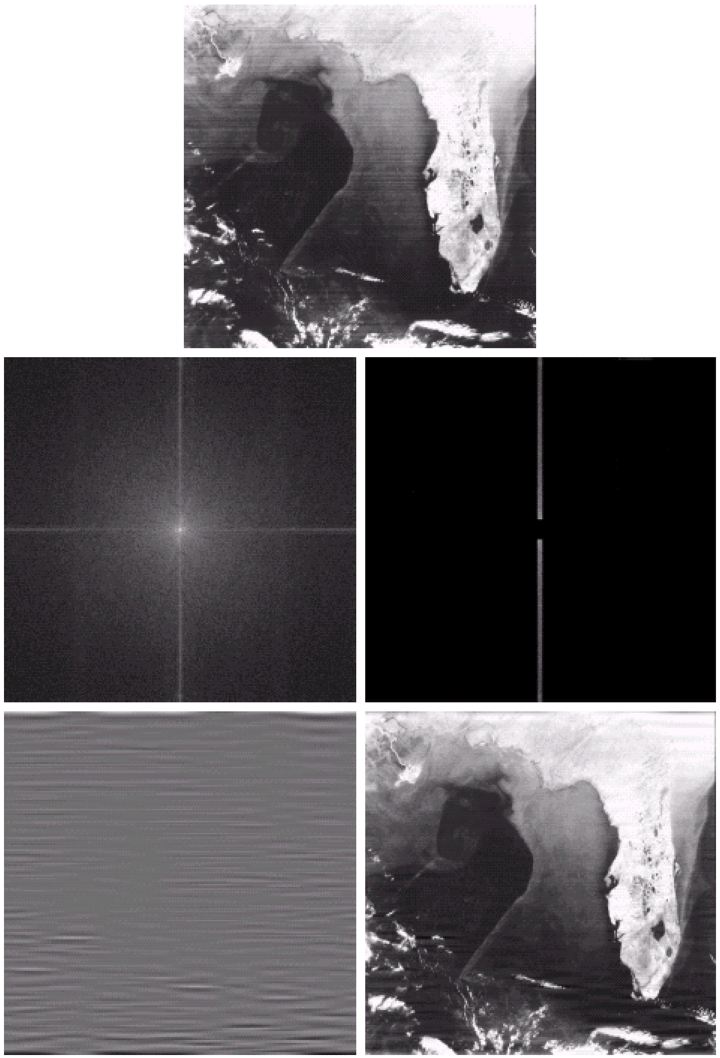


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

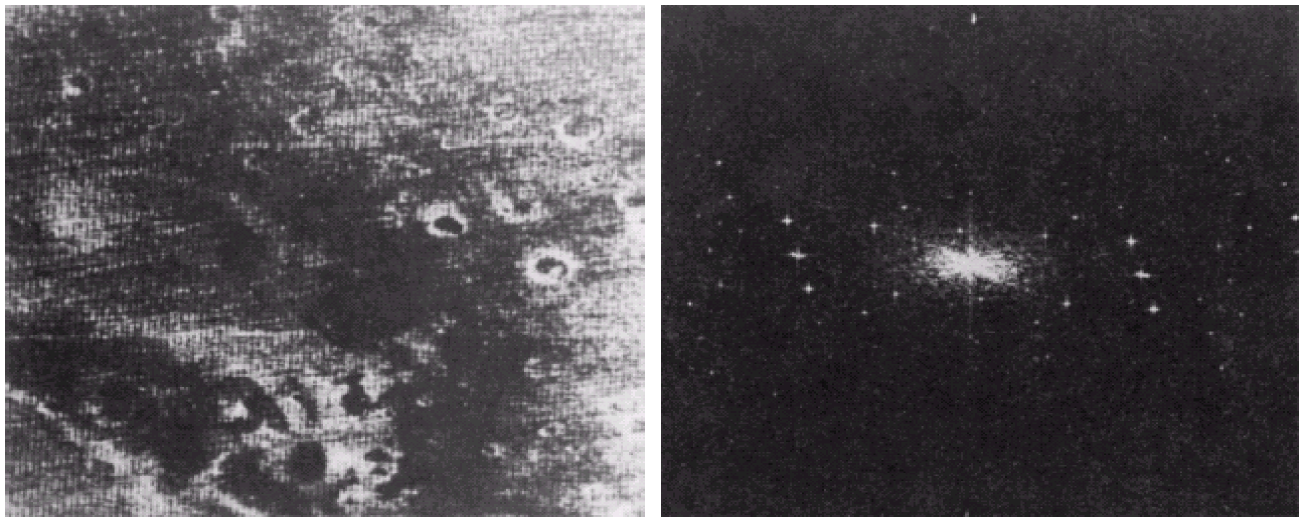
Frequency filtering



a
b c
d e

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico (note horizontal sensor scan lines). (b) Spectrum of (a). (c) Notch pass filter shown superimposed on (b). (d) Inverse Fourier transform of filtered image, showing noise pattern in the spatial domain. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Interference noise

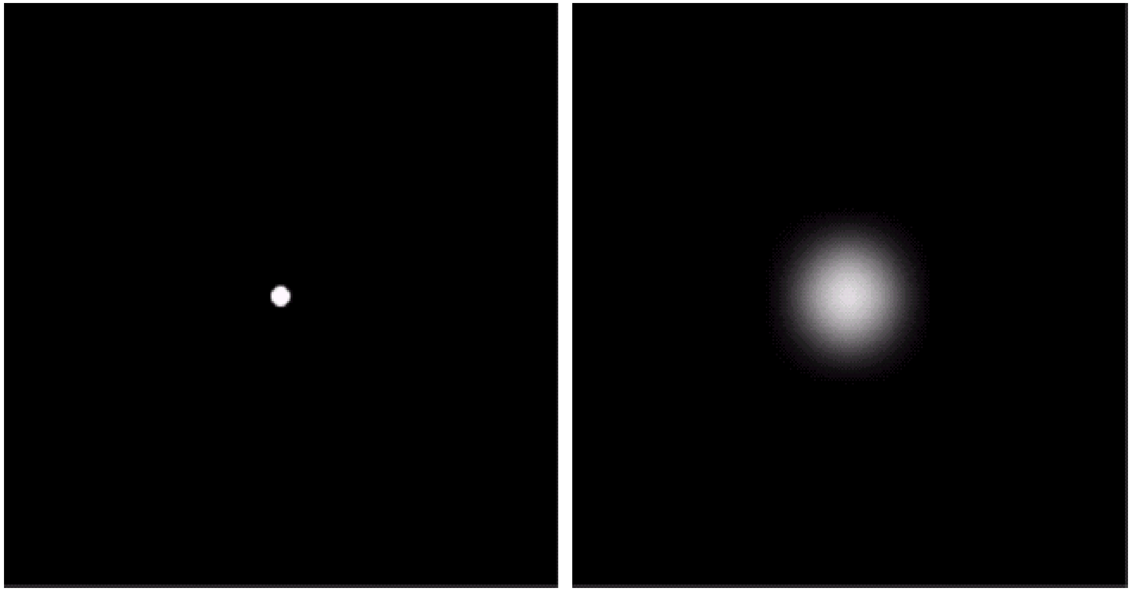


a b

FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6*.
(b) Fourier spectrum showing periodic interference.
(Courtesy of NASA.)

FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a).

Degraded Impulse



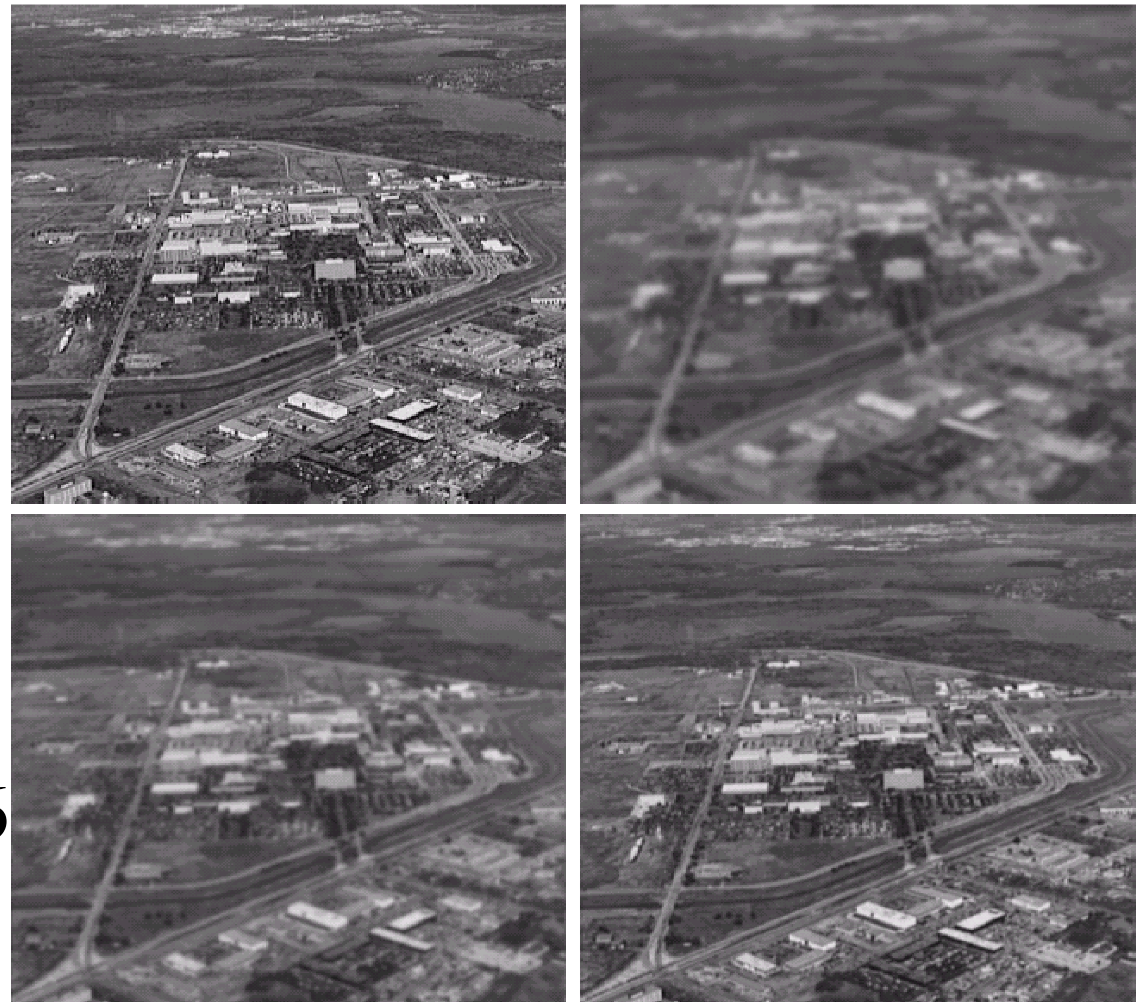
a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Turbulence Noise

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



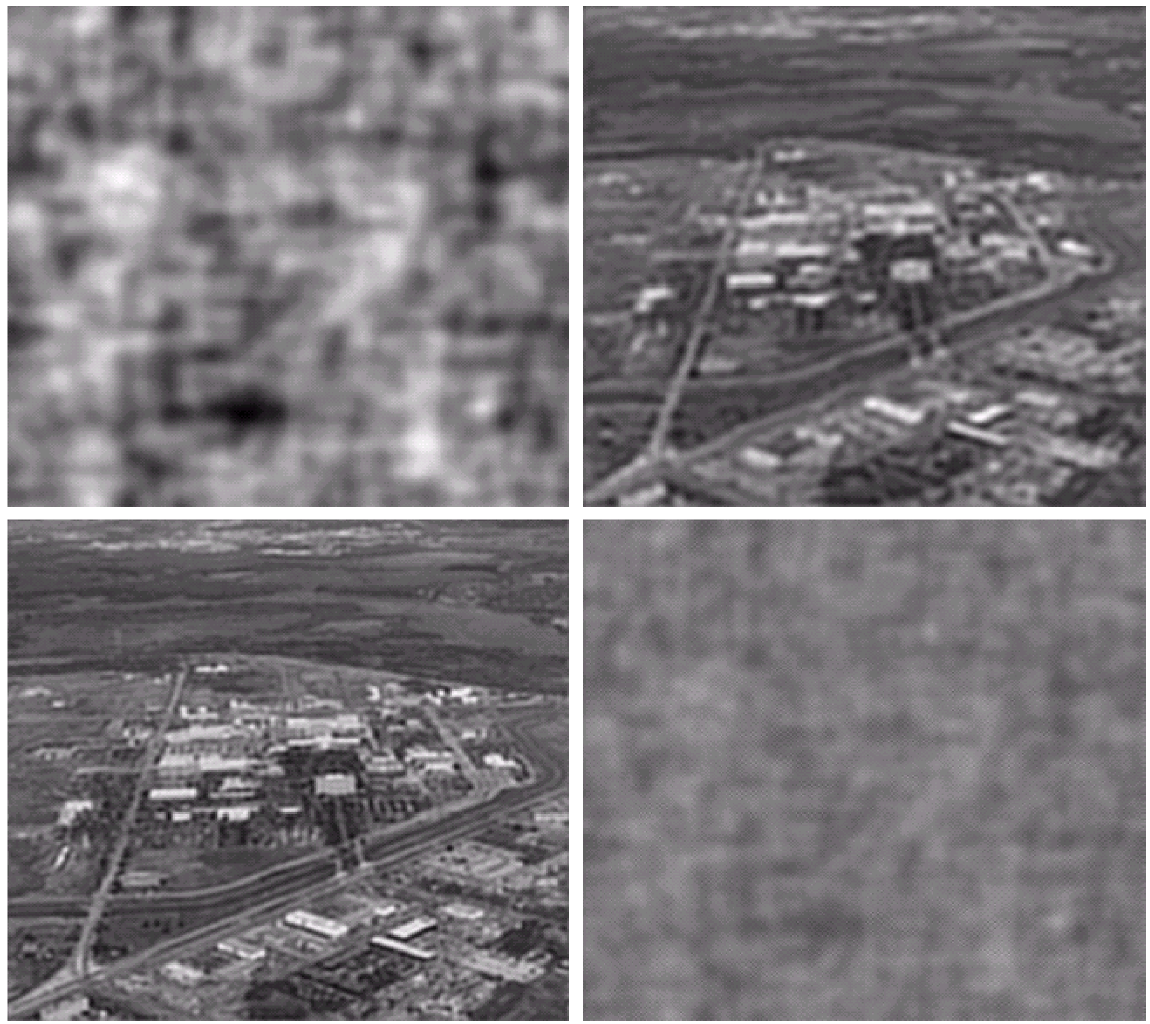
Turbulence Model

$$\exp(-k\{u^\wedge 2+v^\wedge 2)^\wedge 5/6$$

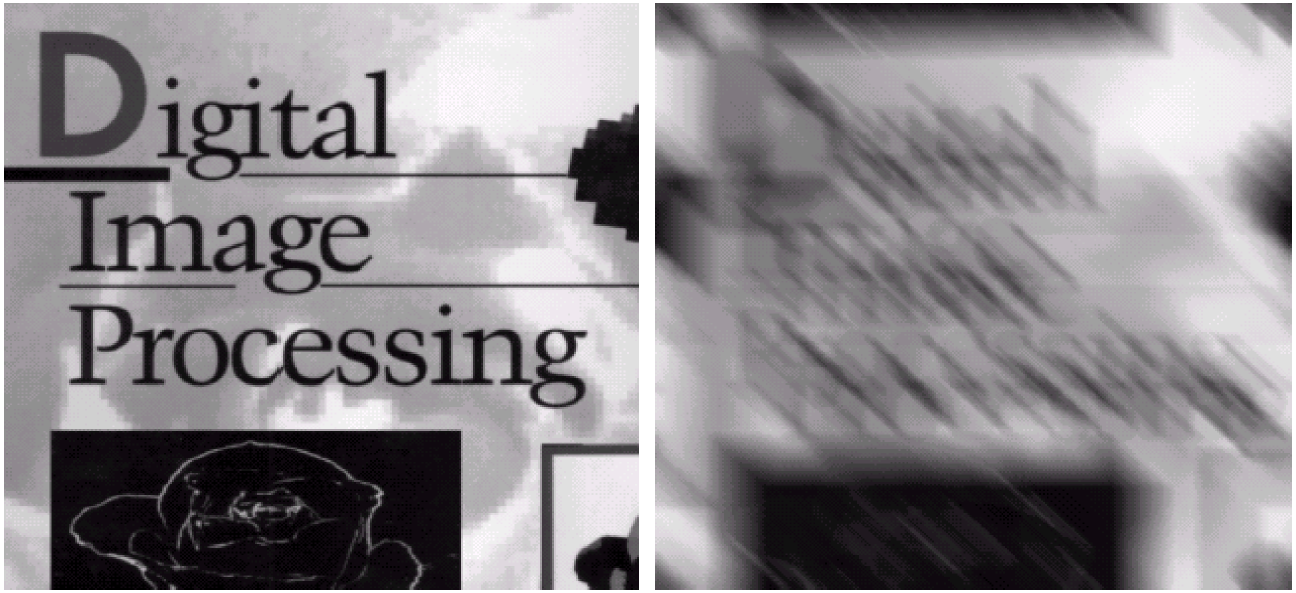
Blurred Image Restoration - IF

a	b
c	d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



Motion Blur only



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Inverse and least mean square (wiener) Filtering

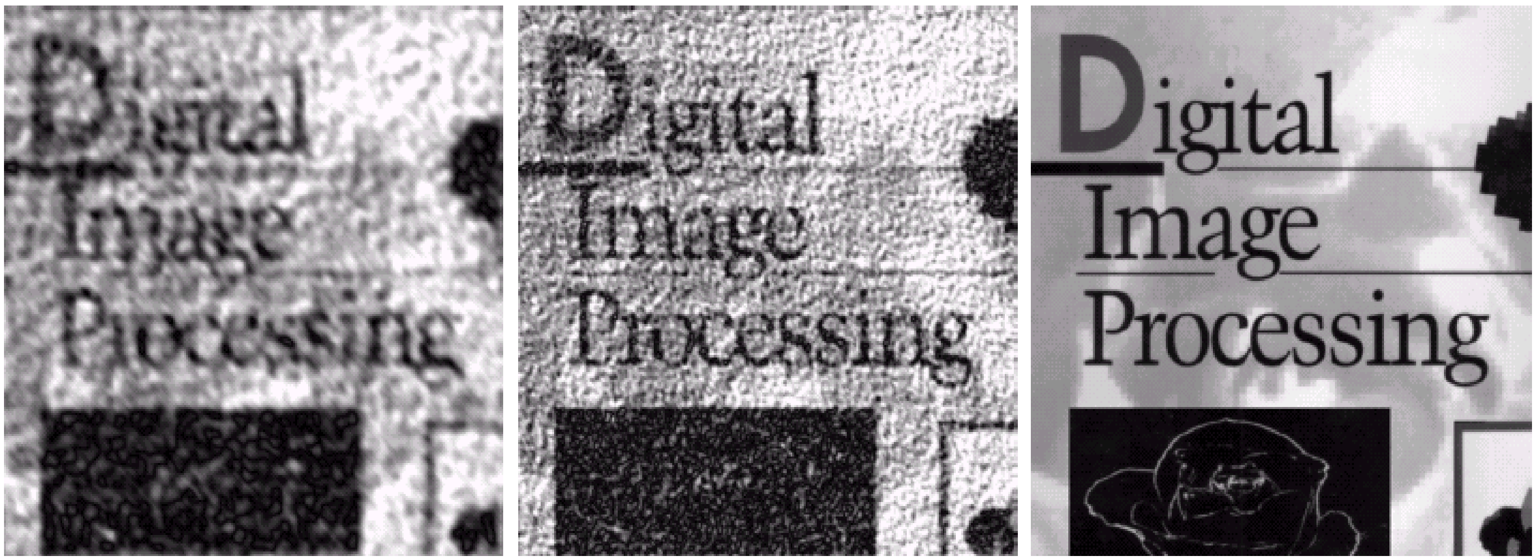
a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Motion Blur+ noise
IF and wiener filtering

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

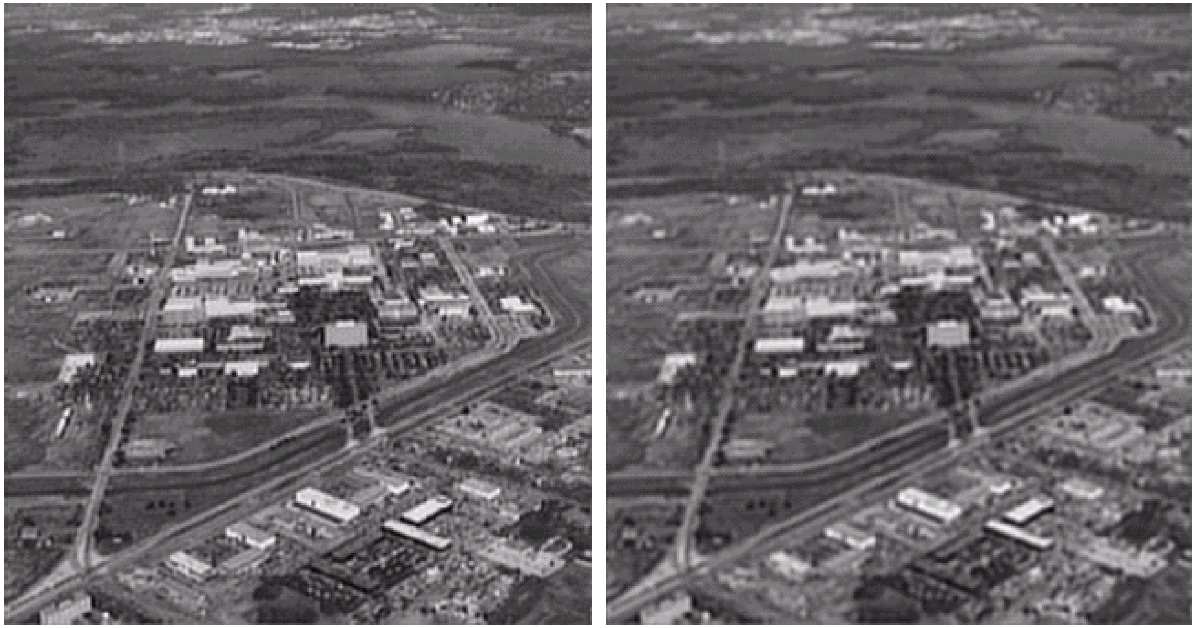
Constrained least square filtering



a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

Iterative Constrained least square filtering



a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.