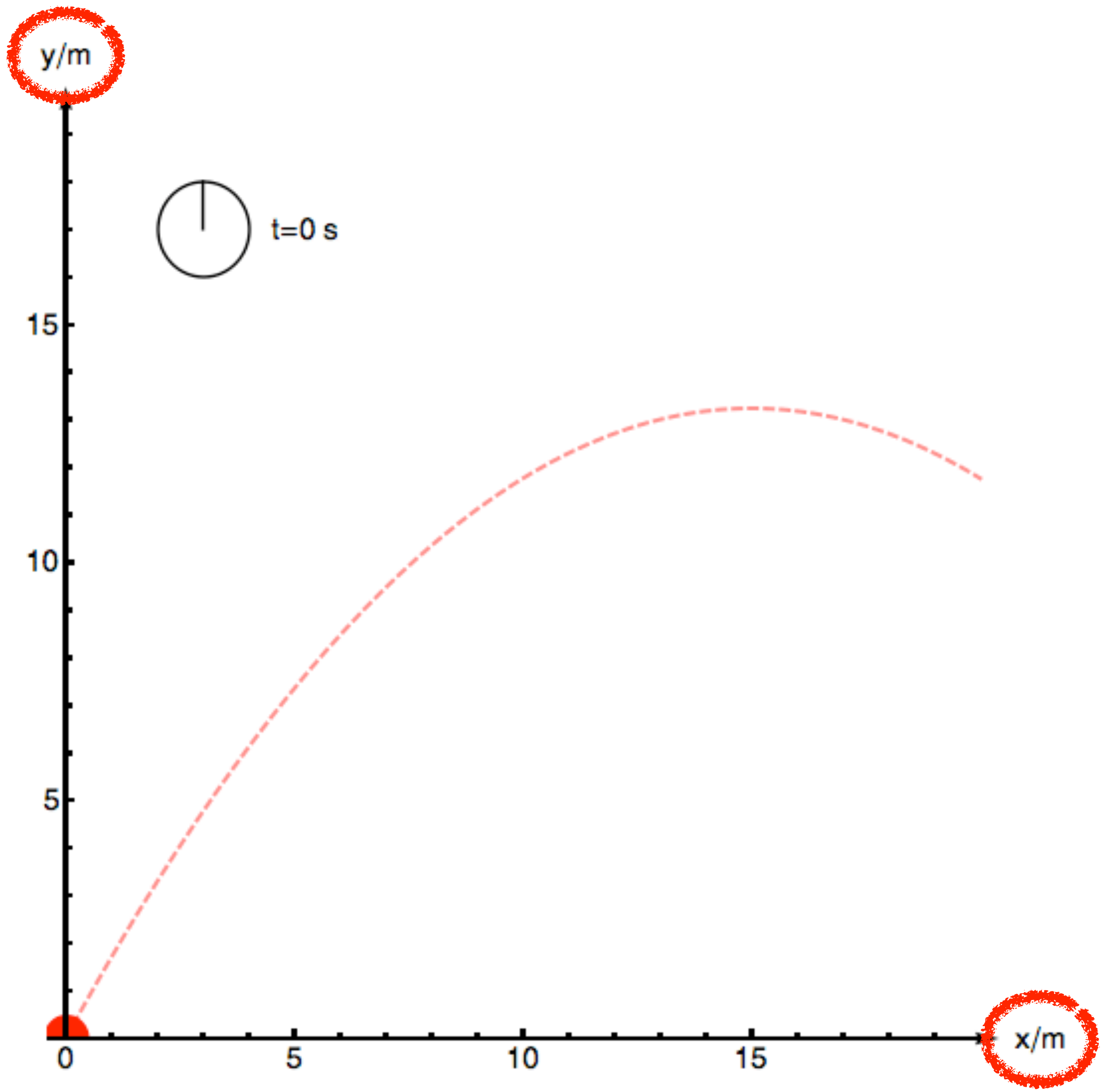
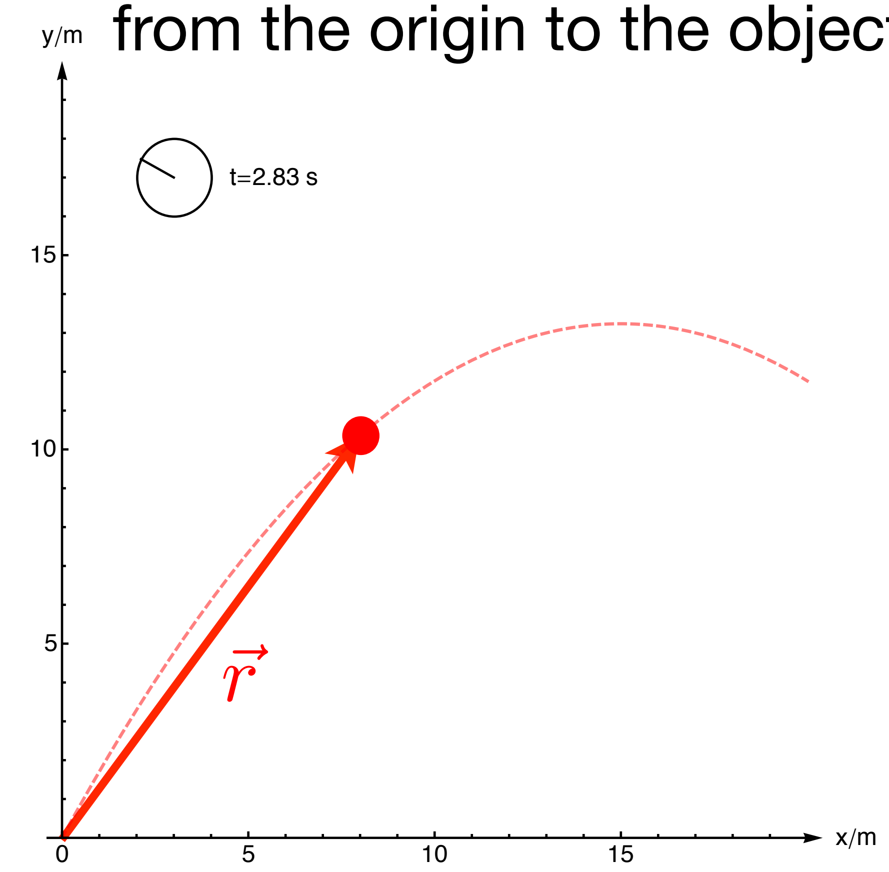
motion in a plane

position & displacement vectors

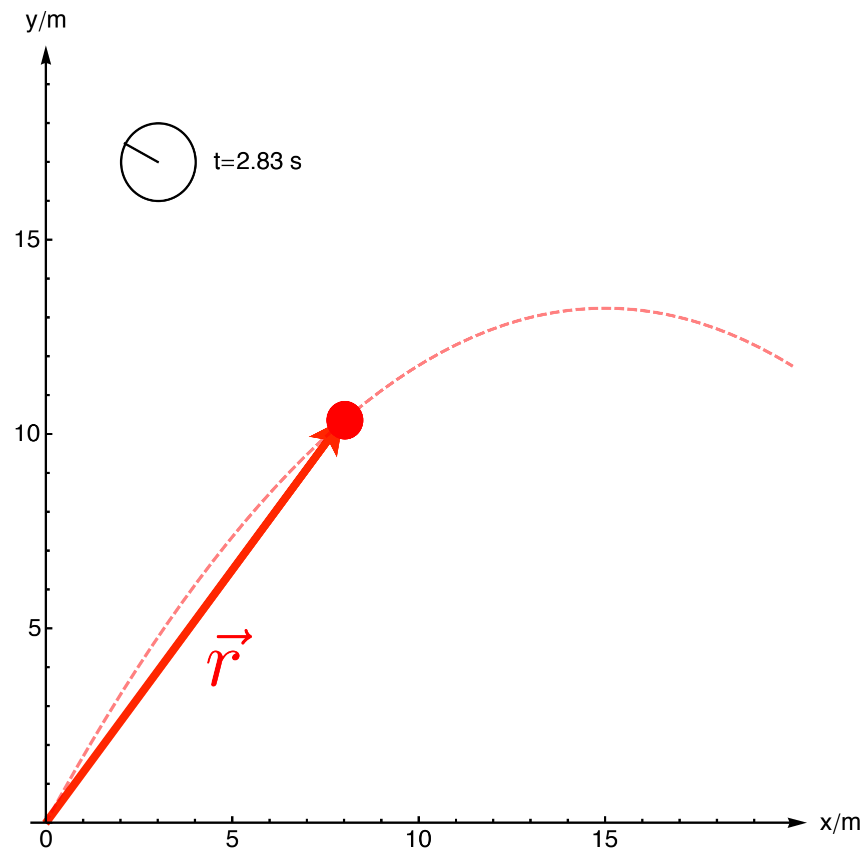


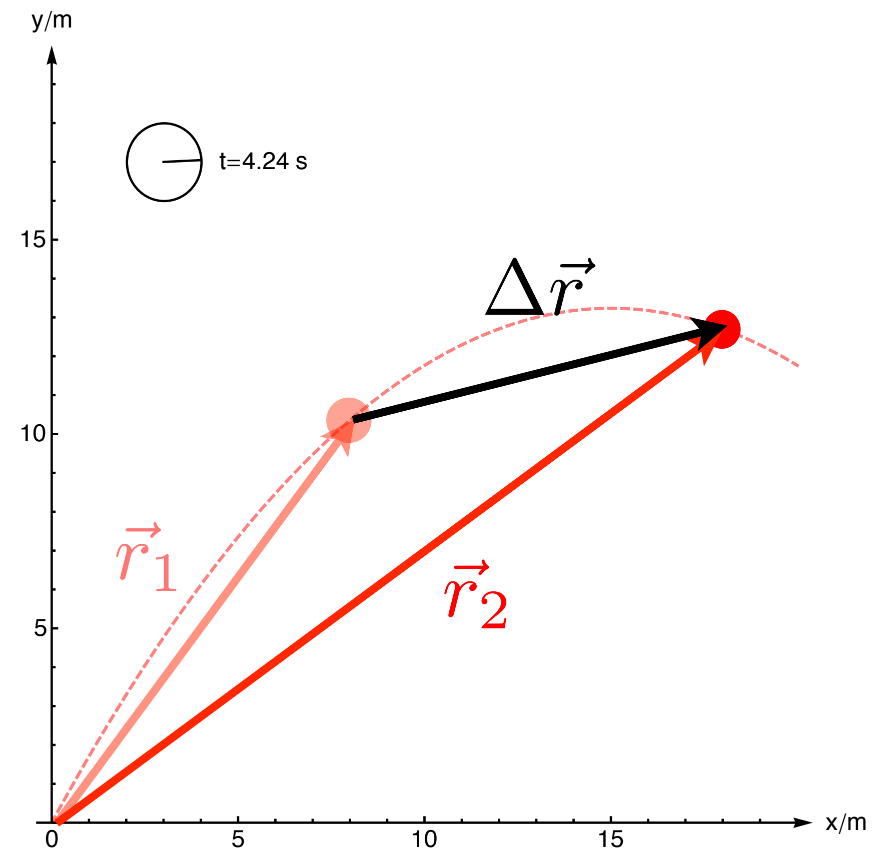
➜ the **position vector** points from the origin to the object



we're plotting the plane
(e.g. billiard table viewed from above)

position & displacement vectors



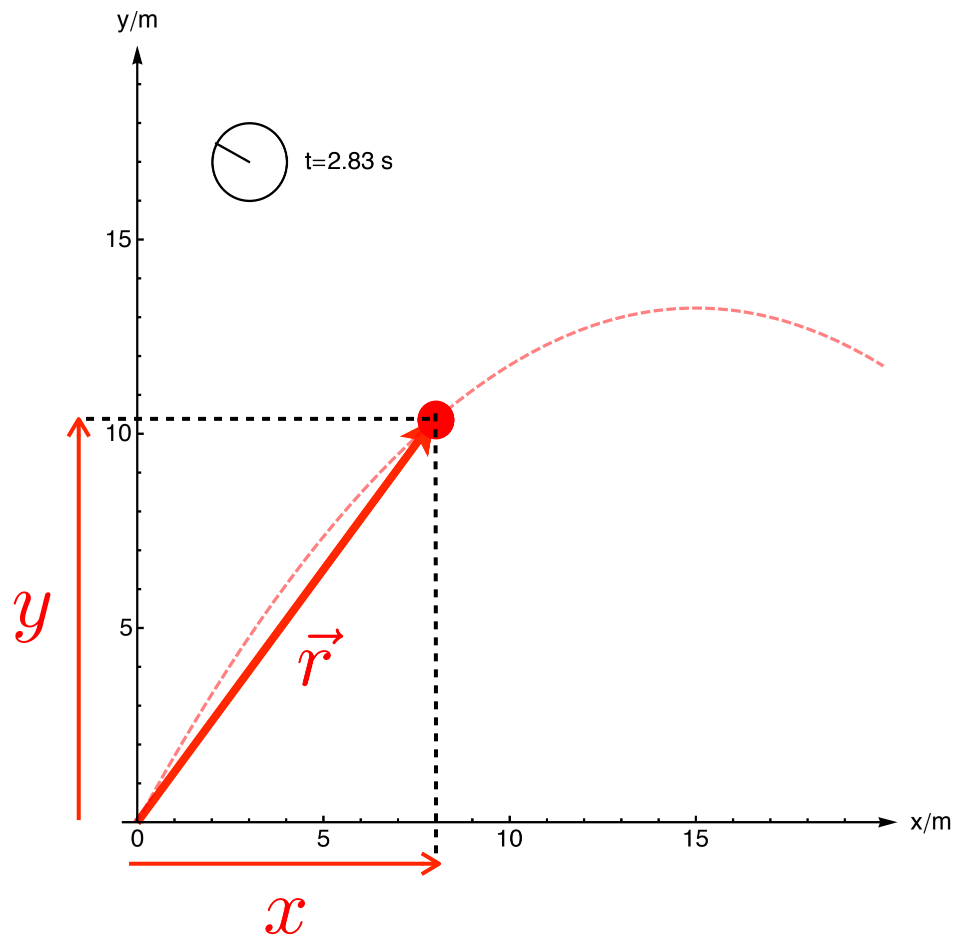


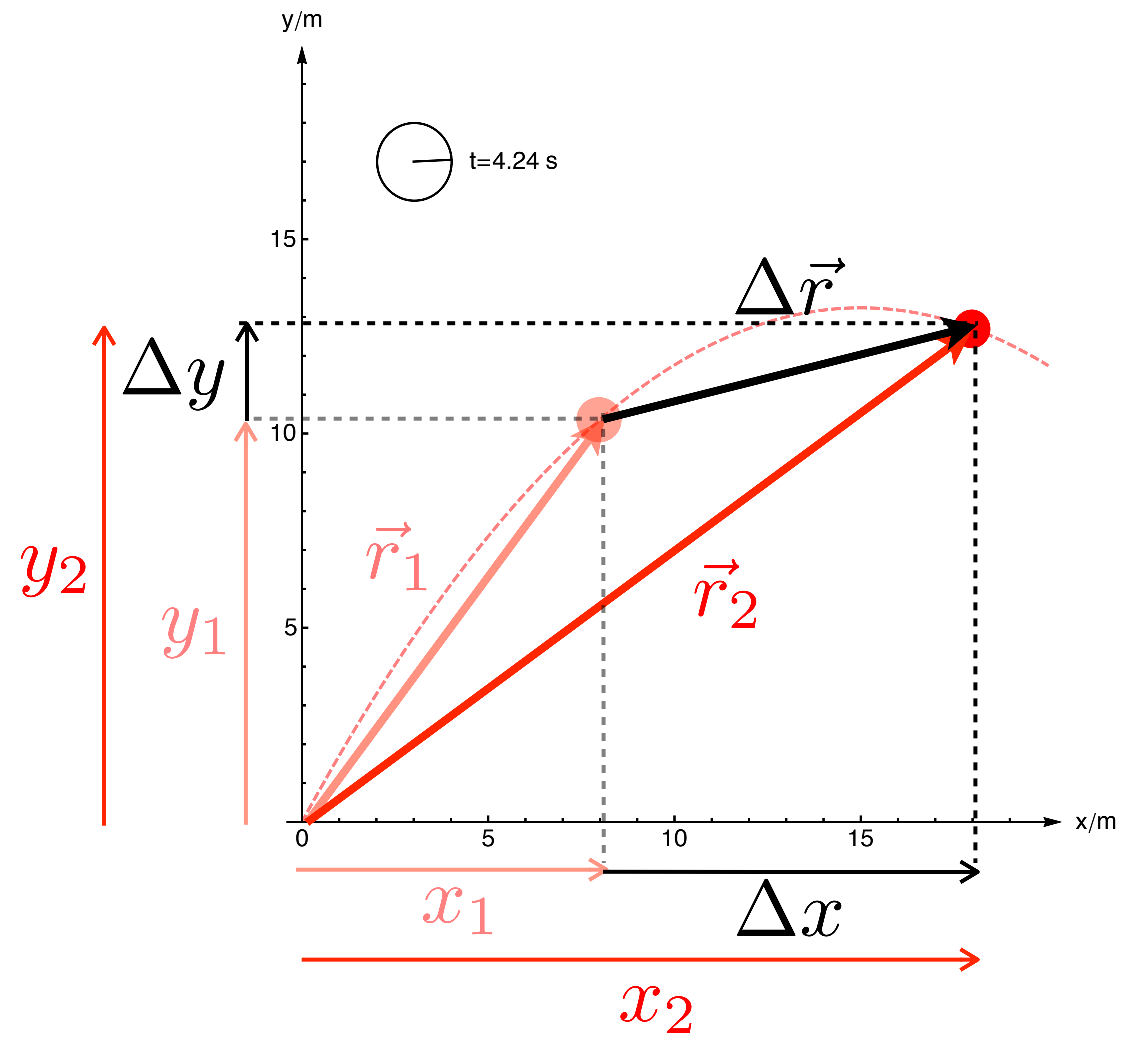
➜ the **displacement vector** points from where the object **was** to where it **is now**

$$\boxed{\Delta\vec{r} = \vec{r}_2 - \vec{r}_1}$$

easier to visualize as $\vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$

position & displacement vectors - components



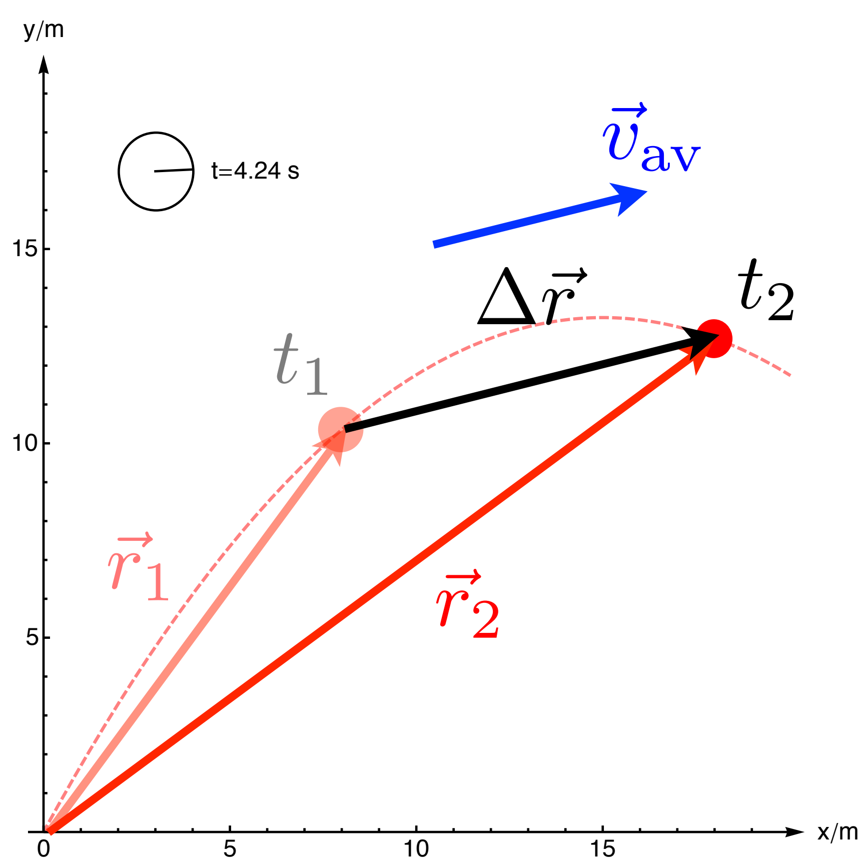


$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta x = x_2 - x_1$$

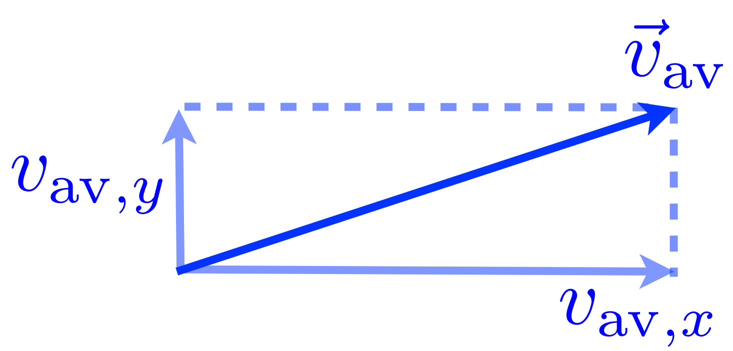
$$\Delta y = y_2 - y_1$$

average velocity



➜ the **average velocity vector** is defined to be the *rate of change of displacement vector*

$$\vec{v}_{\rm av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$



➜ the **components** of the **average velocity vector** are the *rate of change of the components of the displacement vector*

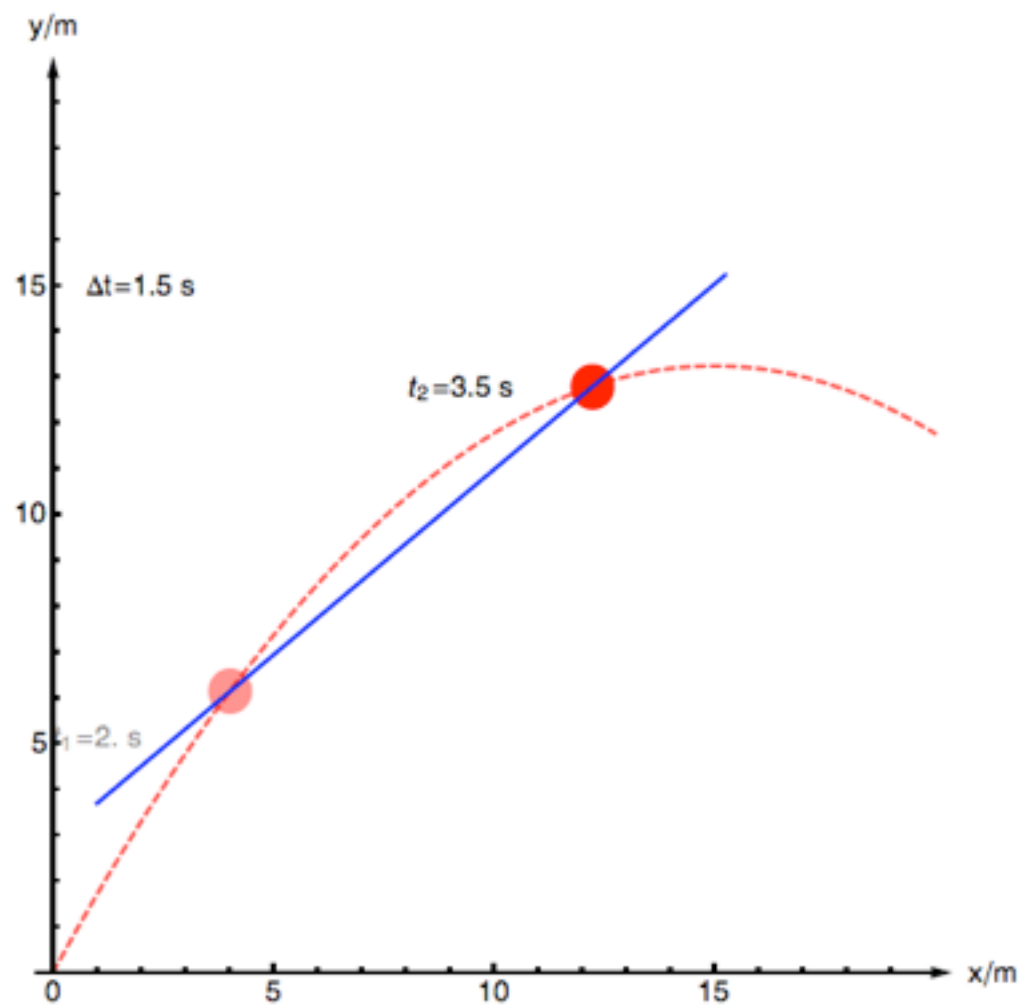
$$v_{{\rm av},x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$v_{{\rm av},y} = \frac{\Delta y}{\Delta t} = \frac{y_2 - y_1}{t_2 - t_1}$$

instantaneous velocity

➜ the **instantaneous velocity vector** is defined analogously to one-dimension

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$



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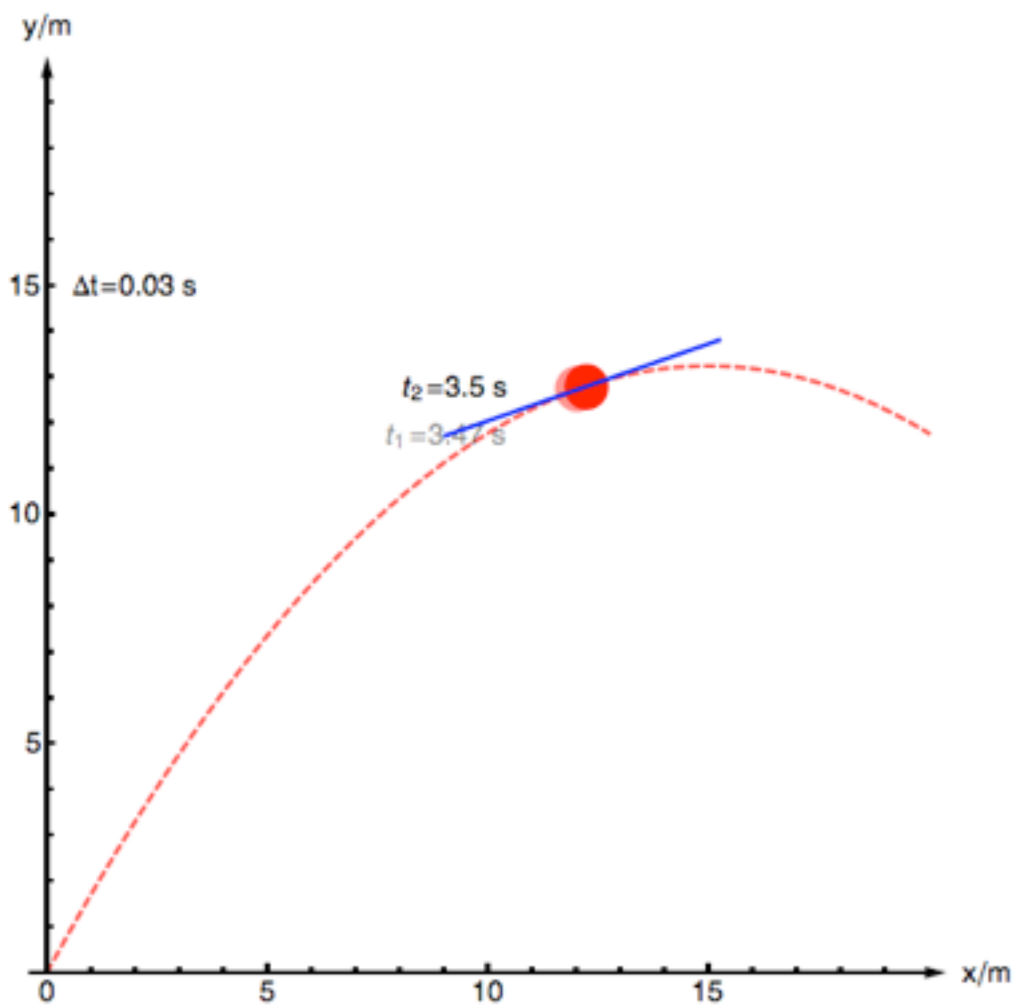
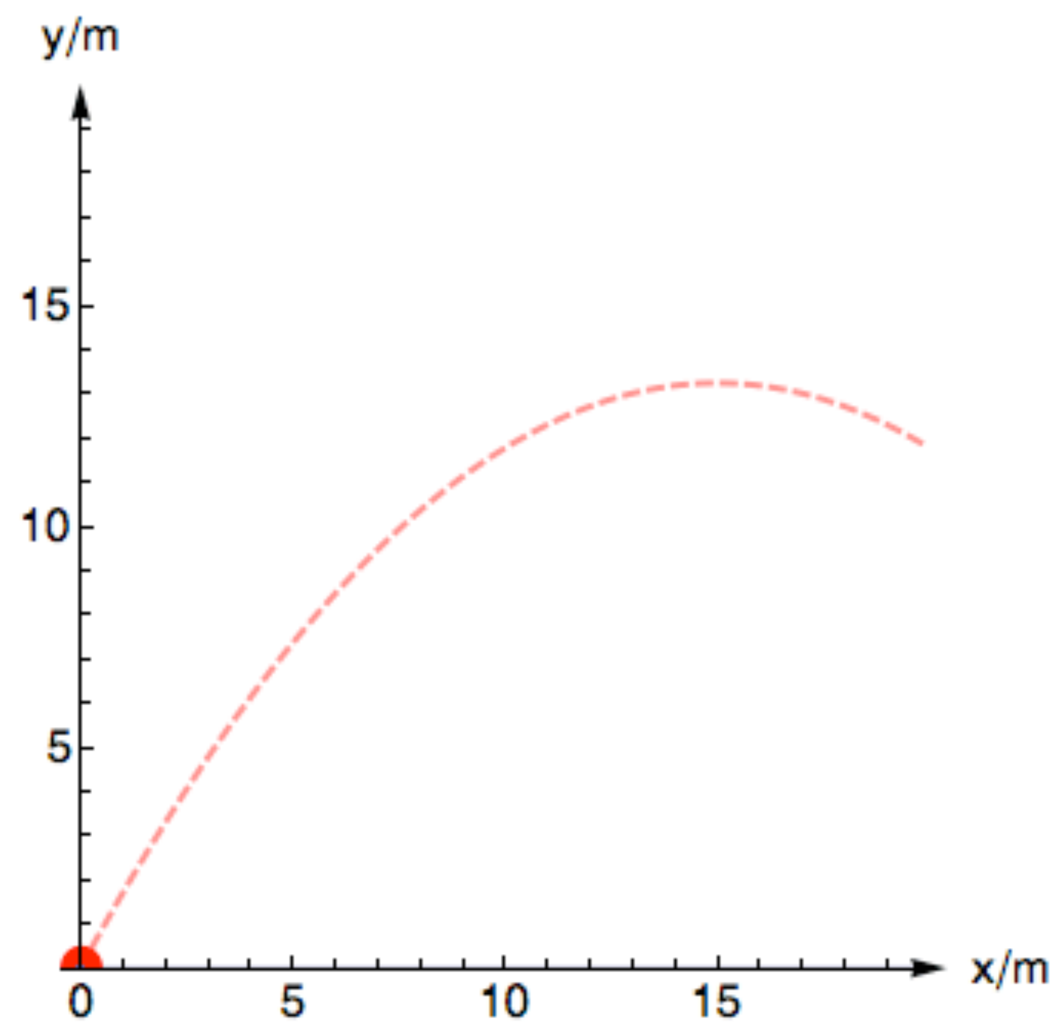
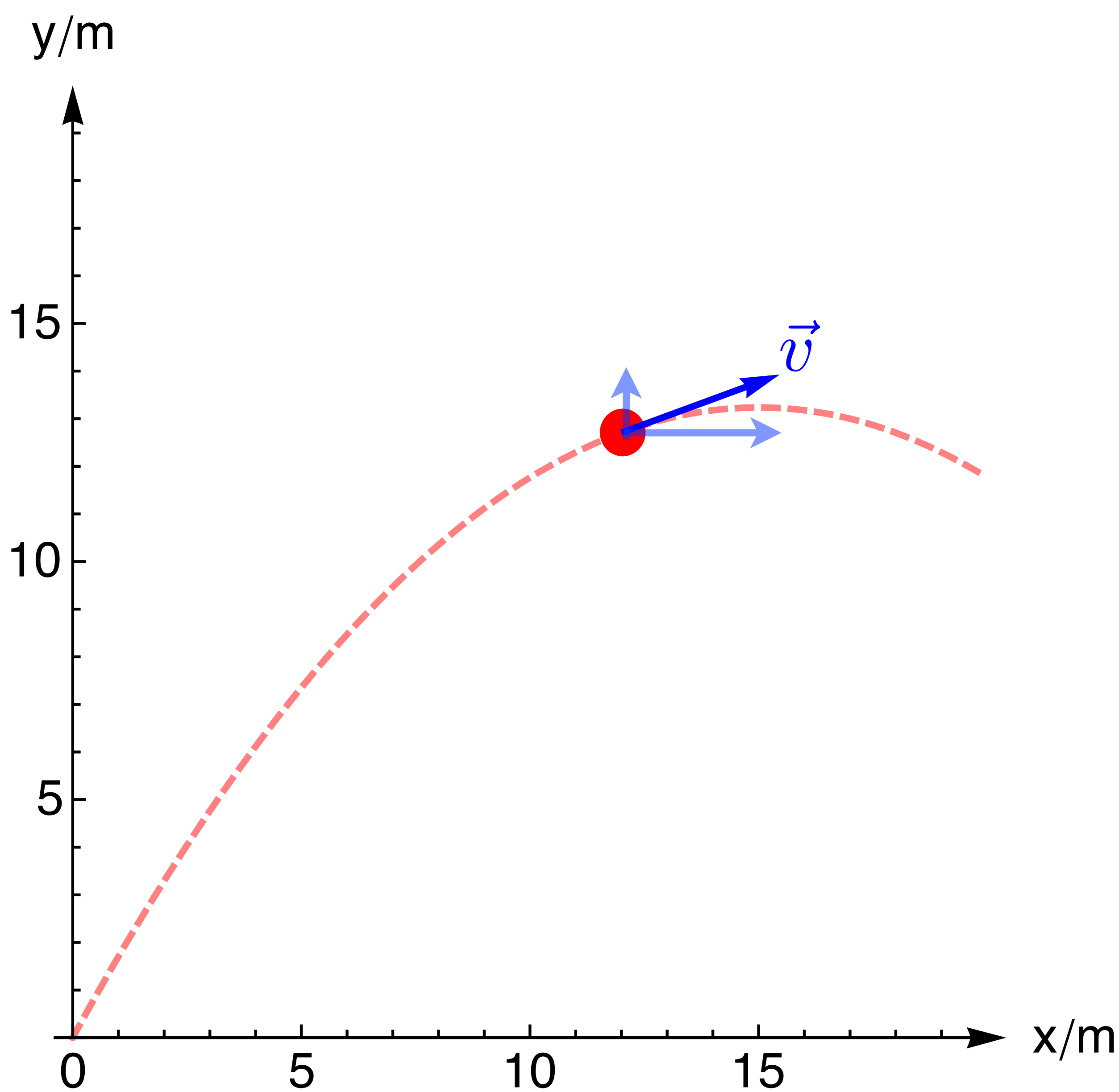


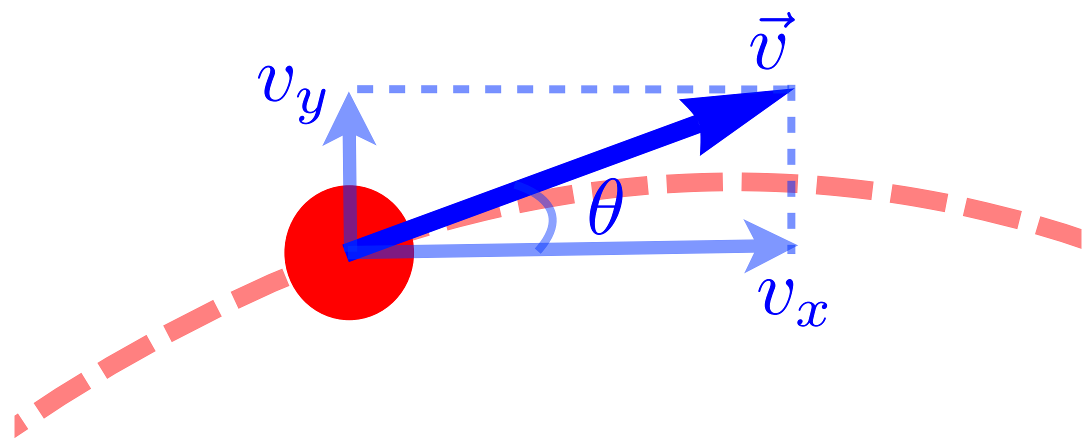
➜ **instantaneous velocity vector** points along the tangent of the *x*-*y* path



instantaneous velocity

➜ the **instantaneous velocity vector** can be expressed via components



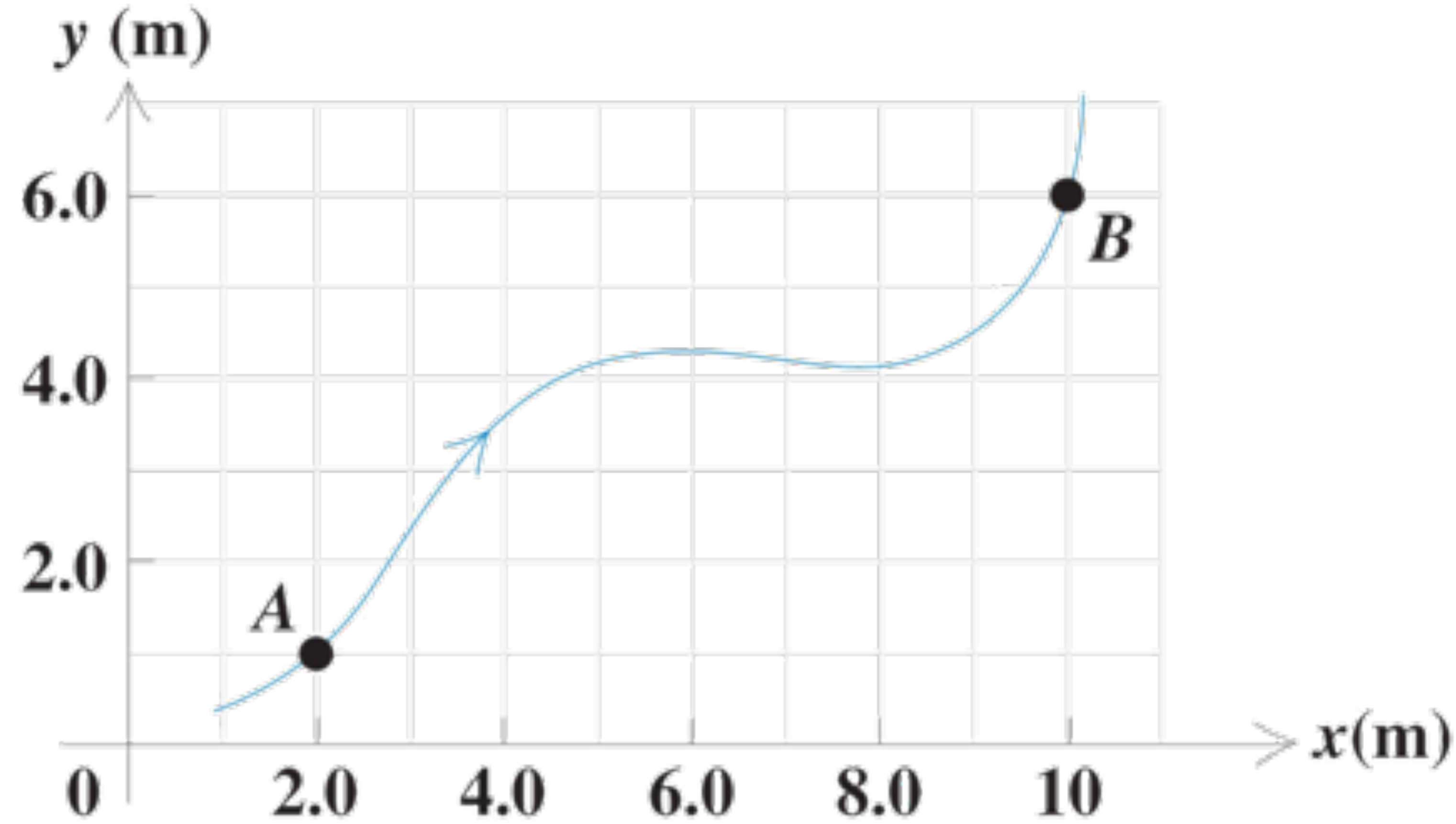


$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1} \frac{v_y}{v_x}$$

dragonfly

A dragonfly follows the path shown, moving from point A to point B in 1.50 s.

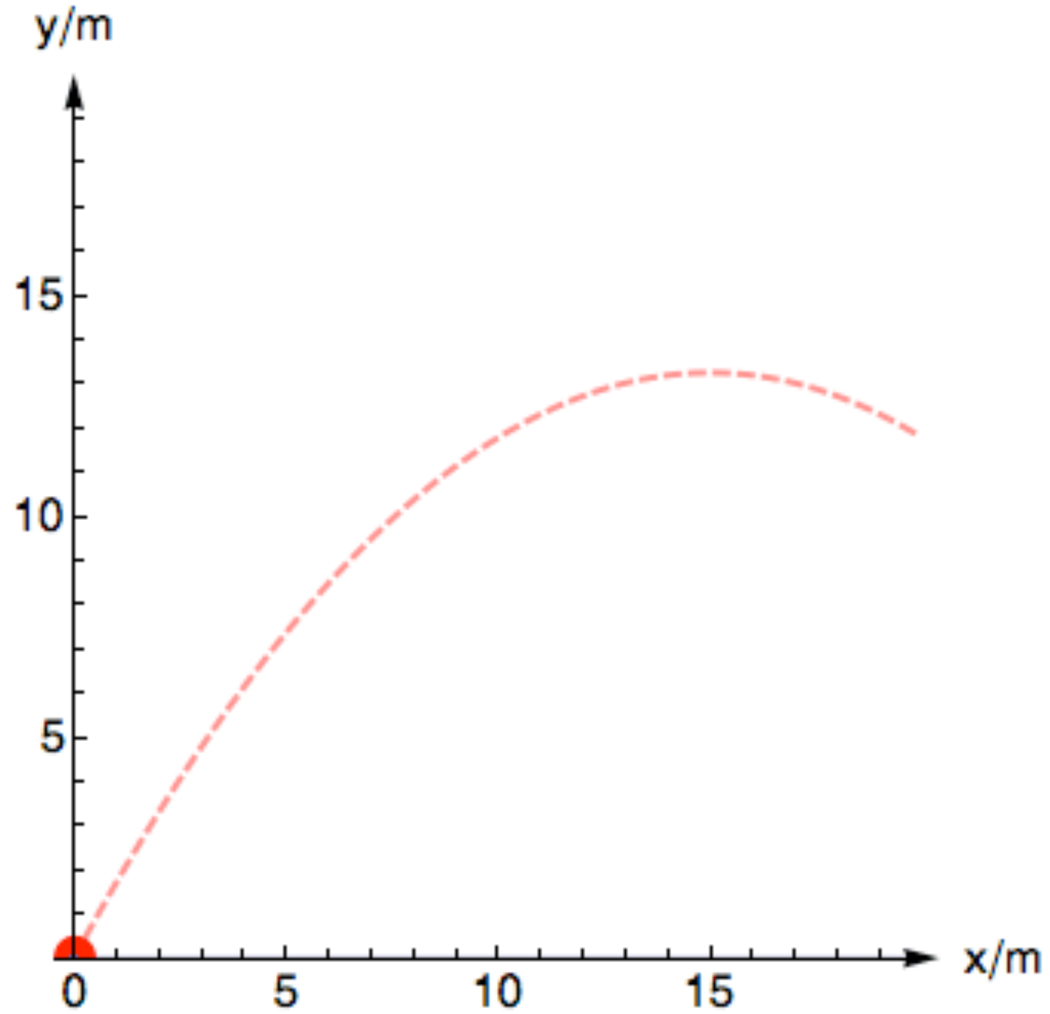


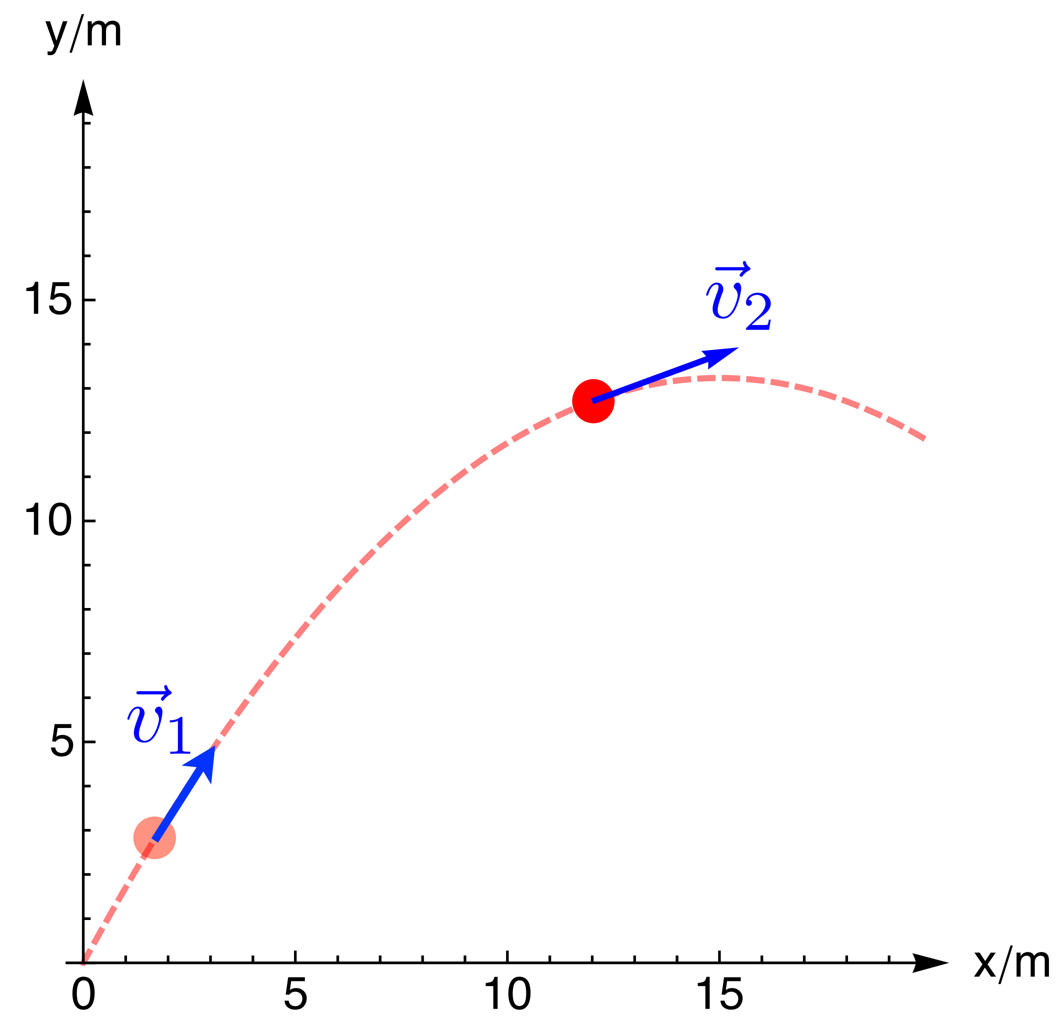
Find the *x* & *y* components of the average velocity between A & B
Find the magnitude and direction of the average velocity between A & B
Indicate the direction of the instantaneous velocity at A and B on the diagram

acceleration in a plane

➜ the definitions of average and instantaneous acceleration are the obvious extensions

$$\vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t} \qquad\qquad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$





acceleration in a plane

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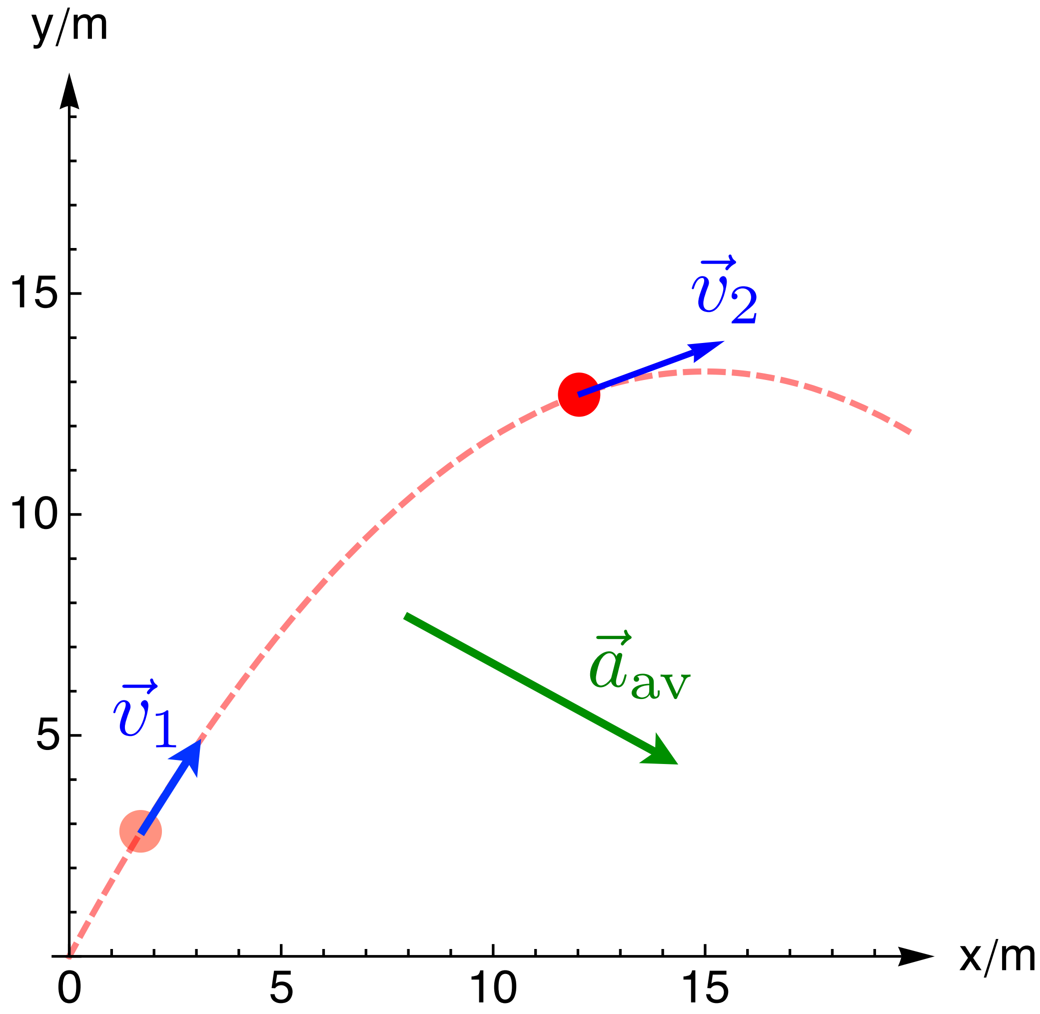
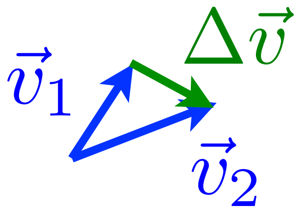
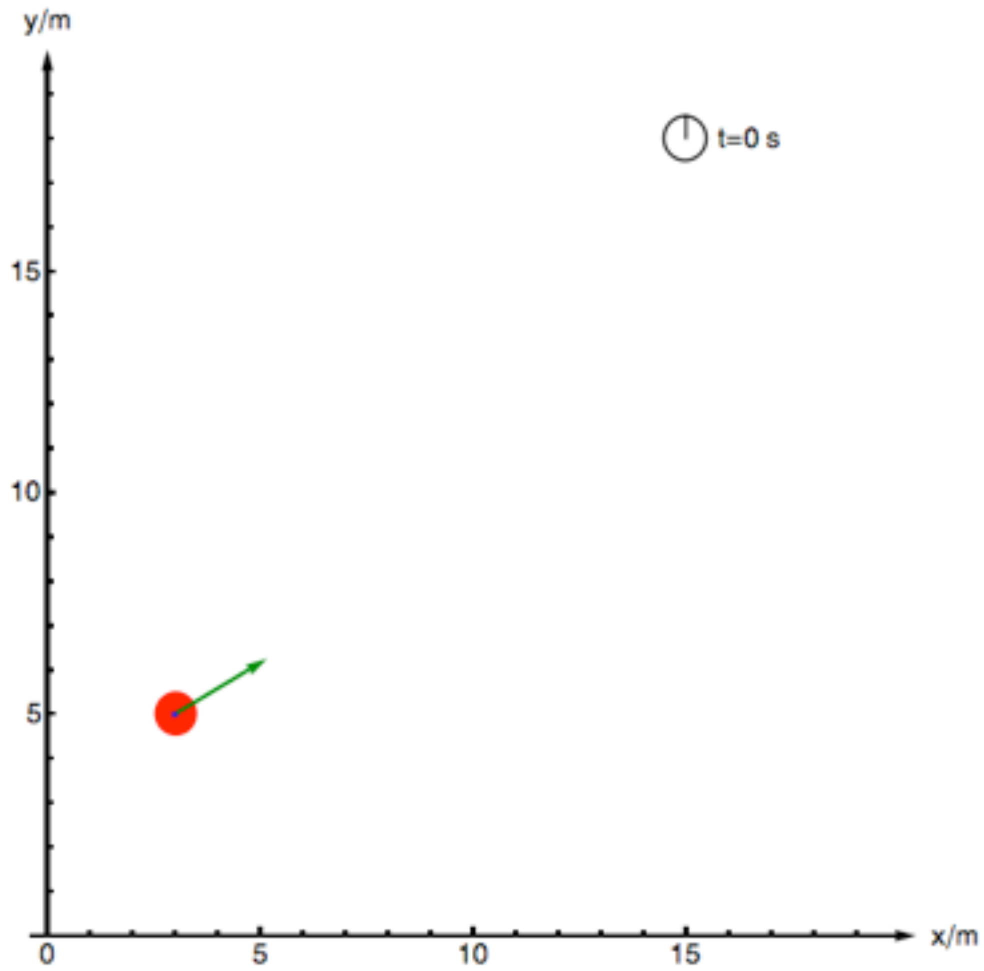


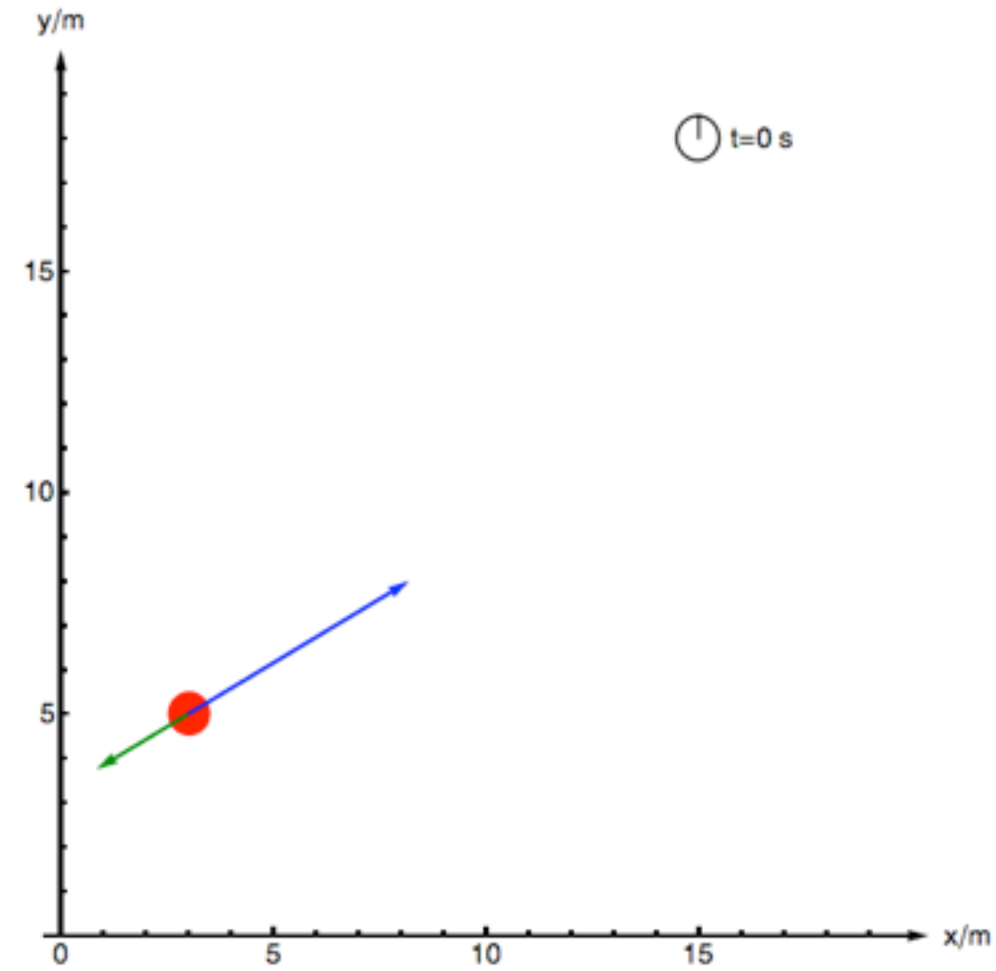
$$\vec{a}_{\rm av} = \frac{\Delta\vec{v}}{\Delta t}$$



acceleration in a plane

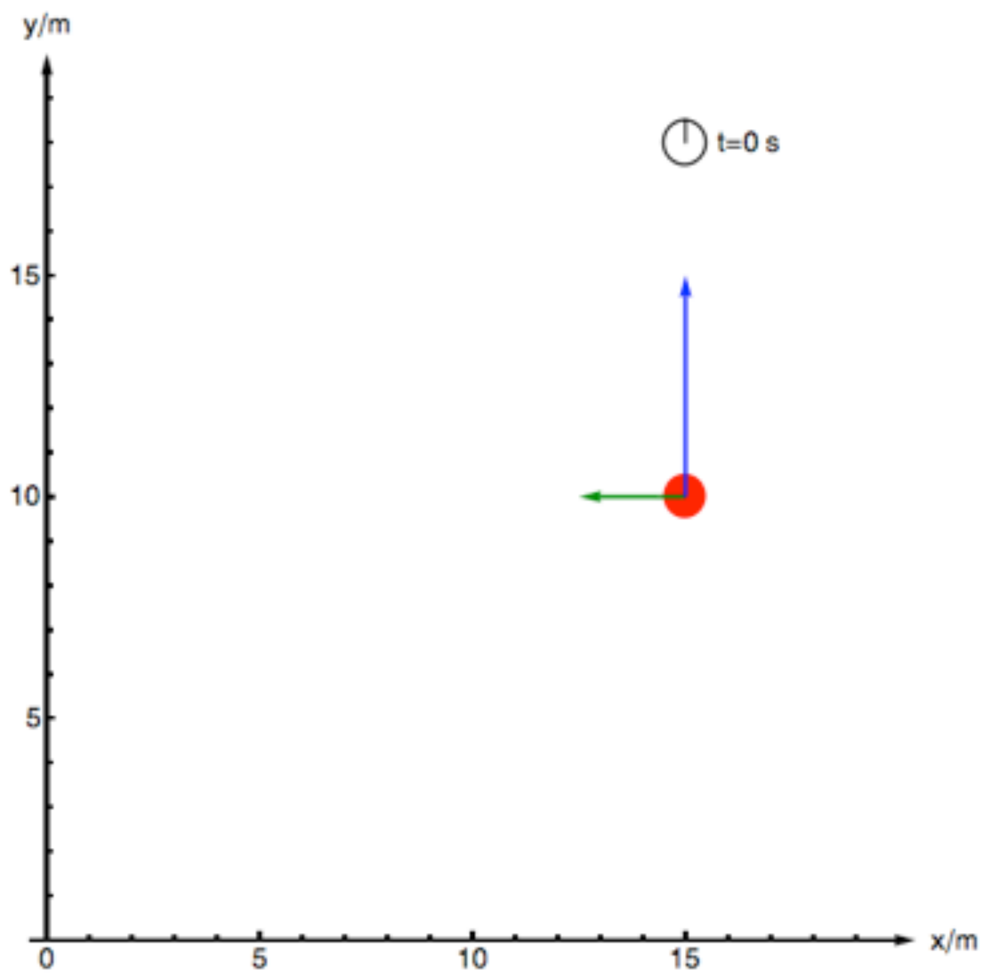
➜ acceleration parallel to the velocity changes the magnitude of velocity
but not the direction





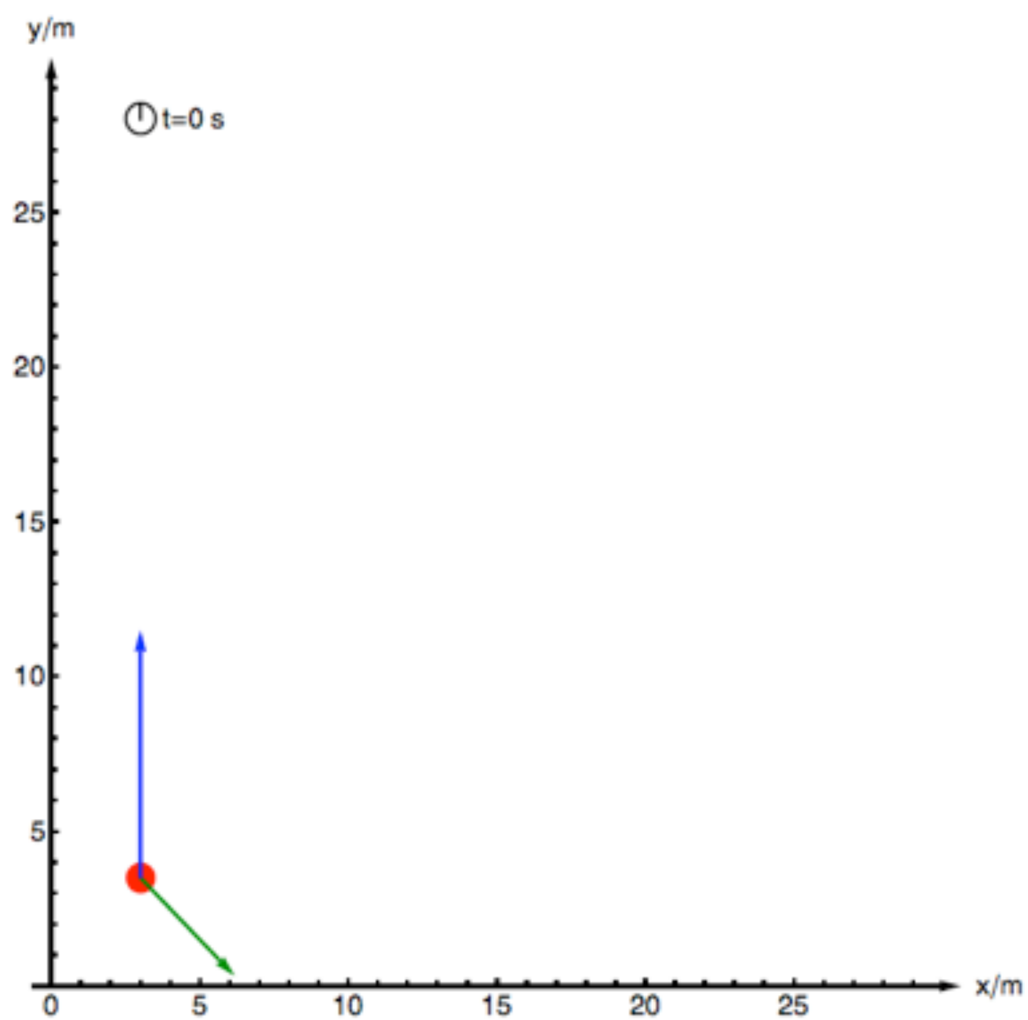
acceleration in a plane

➜ acceleration perpendicular to the velocity changes the direction of velocity but not the magnitude



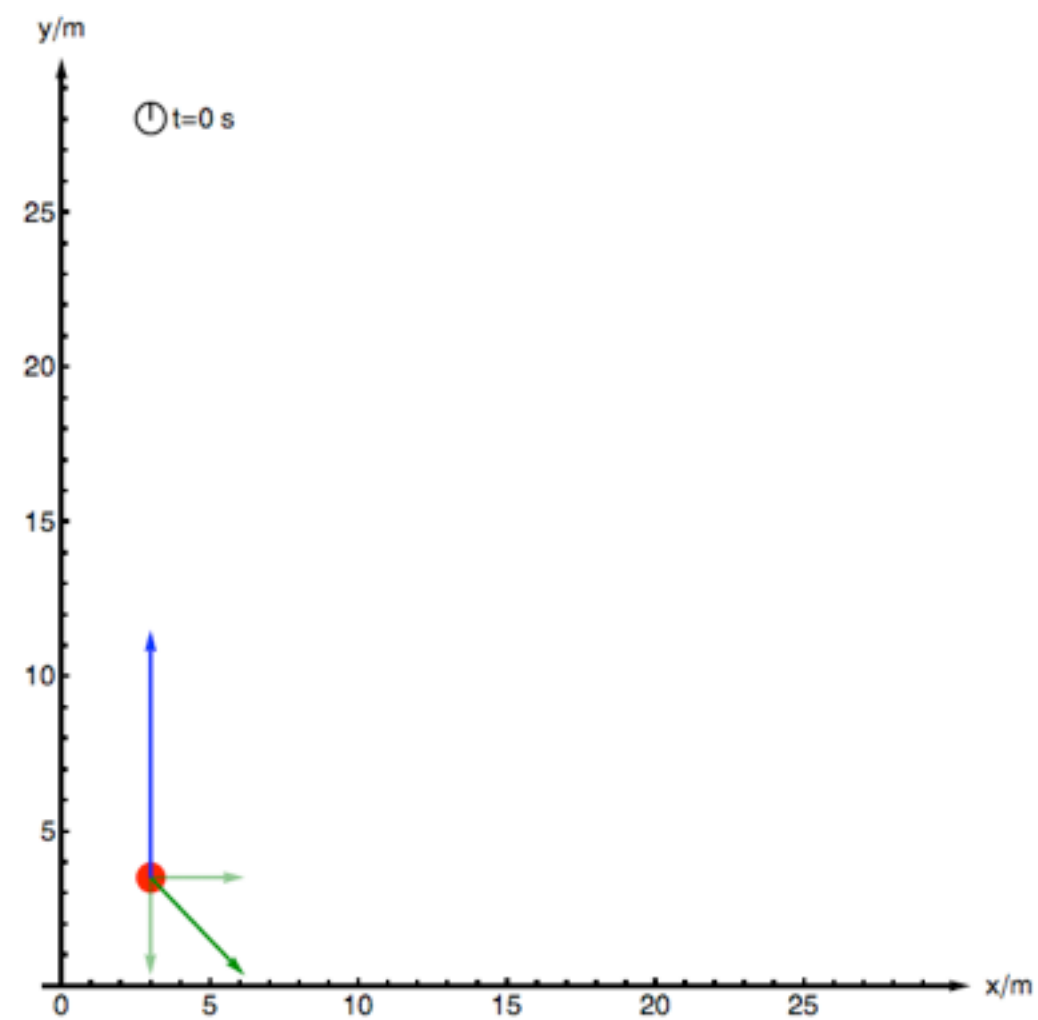
acceleration in a plane

➜ a general acceleration vector can change both the magnitude and direction of the velocity



acceleration in a plane

➜ a general acceleration vector can change both the magnitude and direction of the velocity



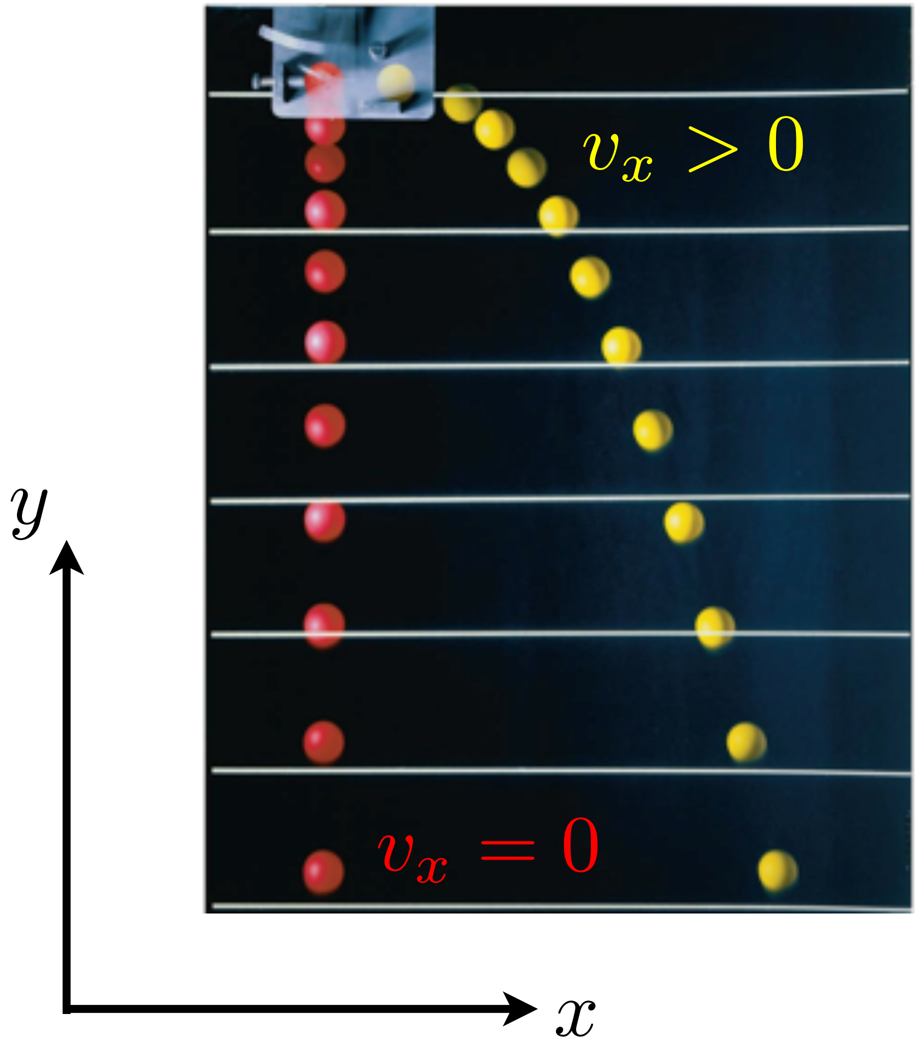
➜ but we can understand the effect by breaking into components parallel and perpendicular to the velocity

projectile motion

we'd like to to be able to describe why
the ball takes this path

projectile motion - independence of horiz. & vertical motion

➜ another experimental observation is useful



notice that the vertical motion is identical

not affected by the horizontal motion

great news! we can consider *x* & *y* separately

➜ the *y*-motion is just free-fall

$$a_y = -9.80\,\mathrm{m/s^2}$$

➜ the *x*-motion a constant velocity

$$a_x = 0\,\mathrm{m/s^2}$$

projectile motion

➜ our theory of projectile motion is that

$$a_y = -9.80\,\mathrm{m/s^2}$$
$$a_x = 0\,\mathrm{m/s^2}$$

➜ and the constant acceleration equations apply to *x* & *y* separately

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
$$v_x = v_{0x} + a_x t$$
$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$
$$v_y = v_{0y} + a_y t$$
$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

projectile motion

➜ our theory of projection motion is that

$$a_y = -9.80\,\mathrm{m/s^2} = -g$$
$$a_x = 0\,\mathrm{m/s^2}$$

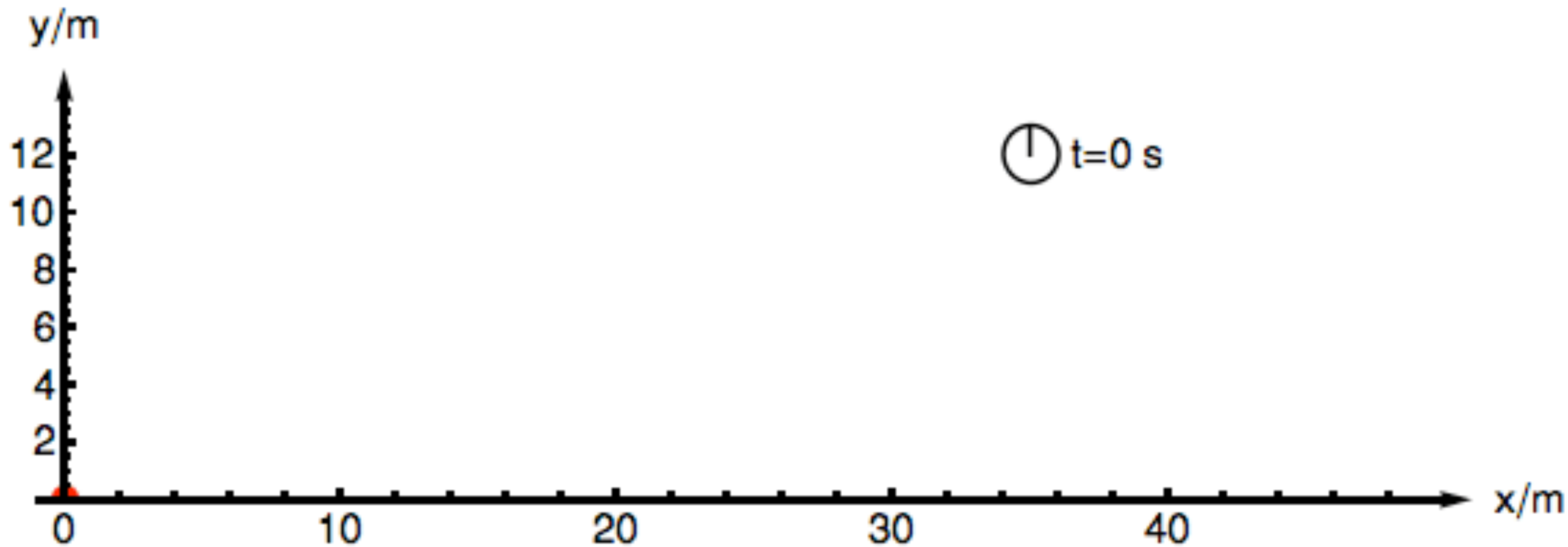
➜ and the constant acceleration equations apply to *x* & *y* separately

$$x = x_0 + v_{0x}t$$
$$v_x = v_{0x}$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$v_y = v_{0y} - gt$$
$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

projectile motion

➜ e.g. a cannonball fired from ground level (y=0m) at x=0m with an initial speed of 20.0 m/s at an angle of 45° to the horizontal

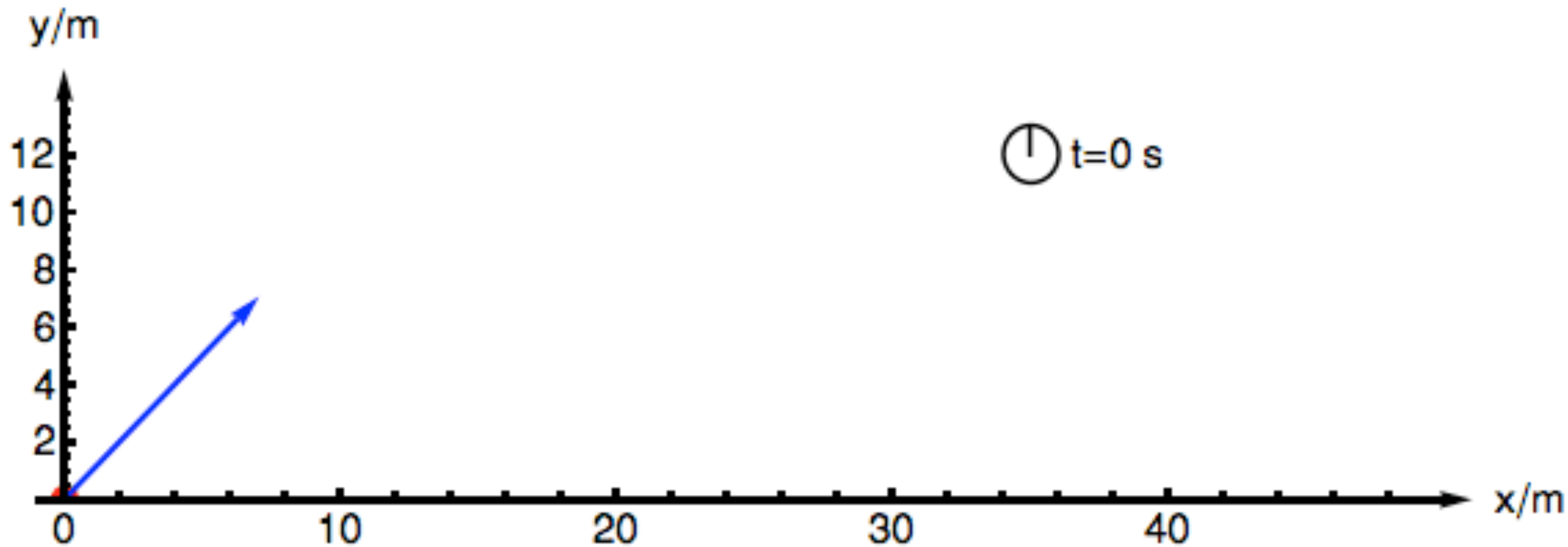


note that it's very similar to the kicked football

projectile motion

➜ e.g. a cannonball fired from ground level (y=0m) at x=0m with an initial speed of 20.0 m/s at an angle of 45° to the horizontal

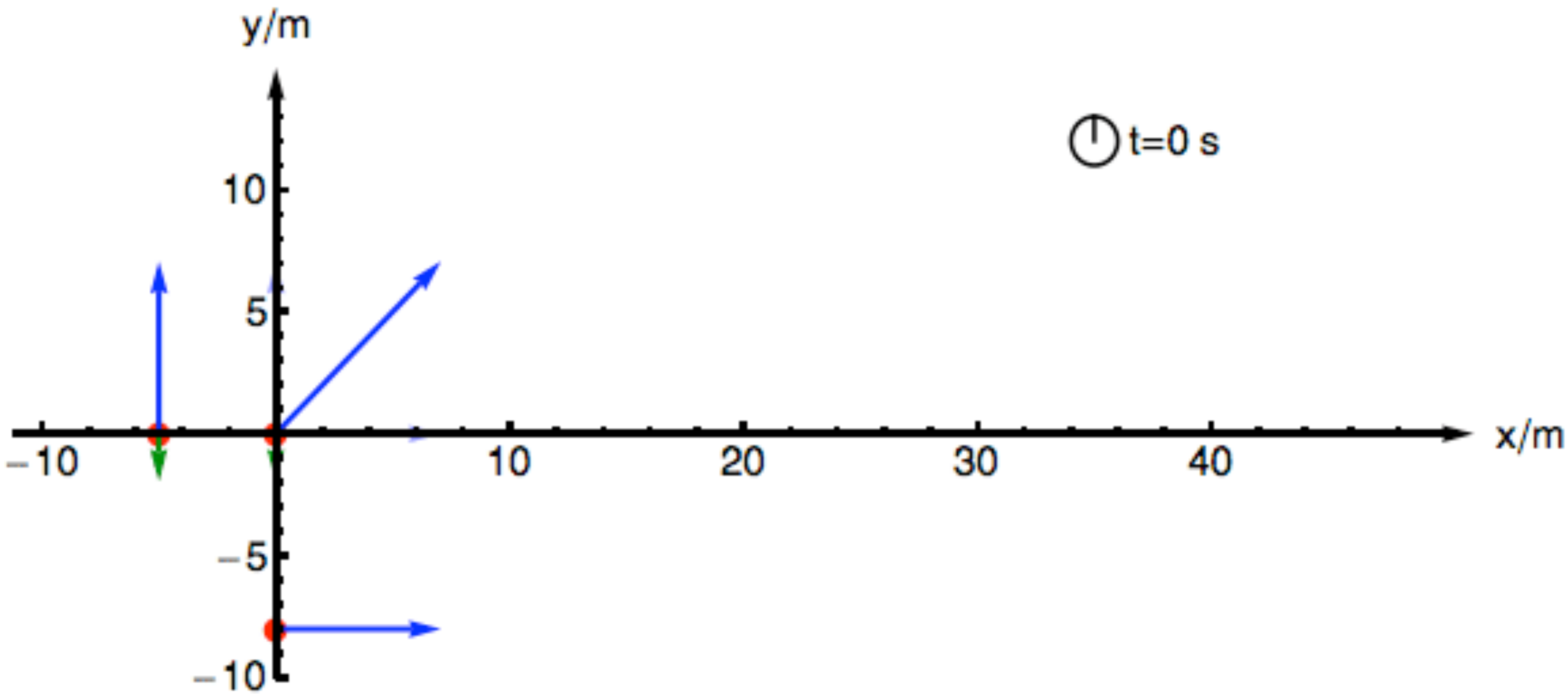
show the velocity vector & the acceleration



projectile motion

➜ e.g. a cannonball fired from ground level (y=0m) at x=0m with an initial speed of 20.0 m/s at an angle of 45° to the horizontal

show the velocity vector components



projectile motion

➜ we can analyse this motion using our equations

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

➜ prove that the shape of the path is a parabola

eliminate *t* from the equations to give *y* as a function of *x*

(choose x_0=0, y_0=0 for simplicity)

$$t = \frac{x}{v_{0x}} \qquad y = \left(\tfrac{v_{0y}}{v_{0x}}\right)x - \left(\frac{g}{2v_{0x}^2}\right)x^2$$

projectile motion

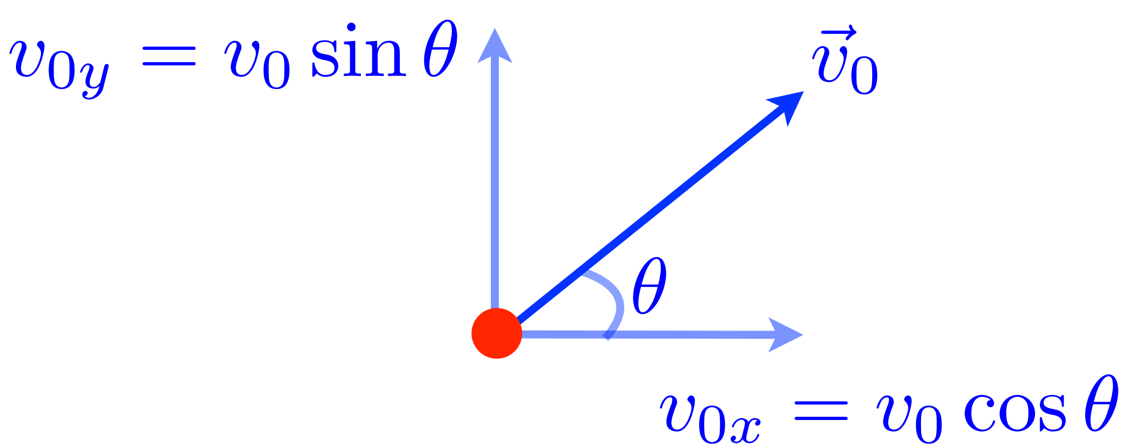
➜ we can analyse this motion using our equations

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

➜ at *t*=0

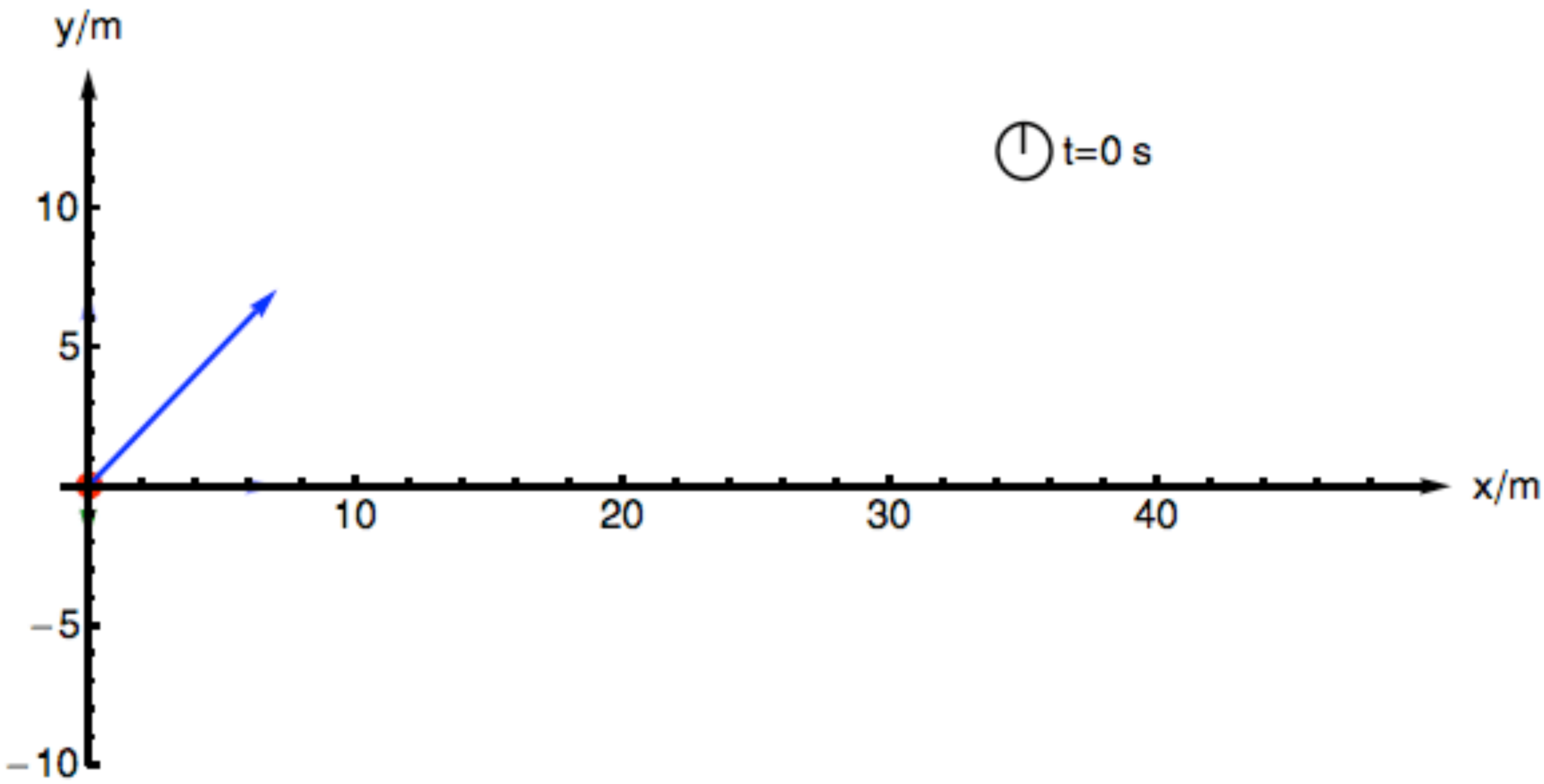


projectile motion

➜ e.g. a cannonball fired from ground level (y=0m) at x=0m with an initial speed of 20.0 m/s at an angle of 45° to the horizontal

Find the time when the ball reaches the highest point of its flight and that height, h .

Find the horizontal range, R, the horizontal distance travelled before hitting the ground



projectile motion

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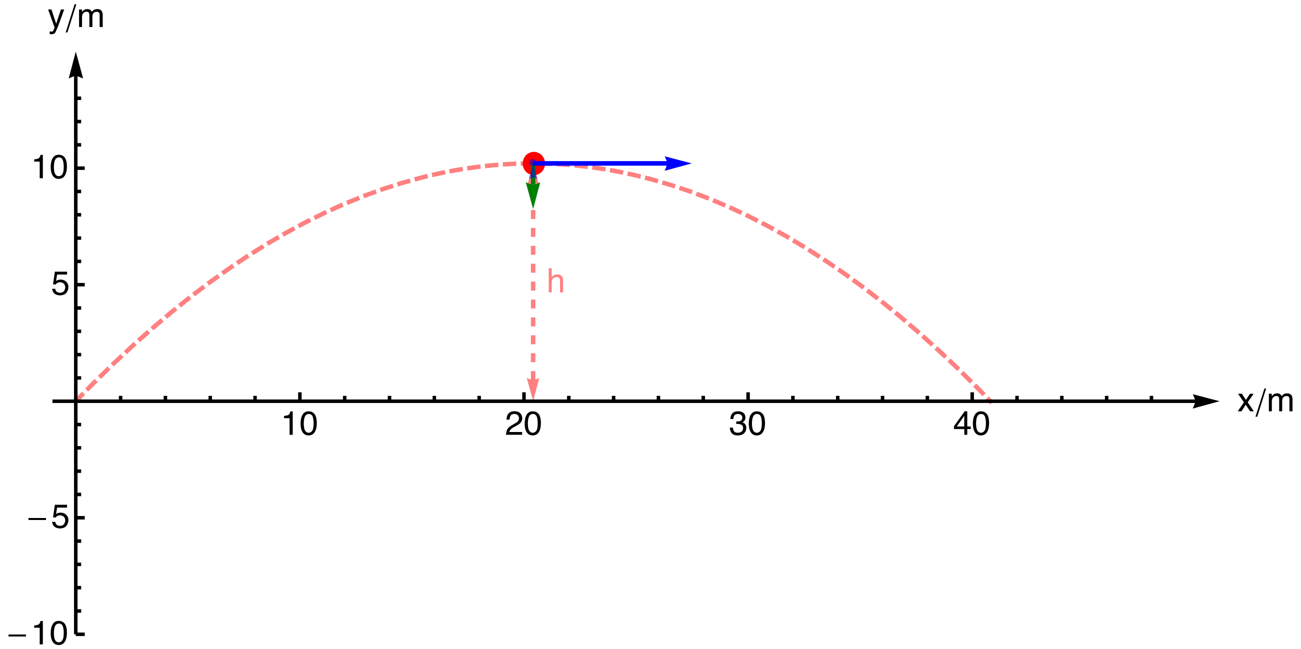

at the highest point $v_y = 0$

$$v_y = v_{0y} - gt$$

$$0 = v_0 \sin\theta - gt$$

$$\implies t = \frac{v_0 \sin\theta}{g}$$

$$t = 1.44\,\mathrm{s}$$

projectile motion

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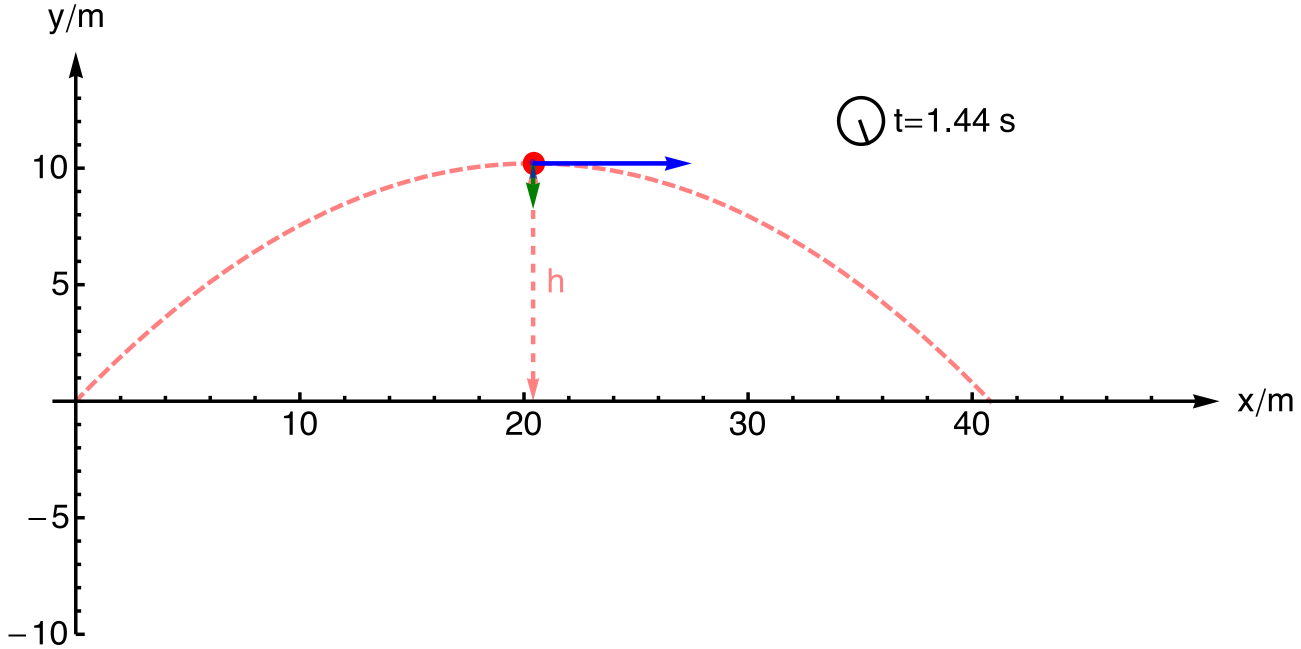
Find the time when the ball reaches the highest point of its flight and that height, h .



at the highest point $v_y = 0$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

$$0 = v_0^2 \sin^2\theta - 2gh$$

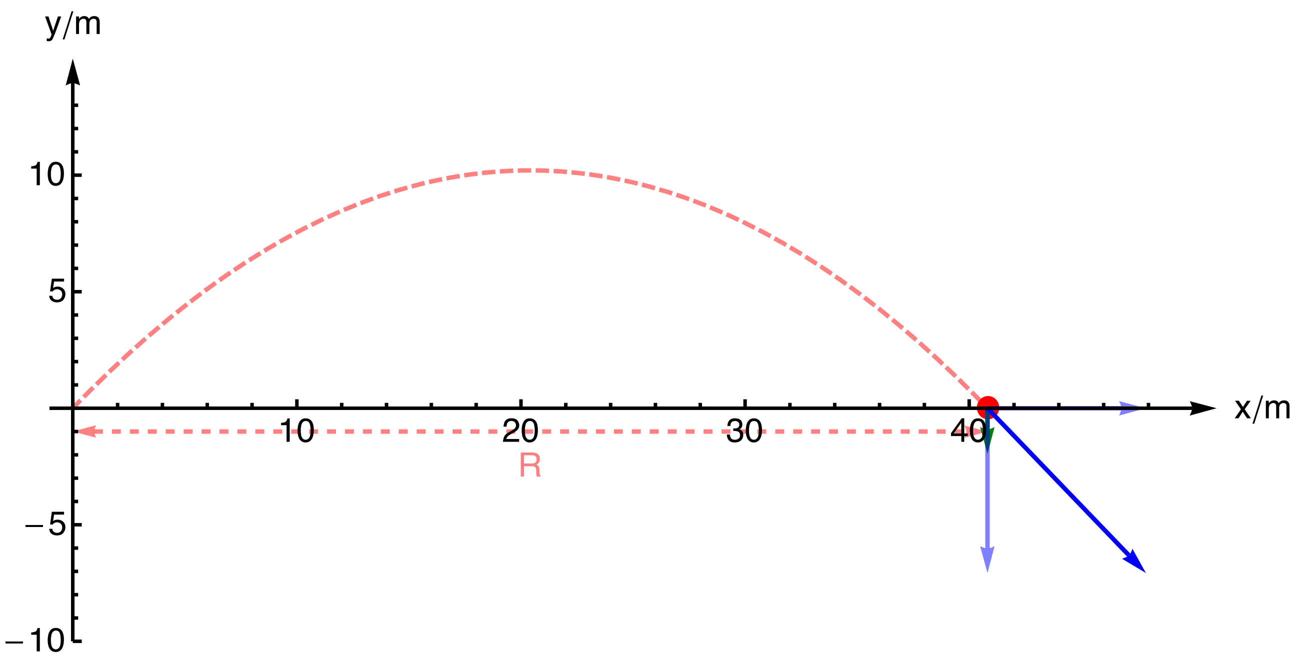
$$\implies h = \frac{v_0^2 \sin^2\theta}{2g}$$

$$\underline{h = 10.2\,\mathrm{m}}$$

projectile motion

➜ e.g. a cannonball fired from ground level (y=0m) at x=0m with an initial speed of 20.0 m/s at an angle of 45° to the horizontal

Find the horizontal range, R, the horizontal distance travelled before hitting the ground



ball hits the ground when $y = 0$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$0 = v_0 \sin\theta t - \tfrac{1}{2}gt^2$$

$$0 = t\left(v_0 \sin\theta - \tfrac{1}{2}gt\right)$$

$$\implies t = \frac{2v_0 \sin\theta}{g}$$

hmmm, twice the time to reach the apex ?

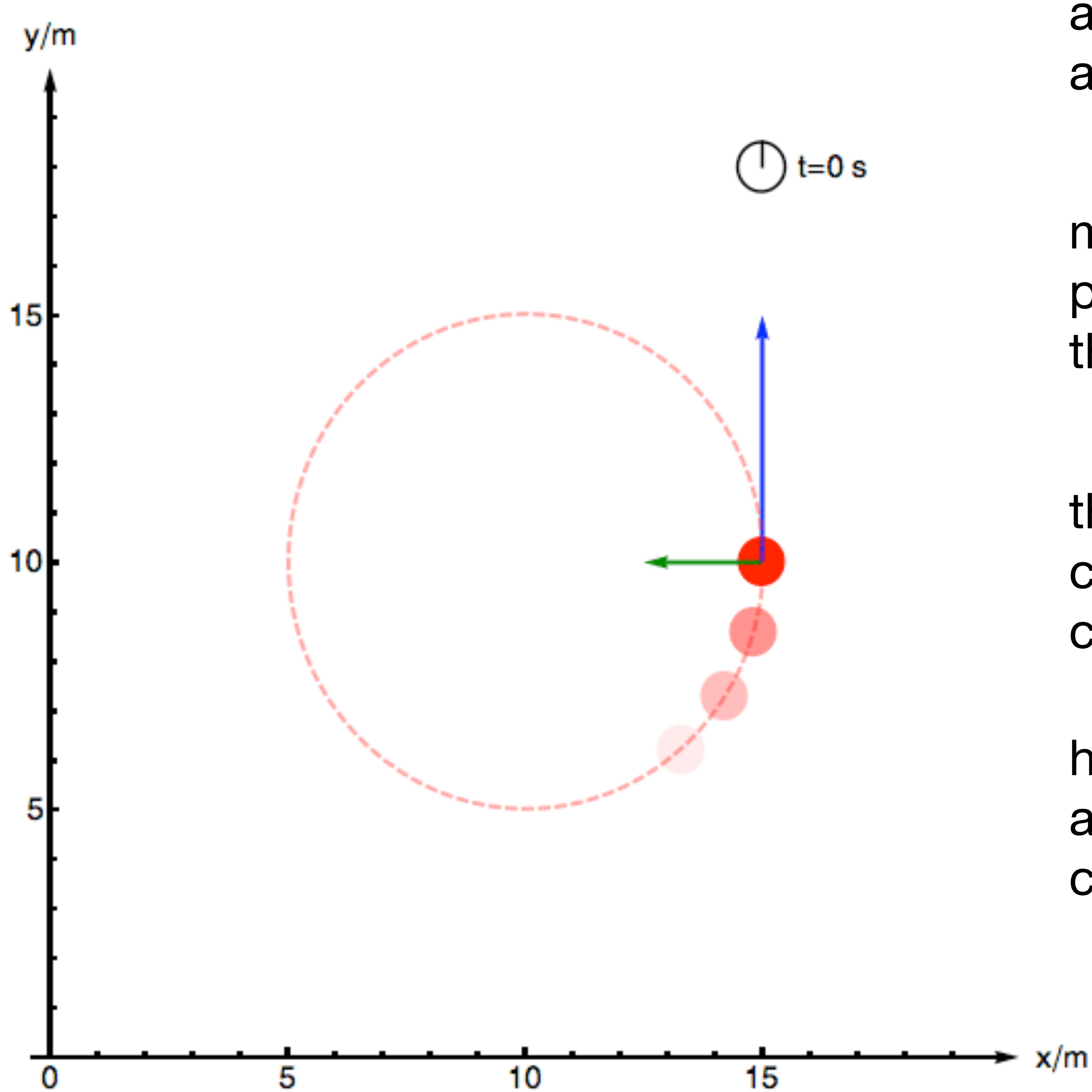
$$x = x_0 + v_{0x}t$$

$$R = v_0 \cos\theta \left(\frac{2v_0 \sin\theta}{g}\right)$$

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

$$R = 40.8\,\mathrm{m}$$

uniform circular motion



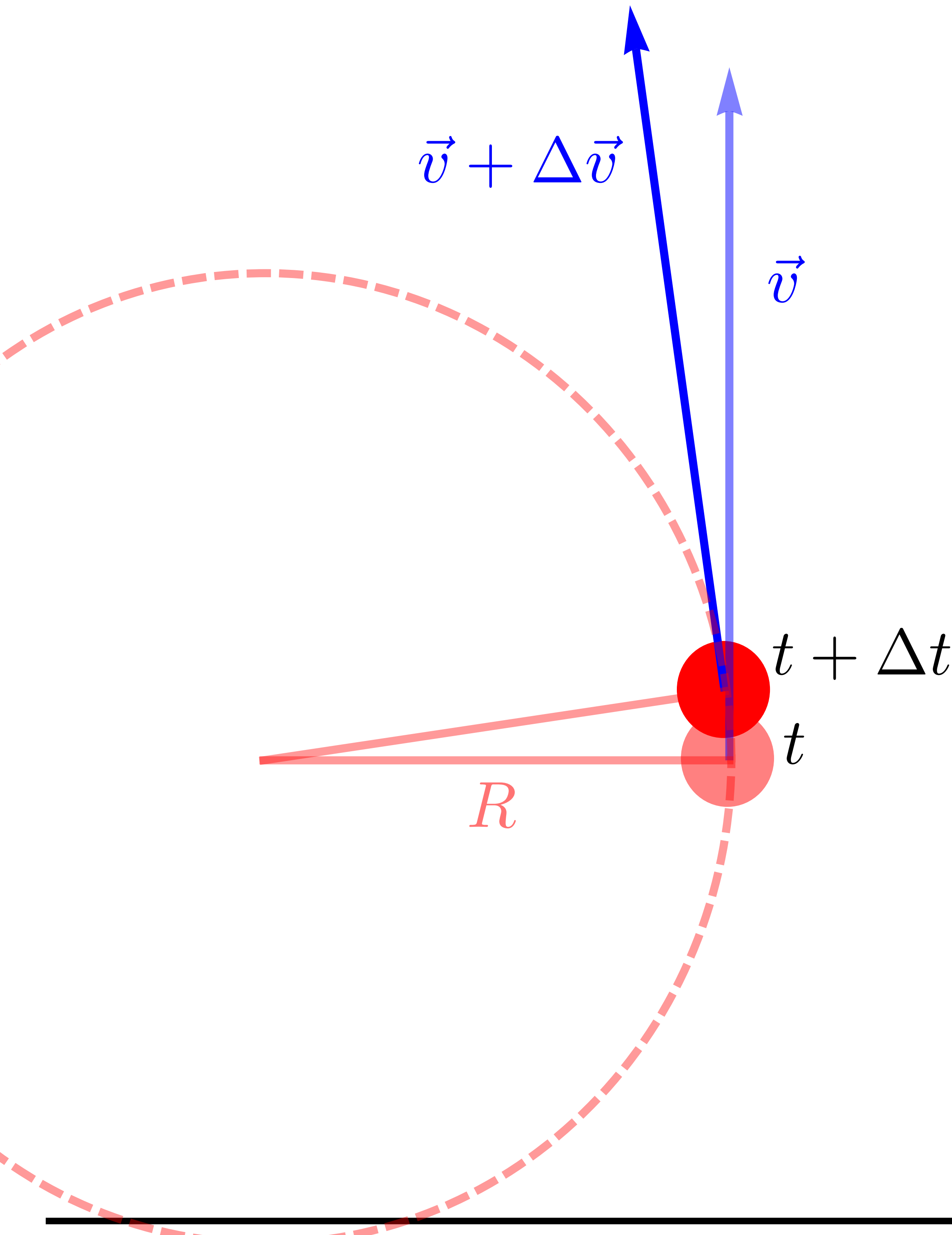
an object moving around a circle at a constant rate

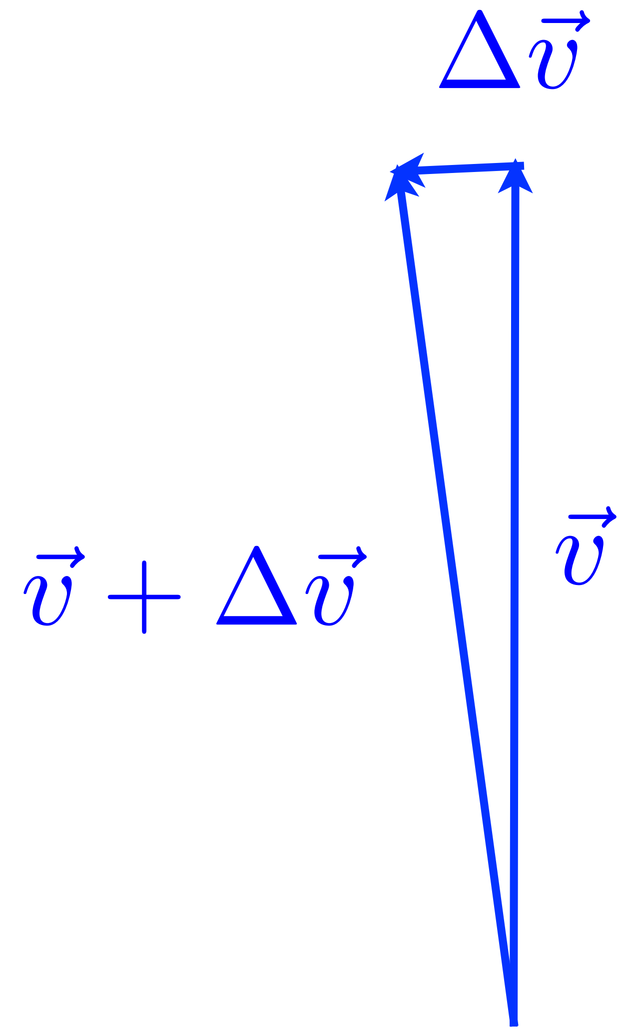
must have an acceleration always perpendicular to the velocity (else the speed would change)

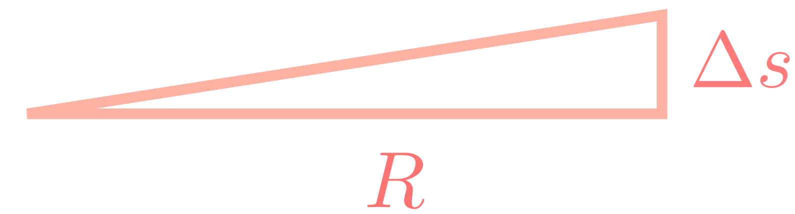
the velocity is clearly tangent to the circle (or it would move off the circle)

hence the acceleration points always toward the center of the circle - “centripetal acceleration”

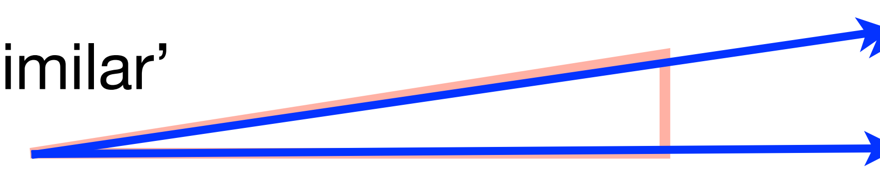
uniform circular motion







the triangles are 'similar'



$$\frac{|\Delta\vec{v}|}{|\vec{v}|} = \frac{\Delta s}{R}$$

$$|\vec{a}| = \frac{|\Delta v|}{\Delta t} = \frac{v}{R}\frac{\Delta s}{\Delta t} = \frac{v^2}{R}$$