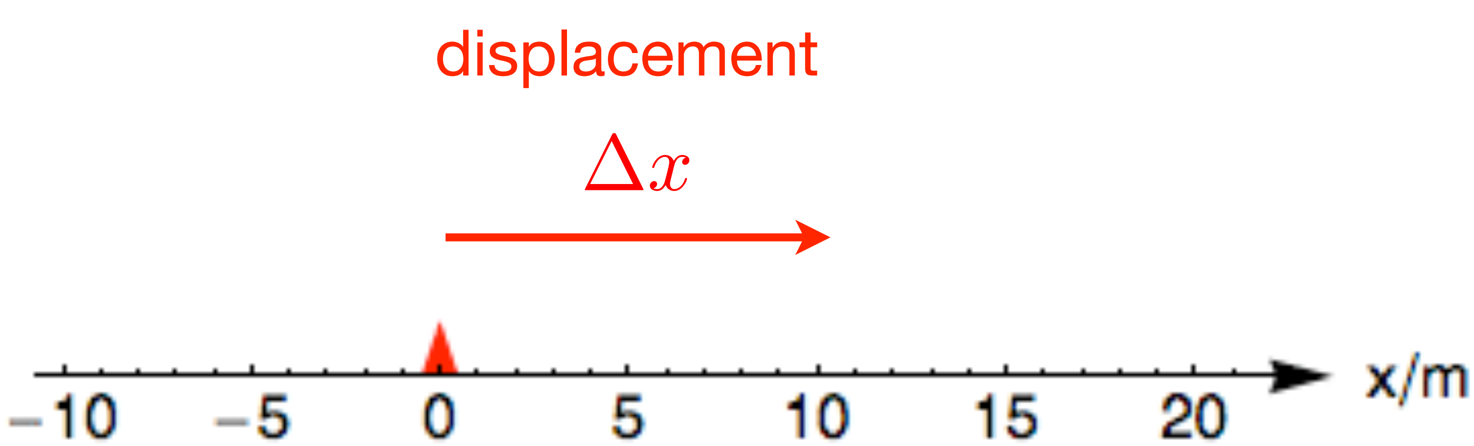
motion along a straight line

“displacement” & “distance”

➜ we need to be a bit pedantic here:

➜ ‘distance’ = total ground covered while traveling, e.g. odometer reading

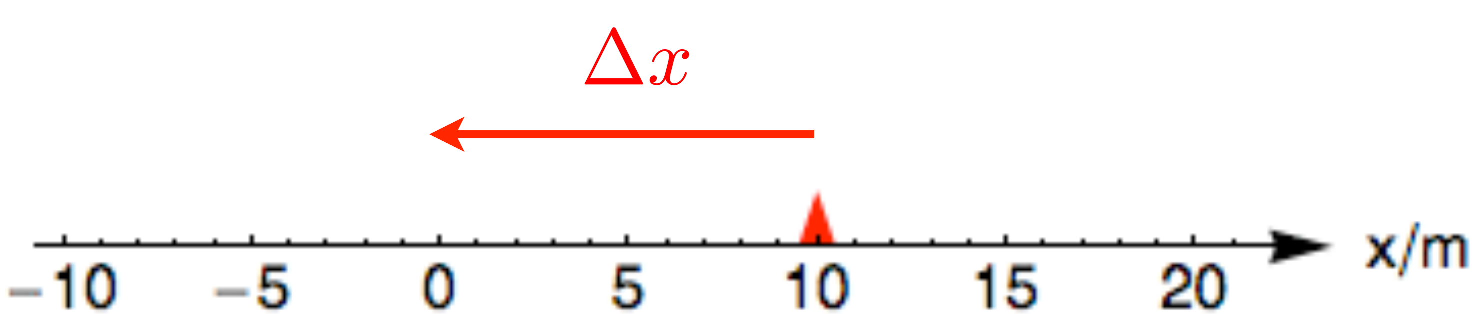
➜ ‘displacement’ = vector from where you started to where you end up



$$\begin{aligned}\Delta x &= x_f - x_i\\ &= 10\,\mathrm{m} - 0\,\mathrm{m}\\ &= +10\,\mathrm{m}\end{aligned}$$

sign indicates the direction

& distance = 10 m



$$\begin{aligned}\Delta x &= x_f - x_i\\ &= 0\,\mathrm{m} - 10\,\mathrm{m}\\ &= -10\,\mathrm{m}\end{aligned}$$

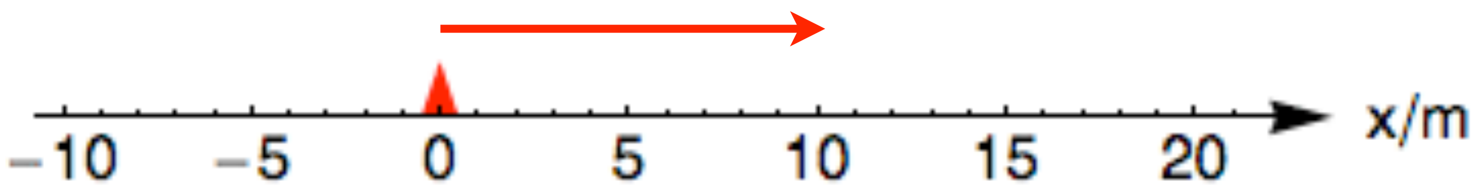
but distance = 10 m

“displacement” & “distance”

➜ displacement and distance can be quite different

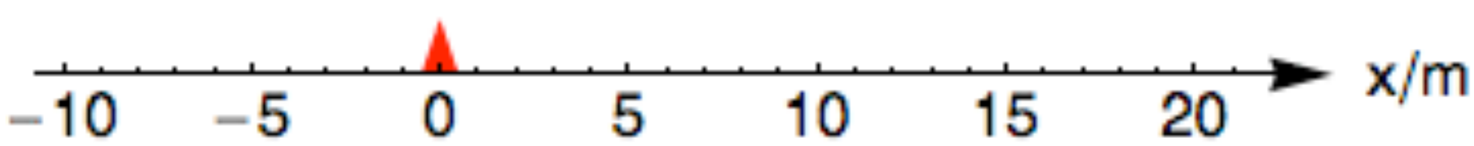
displacement

Δx



$$\Delta x = +10\,\mathrm{m}$$

distance = 8m + 6m + 8m = 22m



$$\begin{aligned}\Delta x &= (+8\,\mathrm{m}) + (-6\,\mathrm{m}) + (+8\,\mathrm{m}) \\ &= +10\,\mathrm{m}\end{aligned}$$

‘vector’ sum (sum with signs)

average velocity

➜ just the displacement divided by the time taken $$v_{\rm av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

e.g. driving a car along a long straight stretch of road

$$\Delta x = 272\,\text{m} - 52\,\text{m} = +220\,\text{m}$$

$$\Delta t = 30.17\,\text{s} - 26.05\,\text{s} = 4.12\,\text{s}$$

$$v_{\rm av} = \frac{\Delta x}{\Delta t} = \frac{+220\,\text{m}}{4.12\,\text{s}} = +53.4\,\text{m/s}$$

average velocity

➜ suppose we go the other way (but define the x-axis the same way)

$$\Delta x = 52\,\mathrm{m} - 272\,\mathrm{m} = -220\,\mathrm{m}$$

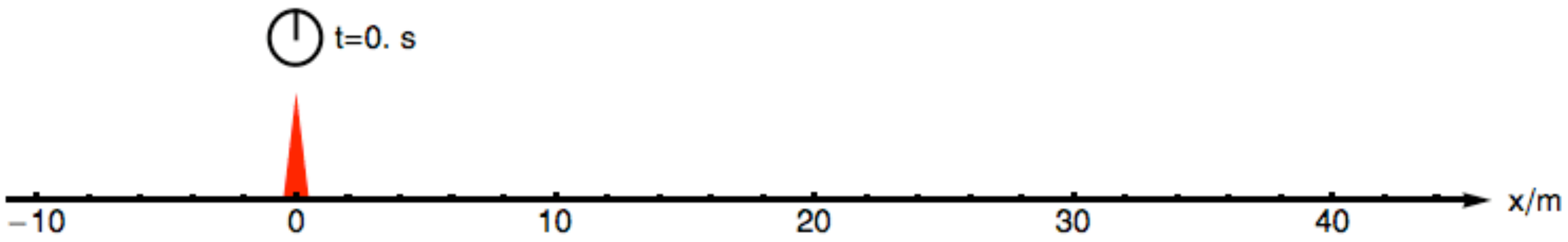
$$\Delta t = 102.05\,\mathrm{s} - 93.88\,\mathrm{s} = 8.17\,\mathrm{s}$$

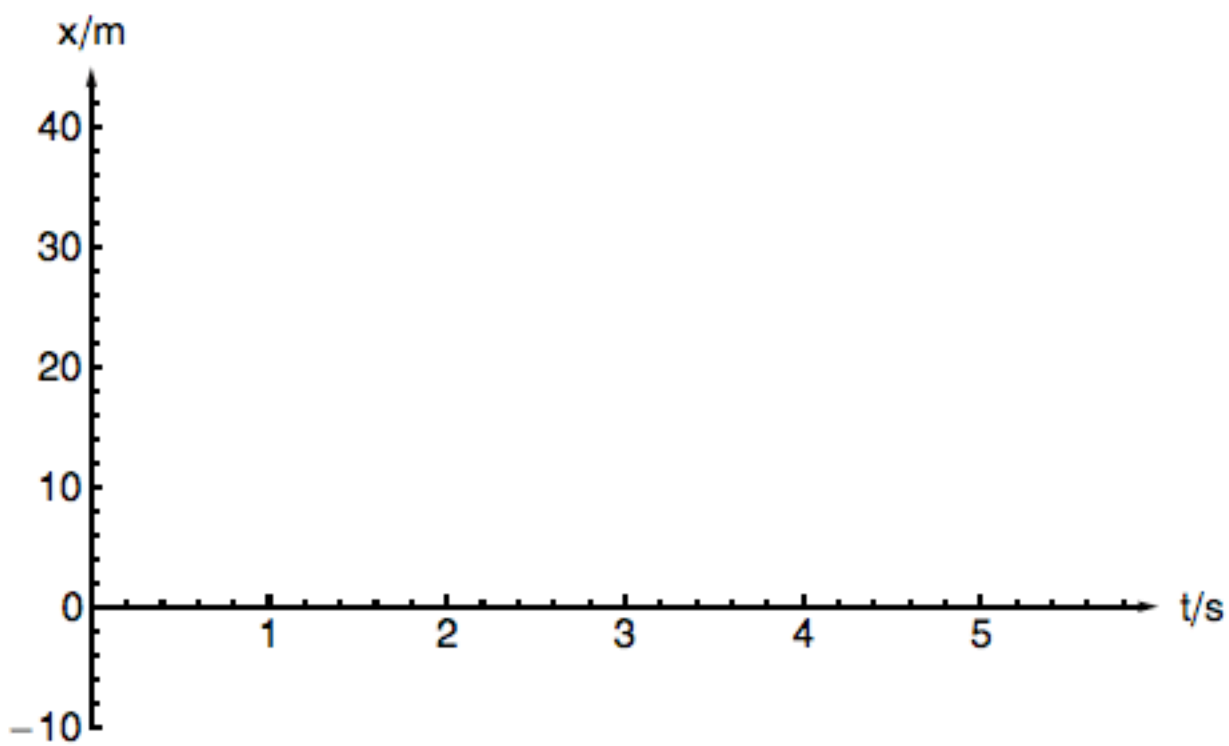
$$v_{\mathrm{av}} = \frac{\Delta x}{\Delta t} = \frac{-220\,\mathrm{m}}{8.17\,\mathrm{s}} = -26.9\,\mathrm{m/s}$$

negative sign means
opposite the axis direction

average & instantaneous velocity

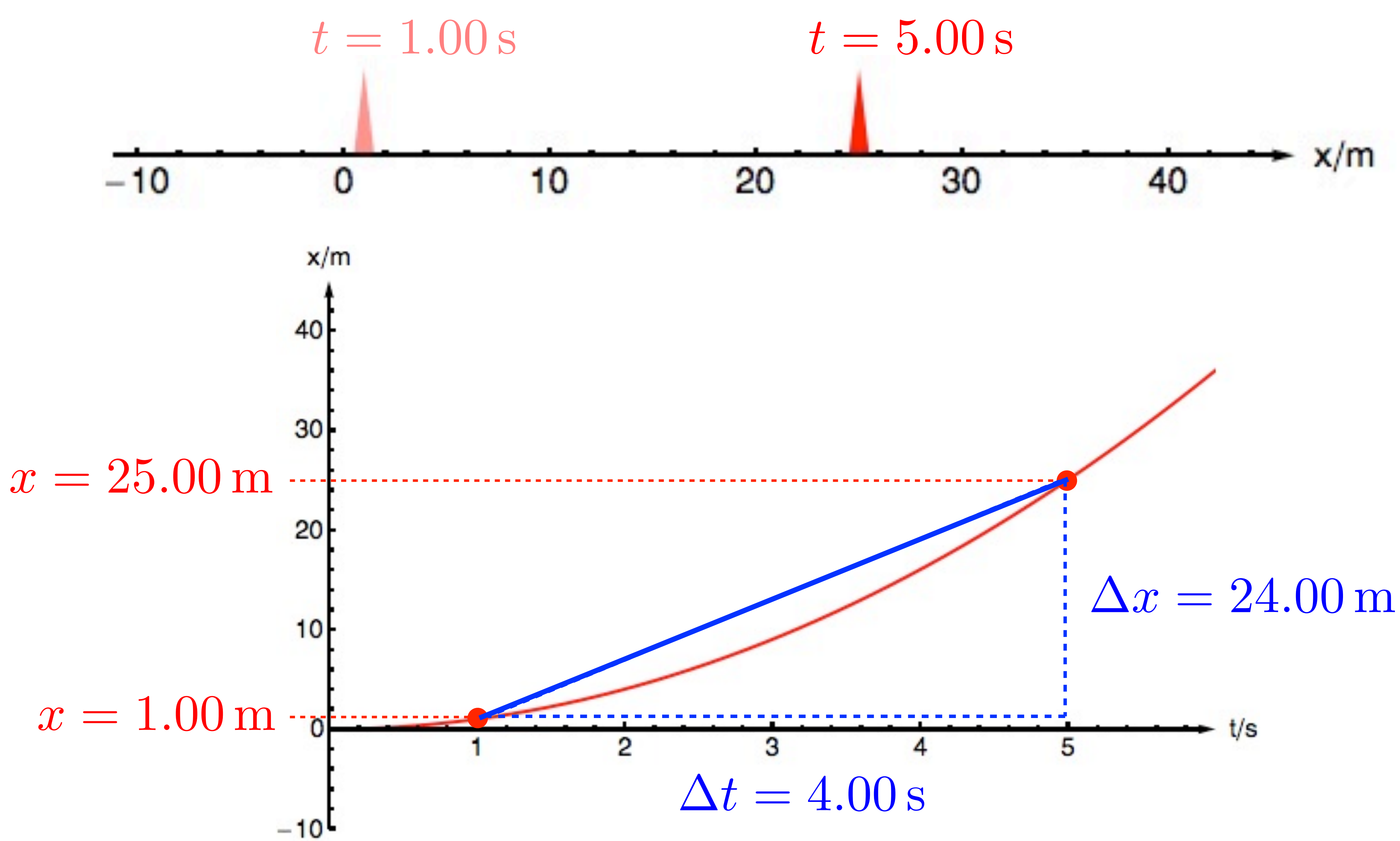
➜ let's make a plot of the position of an object as a function of time





average & instantaneous velocity

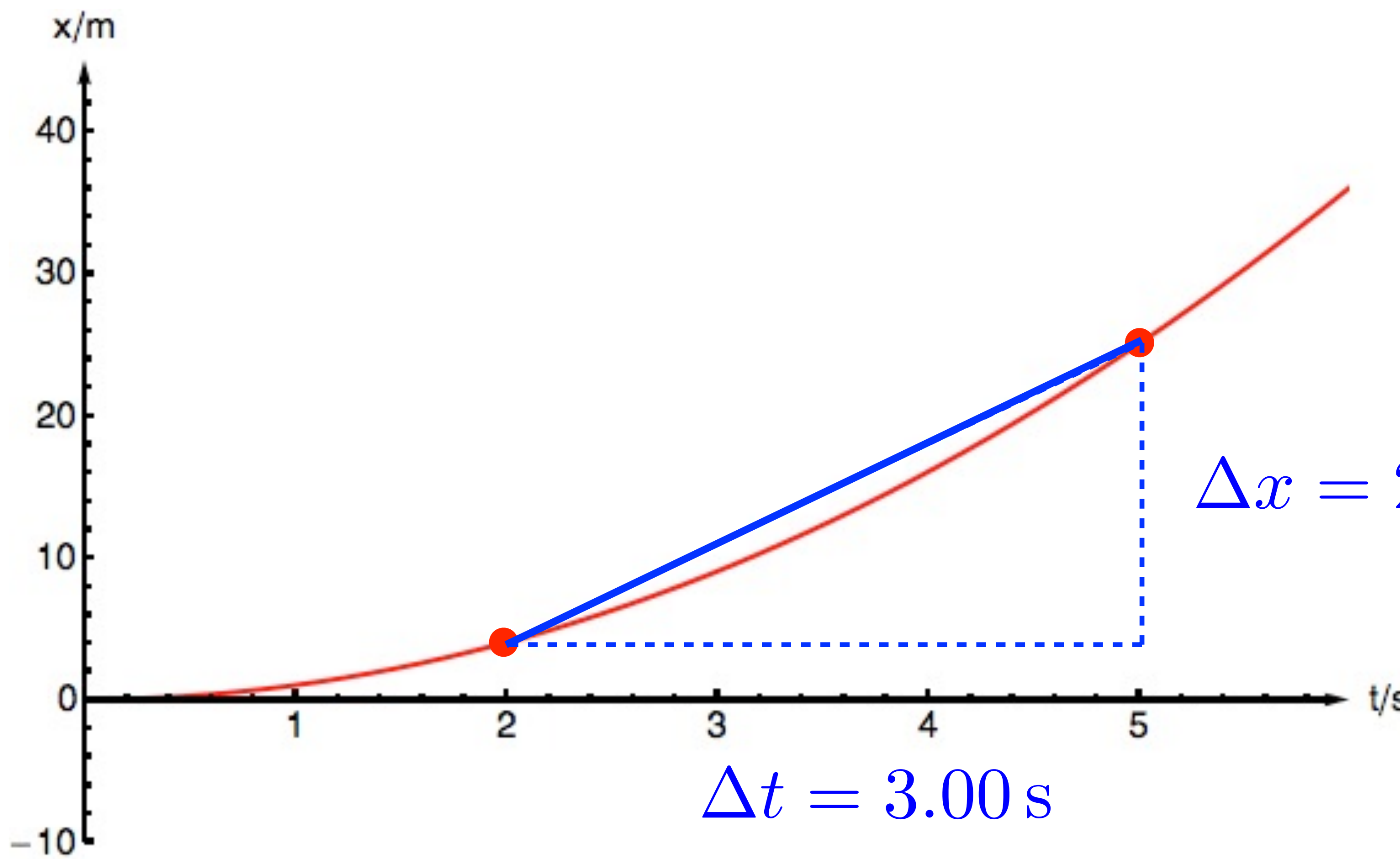
➜ compute the average velocity between *t* = 1.00 s & *t* = 5.00 s



$$v_{\mathrm{av}} = \frac{\Delta x}{\Delta t} = \frac{24.00\,\mathrm{m}}{4.00\,\mathrm{s}} = 6.00\,\mathrm{m/s}$$

average & instantaneous velocity

➜ compute the average velocity between t = 2.00 s & t = 5.00 s



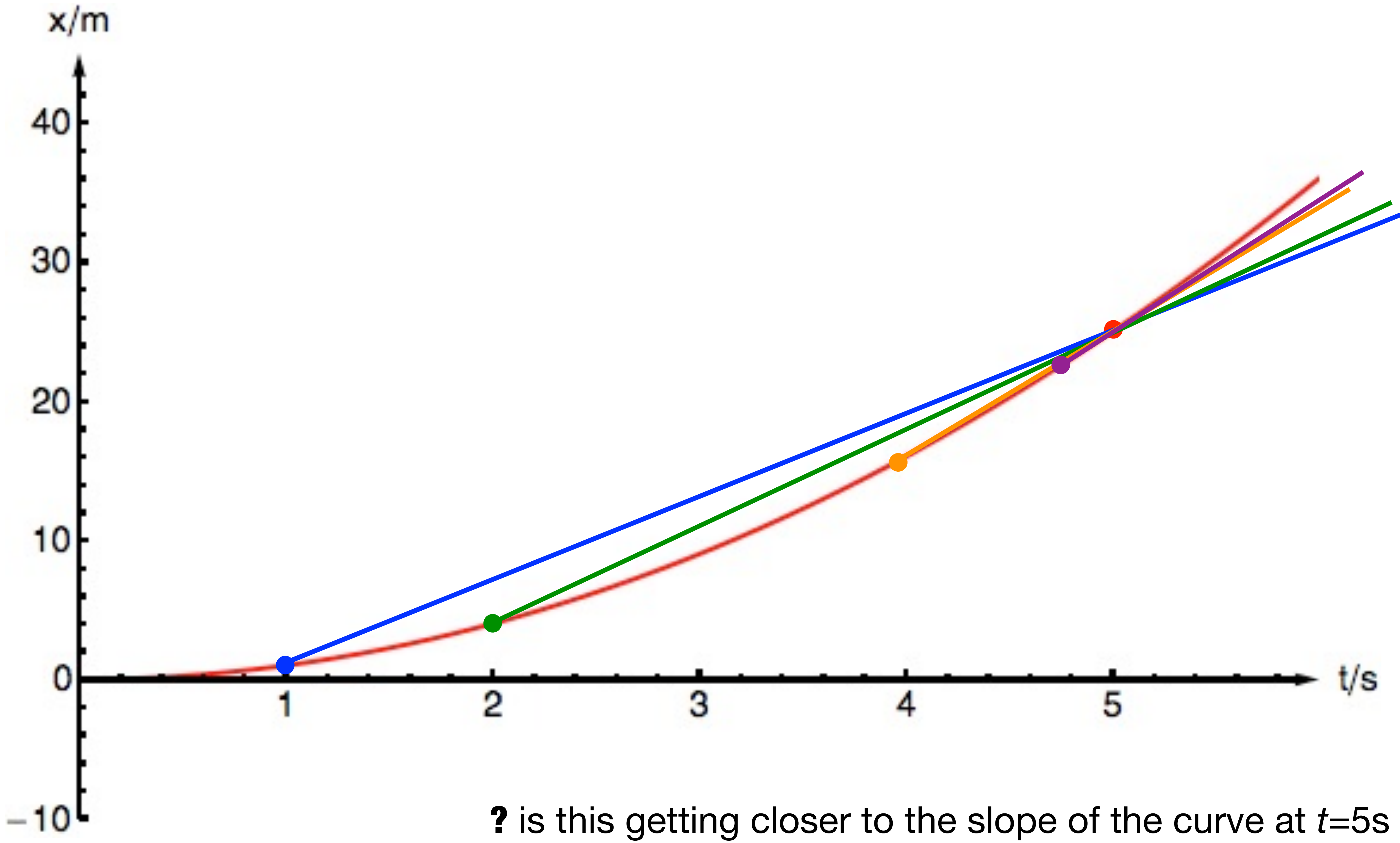
$$\Delta x = 20.00\,\mathrm{m}$$

$$\Delta t = 3.00\,\mathrm{s}$$

$$v_{\mathrm{av}} = \frac{\Delta x}{\Delta t} = \frac{20.00\,\mathrm{m}}{3.00\,\mathrm{s}} = 6.67\,\mathrm{m/s}$$

average & instantaneous velocity

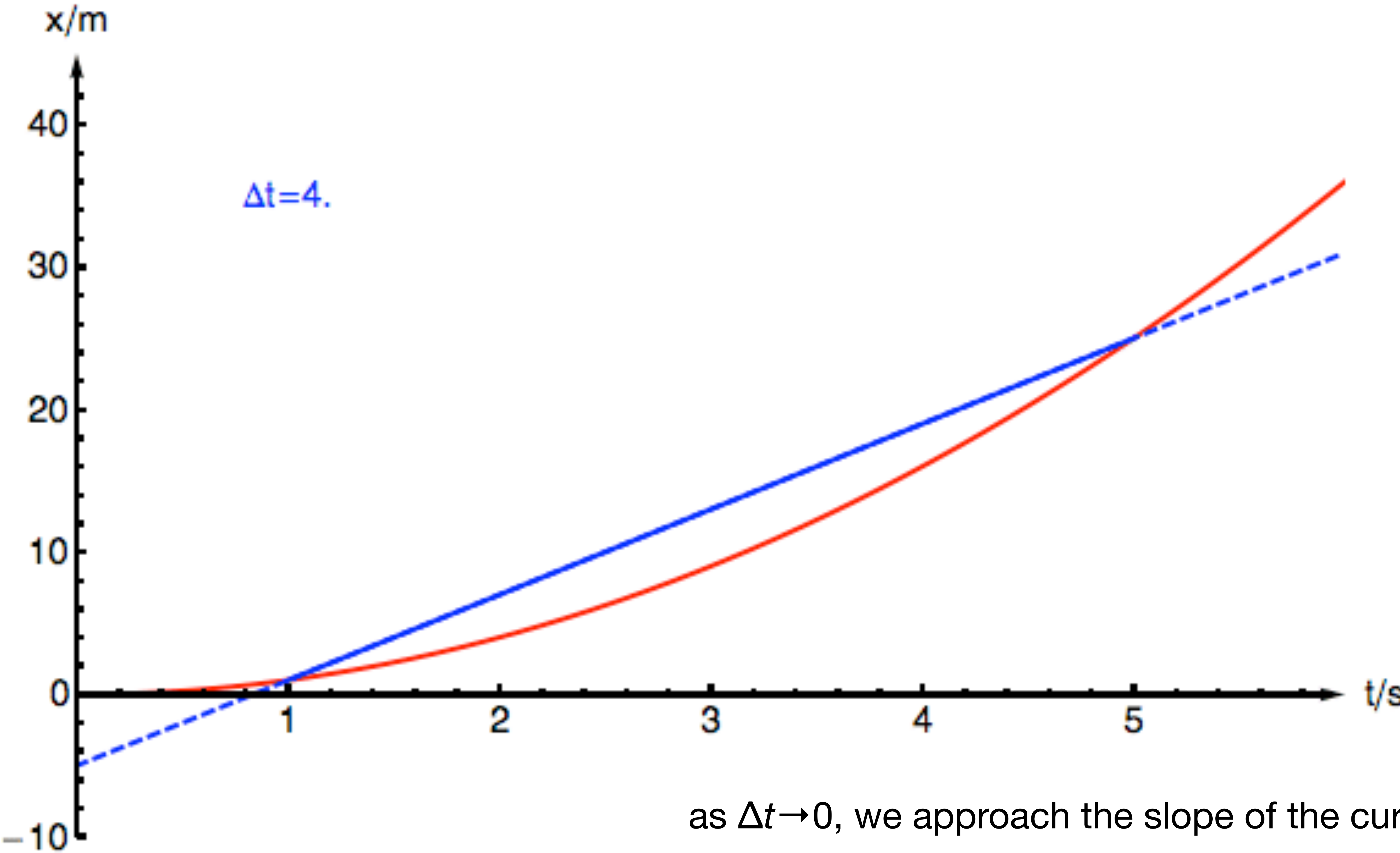
➜ compute the average velocity between t = ? & t = 5.00 s



? is this getting closer to the slope of the curve at t=5s **?**

average & instantaneous velocity

➜ compute the average velocity between t = ? & t = 5.00 s

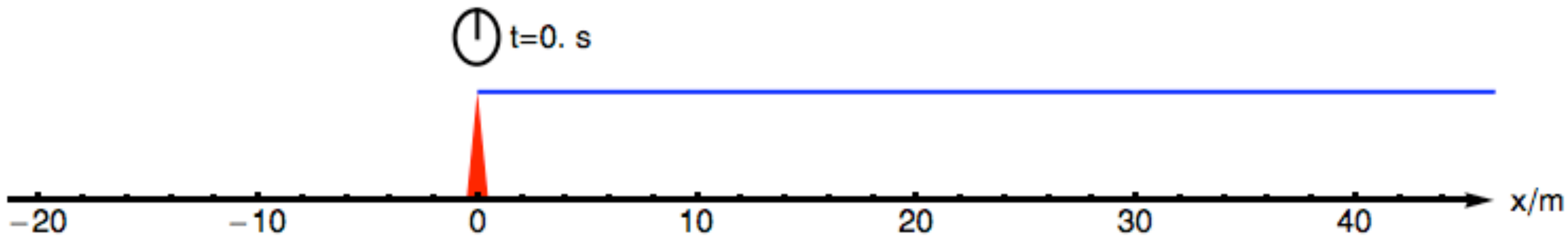


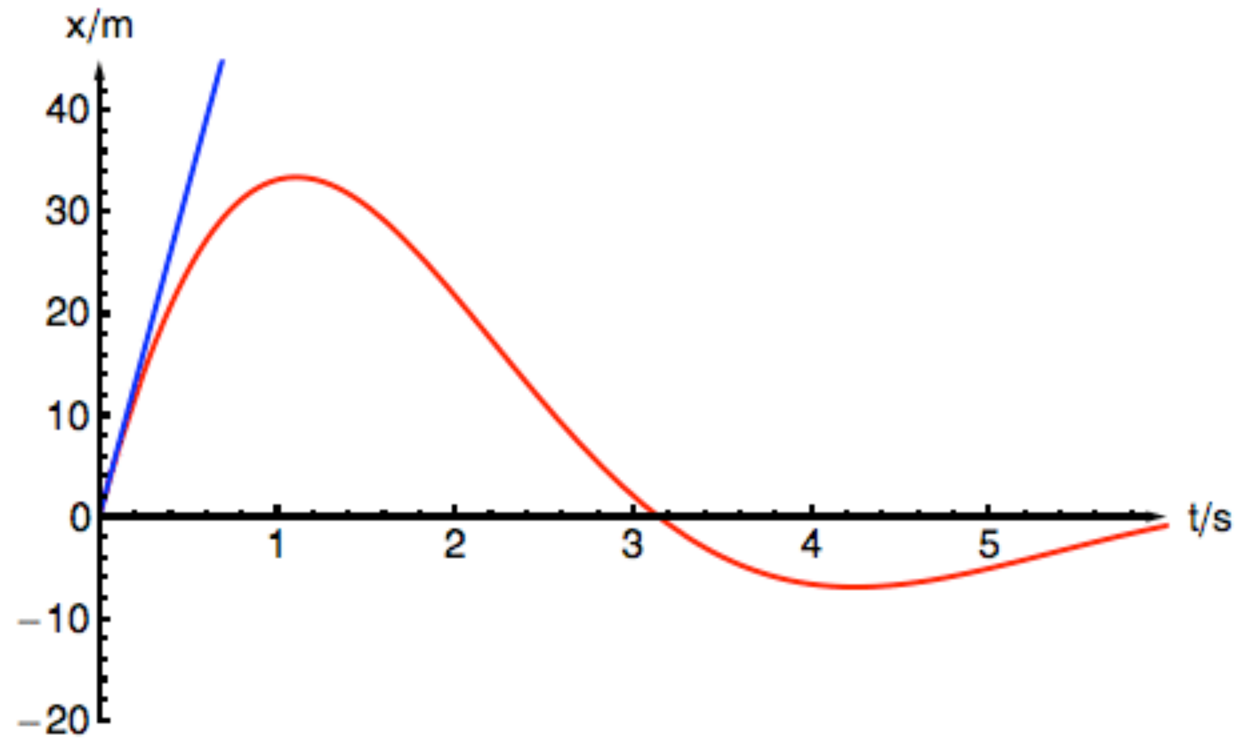
as $\Delta t \rightarrow 0$, we approach the slope of the curve

instantaneous velocity

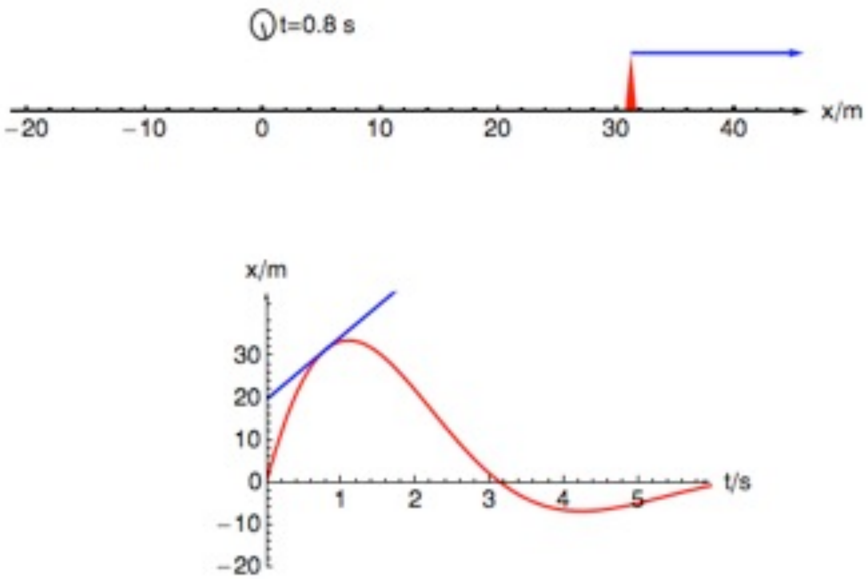
➜ the velocity "at an instant in time" is defined to be $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

and we just saw that it corresponds to the slope of the *x*-*t* curve



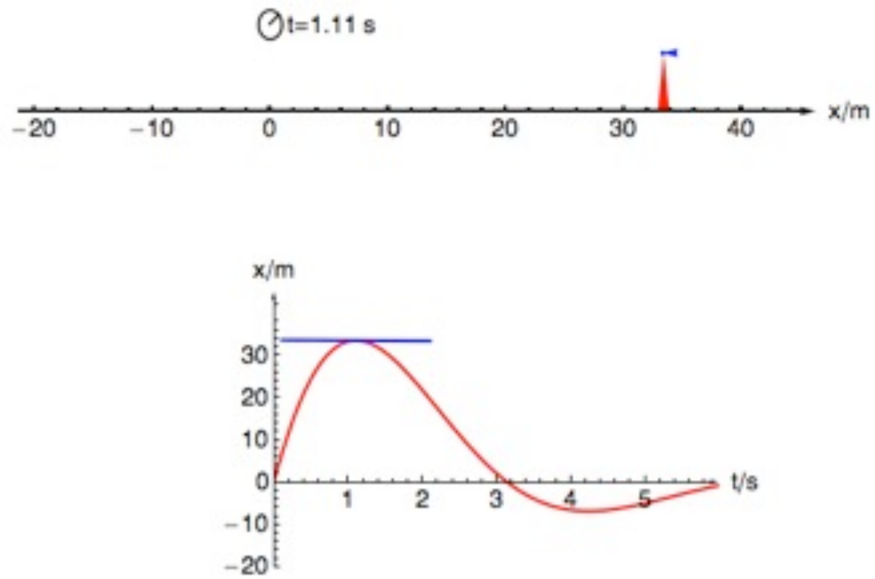


instantaneous velocity



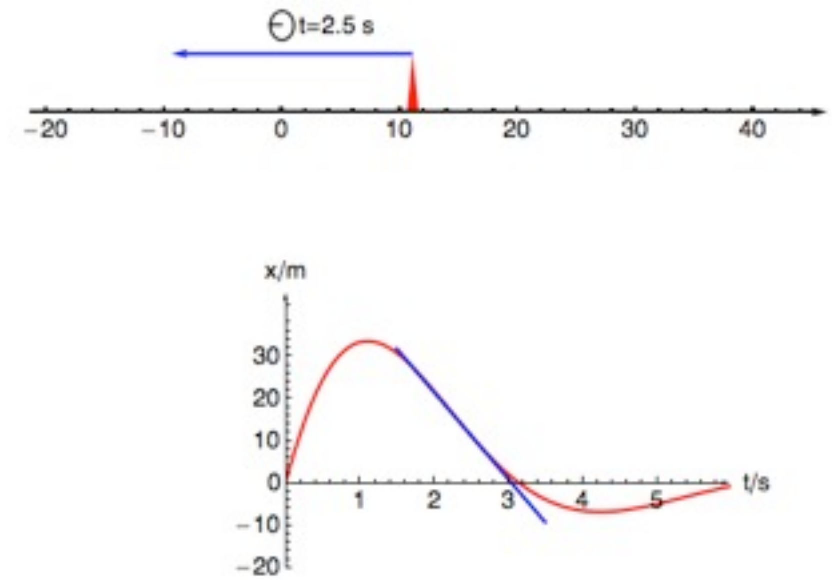
$v > 0$

“moving to larger x”



$v = 0$

“not moving”



$v < 0$

“moving to smaller x”

acceleration

➜ if an object's velocity changes, it has undergone an acceleration

➜ we can define average acceleration

$$a_{\rm av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

e.g. driving a car along a long straight stretch of road

$$v_f = 60\,\mathrm{m.p.h} = 60\,\frac{\mathrm{mi}}{\mathrm{hr}} \times \left(\frac{1609\,\mathrm{m}}{1\,\mathrm{mi}}\right) \times \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right) = 27\,\mathrm{m/s}$$

$$v_i = 0\,\mathrm{m/s}$$

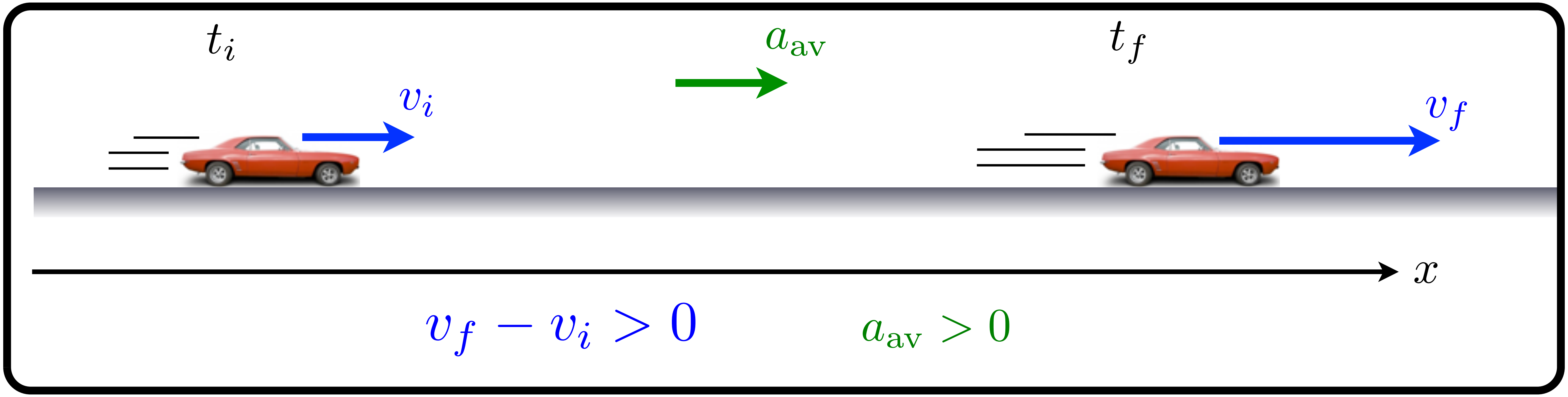
$$\Delta t = 5.50\,\mathrm{s}$$

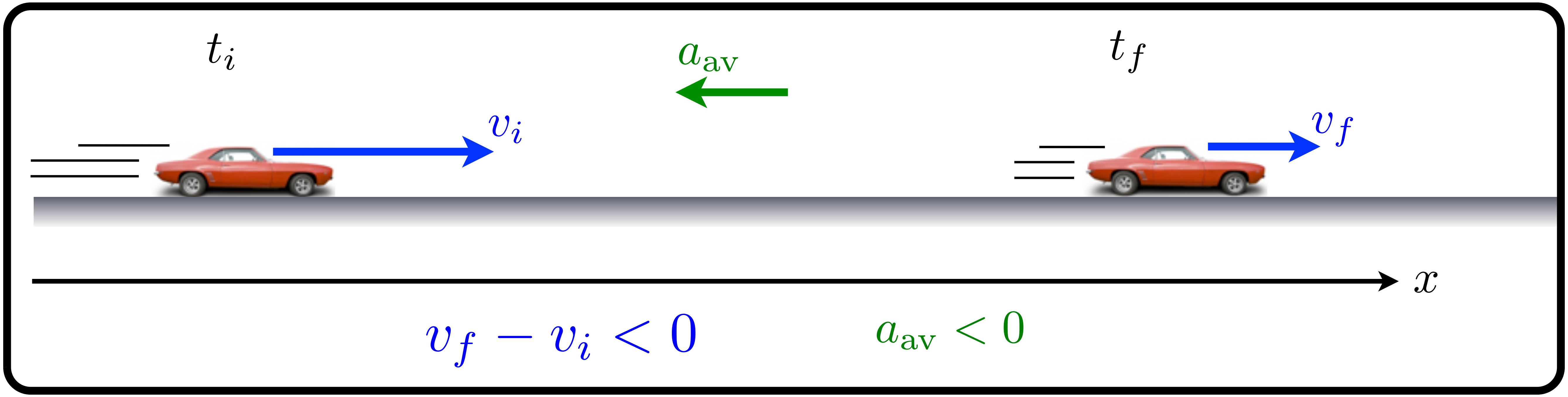
$$a_{\rm av} = +4.9\,\mathrm{m/s^2}$$

positive sign means velocity is increasing

acceleration - meaning of the sign

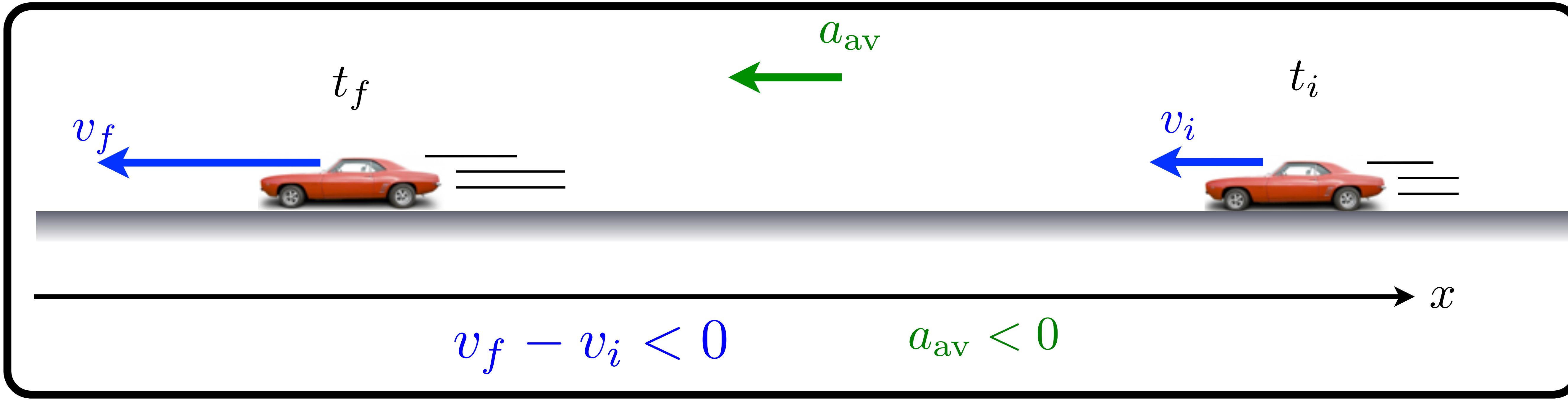
$$a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

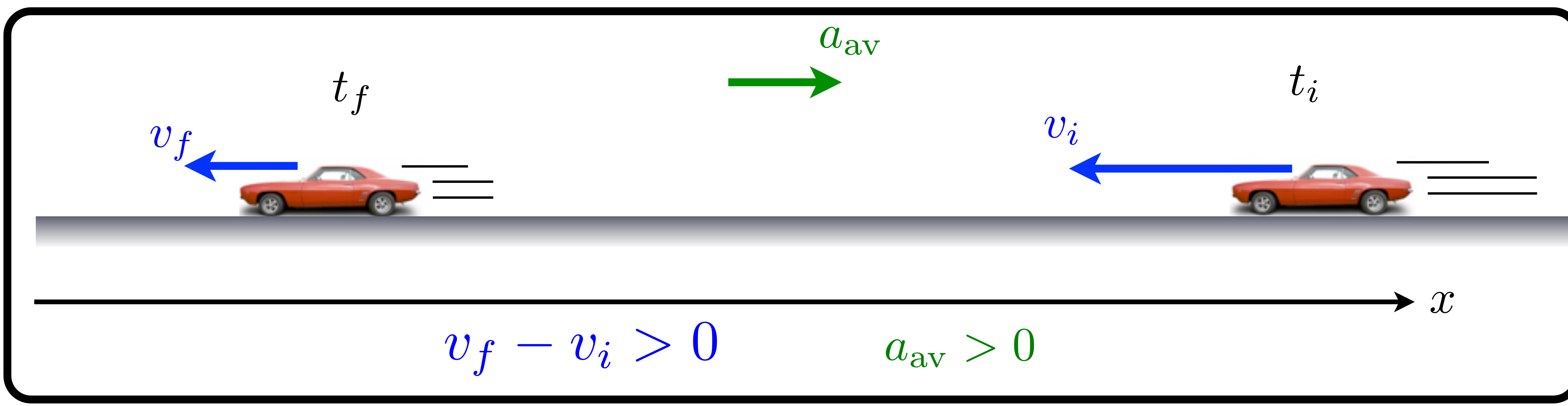




acceleration - meaning of the sign

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$



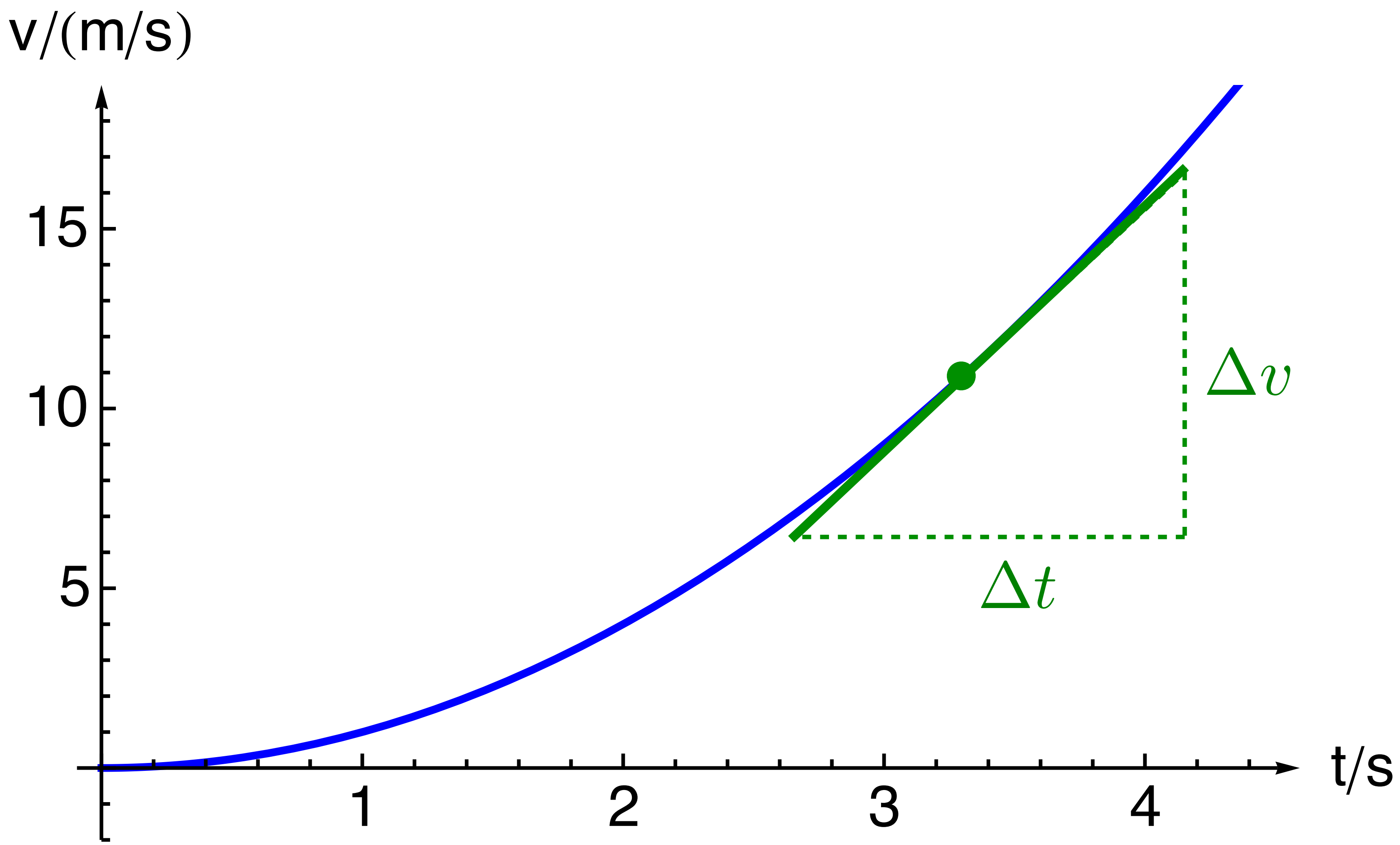


so be careful, the sign of the acceleration doesn't just mean speeding up / slowing down

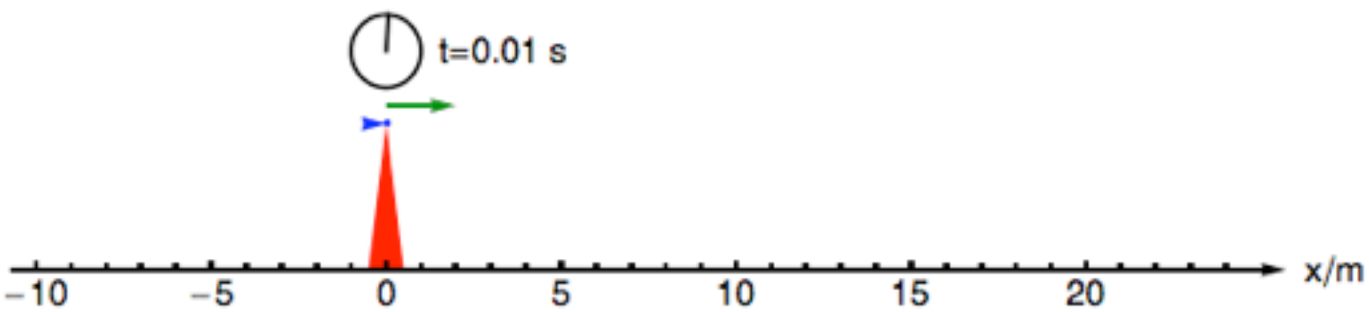
instantaneous acceleration

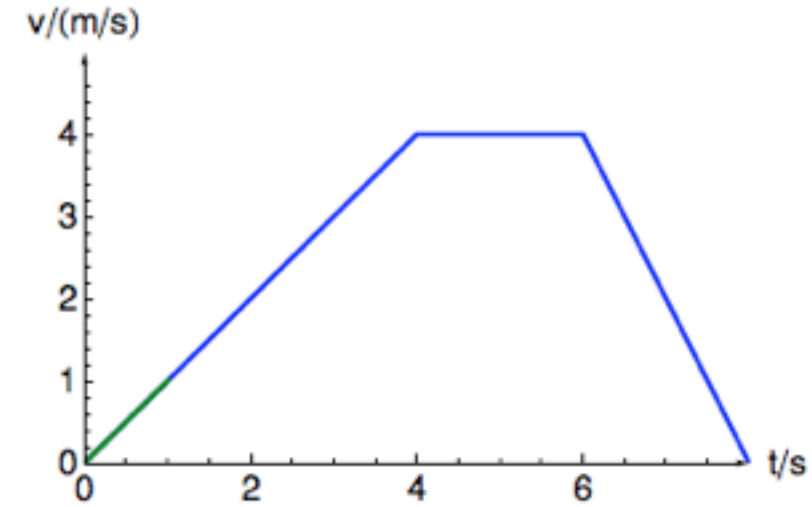
➜ we define instantaneous acceleration $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

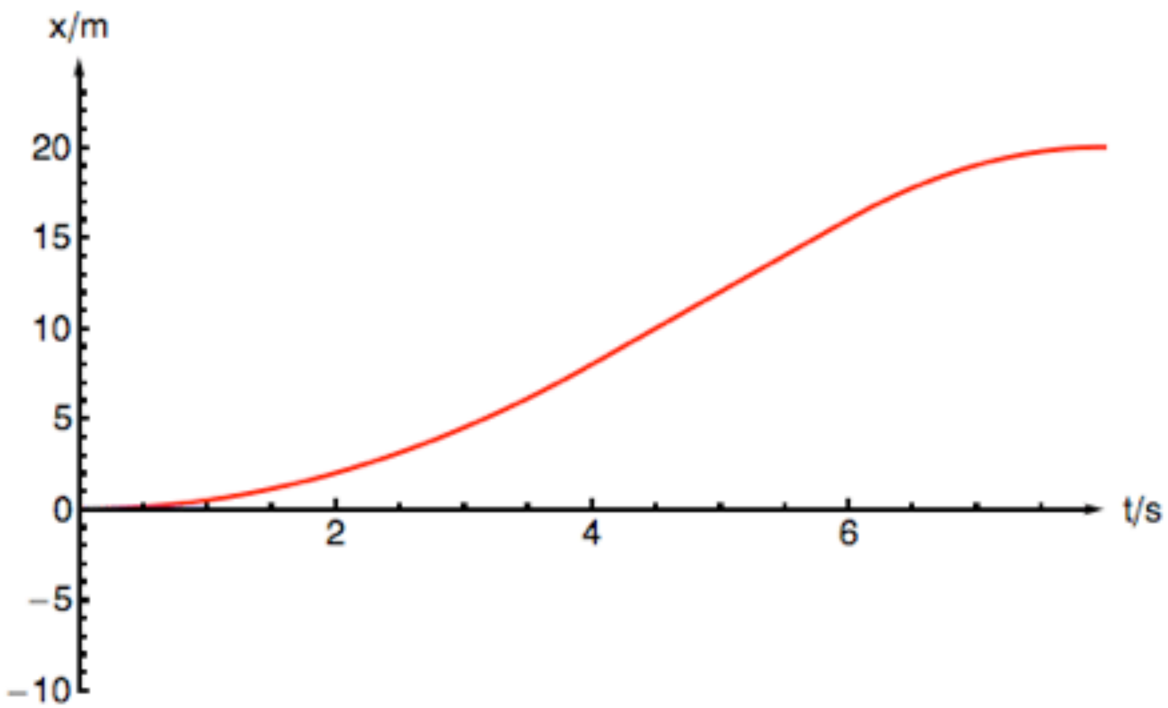
which is the slope of the *v*-*t* curve

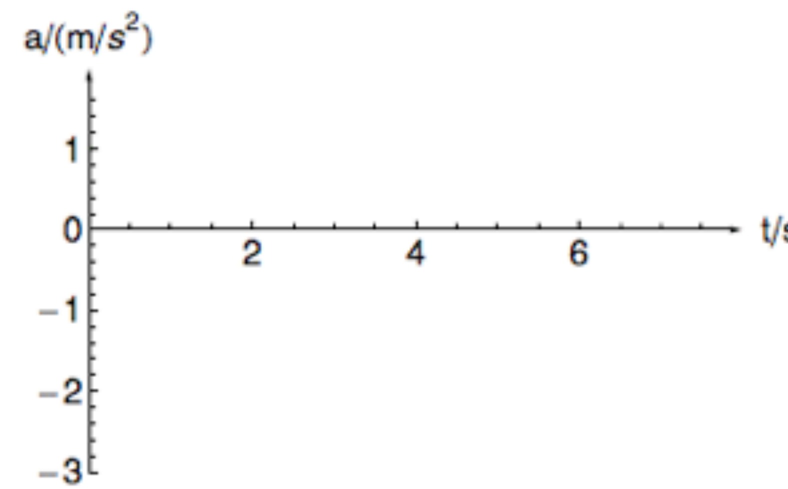


instantaneous acceleration









motion with constant acceleration

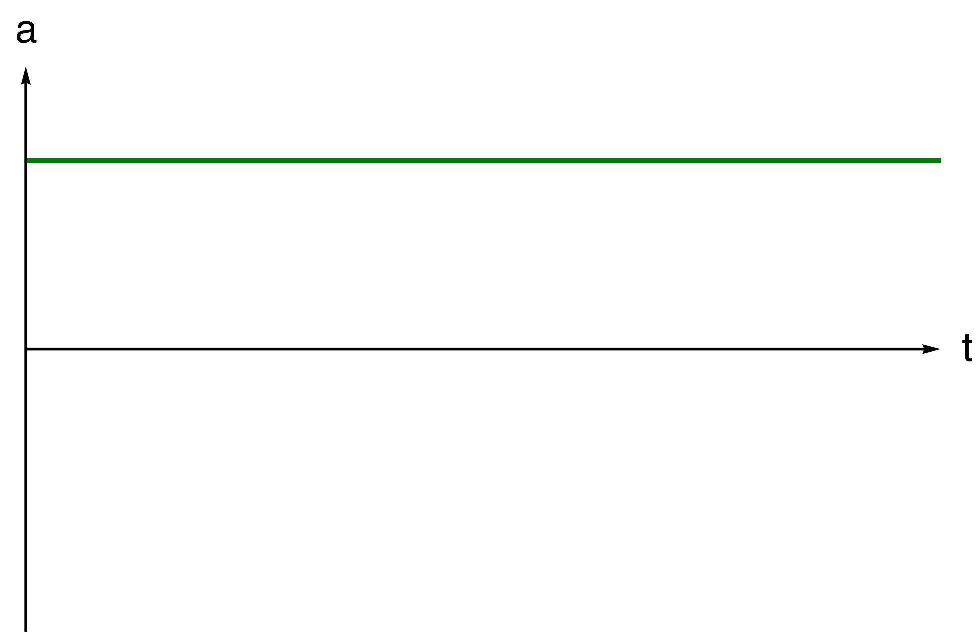
➜ simple, but very important, example of a particle being accelerated $a = \text{const}$

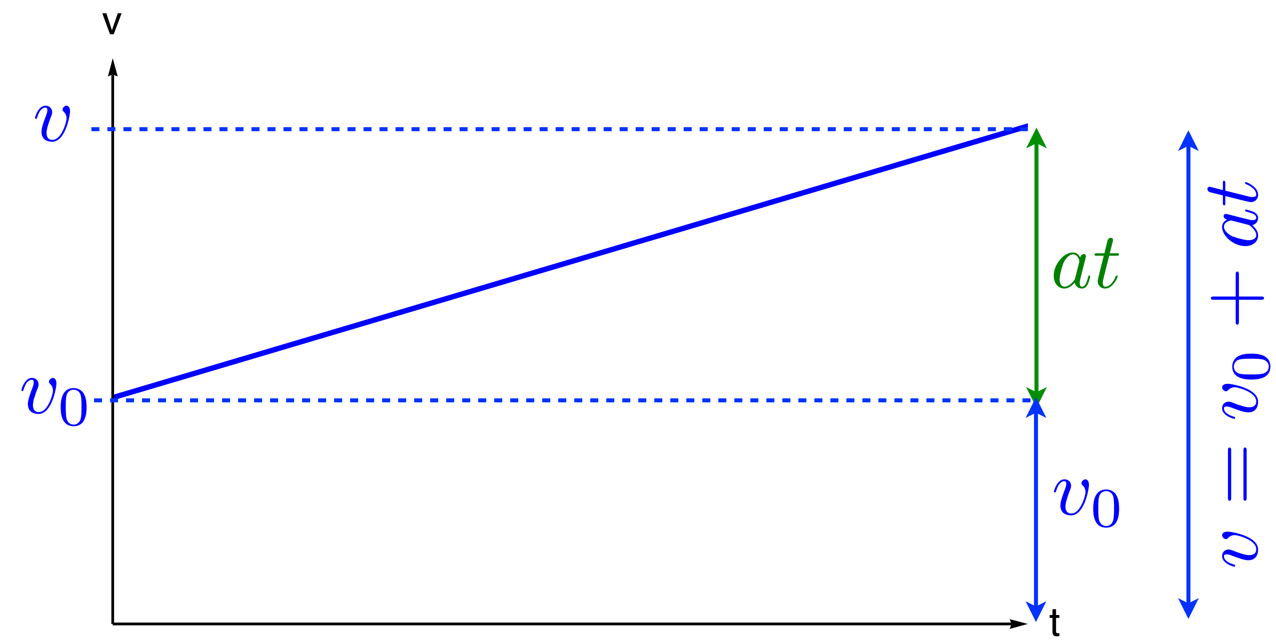
$$a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

@ t=0, velocity = v_0

@ t, velocity = v

$$v = v_0 + at$$



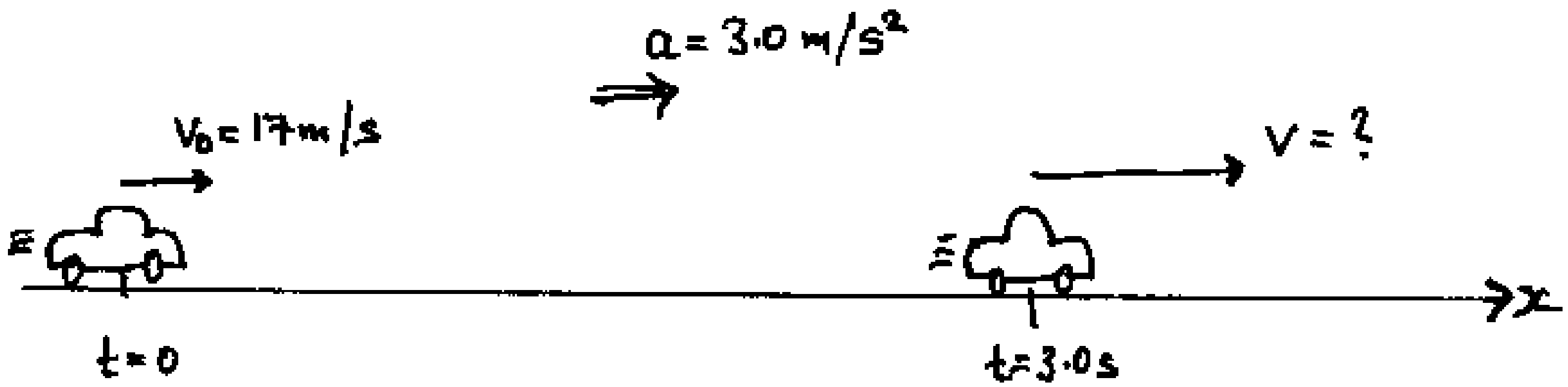


➜ the constant (positive) acceleration is causing the velocity to increase at a constant rate

e.g. if a=+10 m/s^2,
in 1 sec, v increases by 10 m/s
in another 1 sec, v increases by another 10 m/s ...

pedal to the metal

➜ suppose you're driving on the highway at 17 m/s and you press the accelerator to accelerate at a constant 3.0 m/s^2. After 3.0 seconds, what is your speed?



$$v = v_0 + at$$

$$= 17\,\mathrm{m/s} + \left(3.0\,\mathrm{m/s^2} \times 3.0\,\mathrm{s}\right)$$

$$= 17\,\mathrm{m/s} + 9.0\,\mathrm{m/s}$$

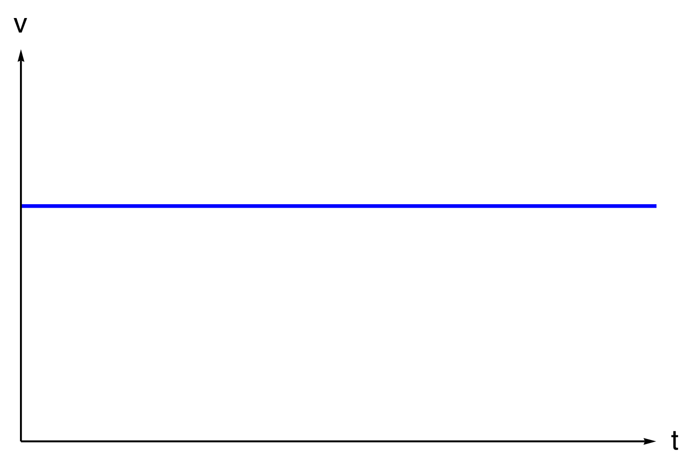
$$\underline{v = 26\,\mathrm{m/s}}$$

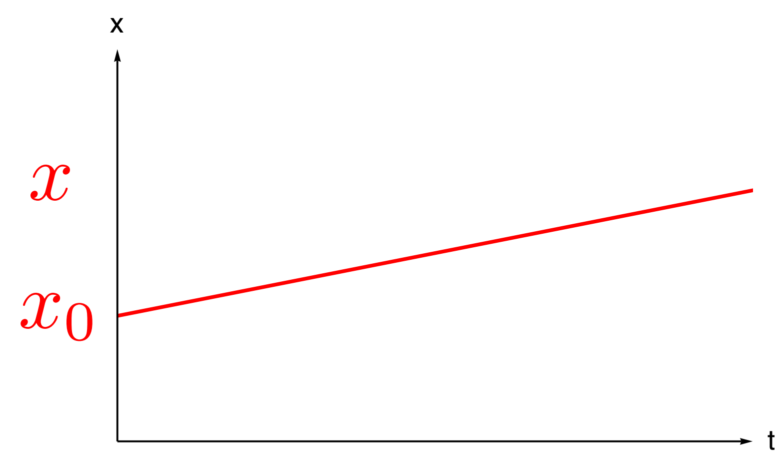
position at constant acceleration

➜ we'd like to get an equation for the position as a function of time

we can figure it out ("derive" it)

first a simpler example - constant velocity

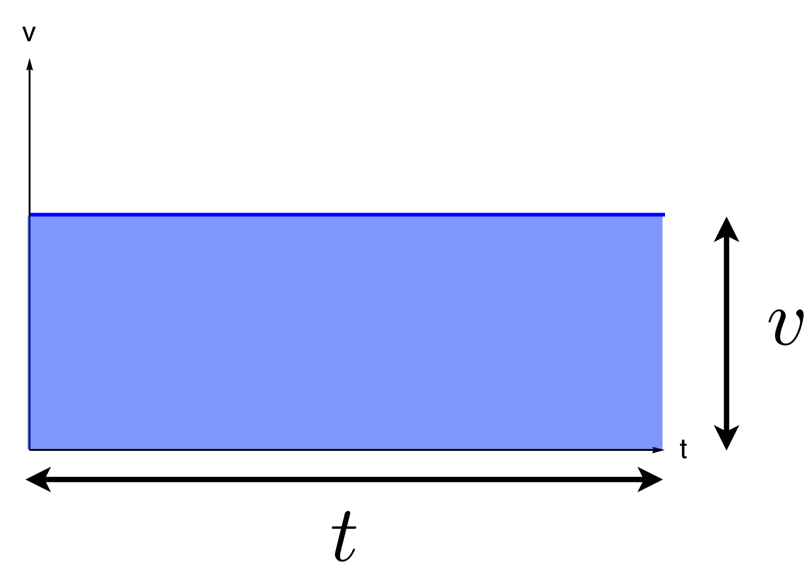




$$v = \text{const} = \frac{x - x_0}{t - 0}$$

$$x = x_0 + vt$$

notice that the change in *x* is the area under the *v*-*t* graph

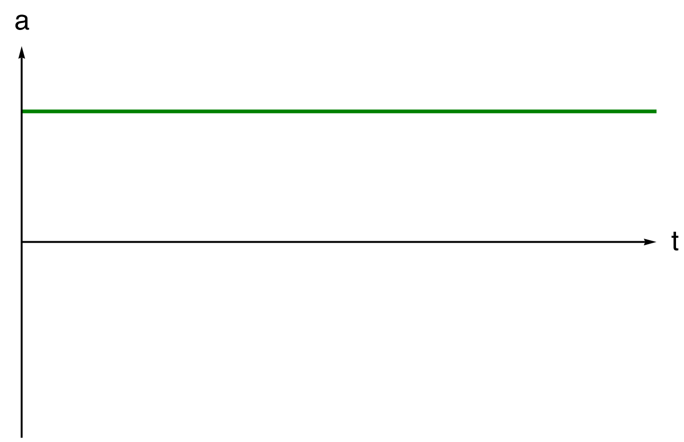


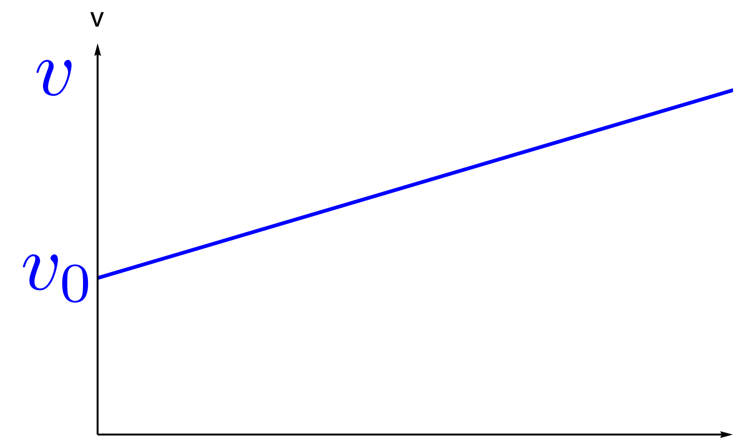
area = *v*×*t*

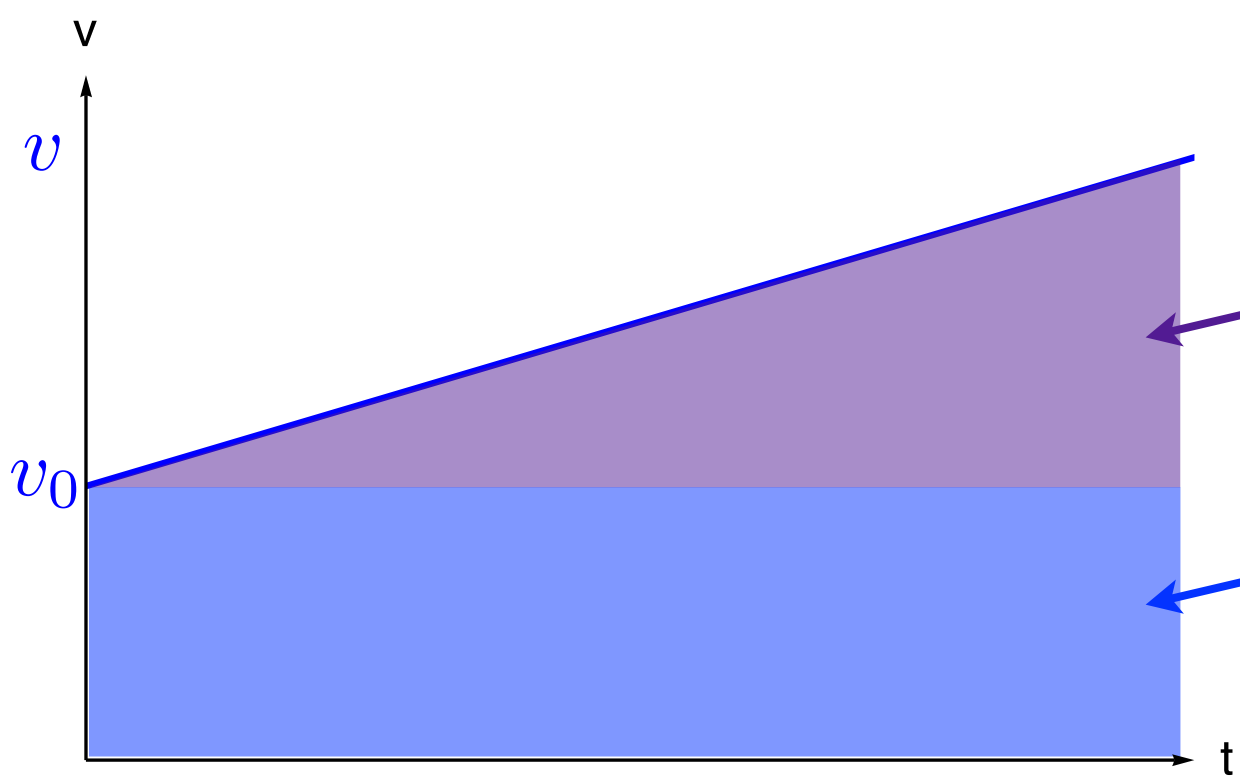
position at constant acceleration

➜ it is generally true that the change in position is the area under the *v*-*t* graph

constant acceleration







area = $\frac{1}{2}(v - v_0)t$

total area = $\frac{1}{2}(v + v_0)t$

area = $v_0 t$

$$x = x_0 + \frac{1}{2}(v + v_0)t$$

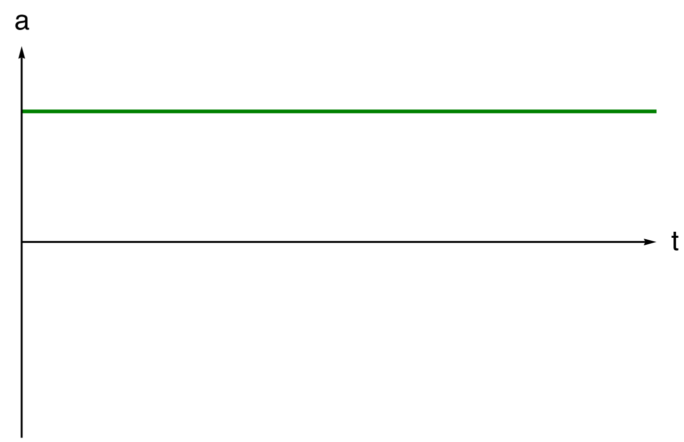
$$v = v_0 + at$$

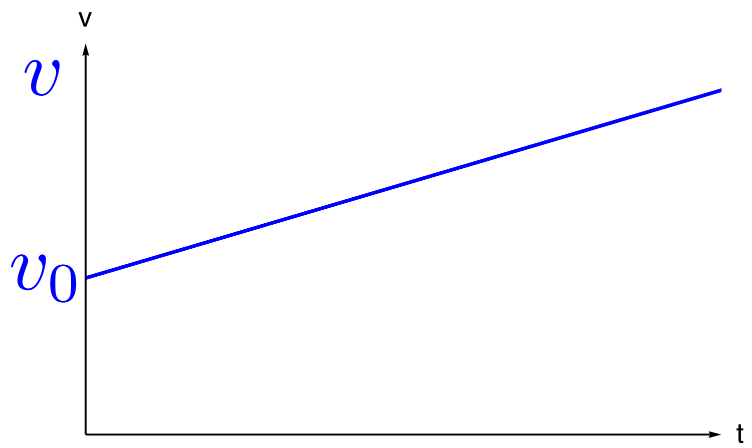
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

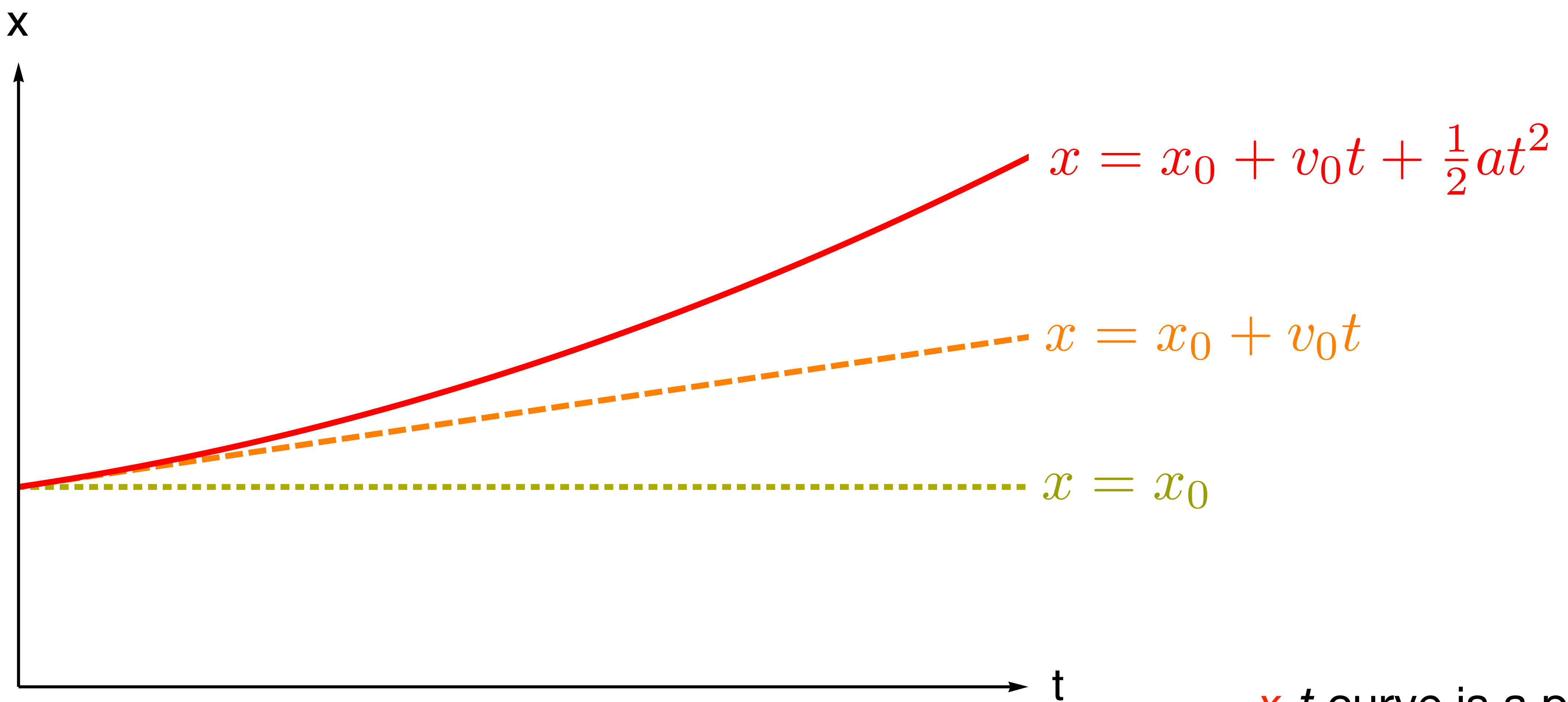
position at constant acceleration

➜ it is generally true that the change in position is the area under the v-t graph

constant acceleration







x-t curve is a parabola

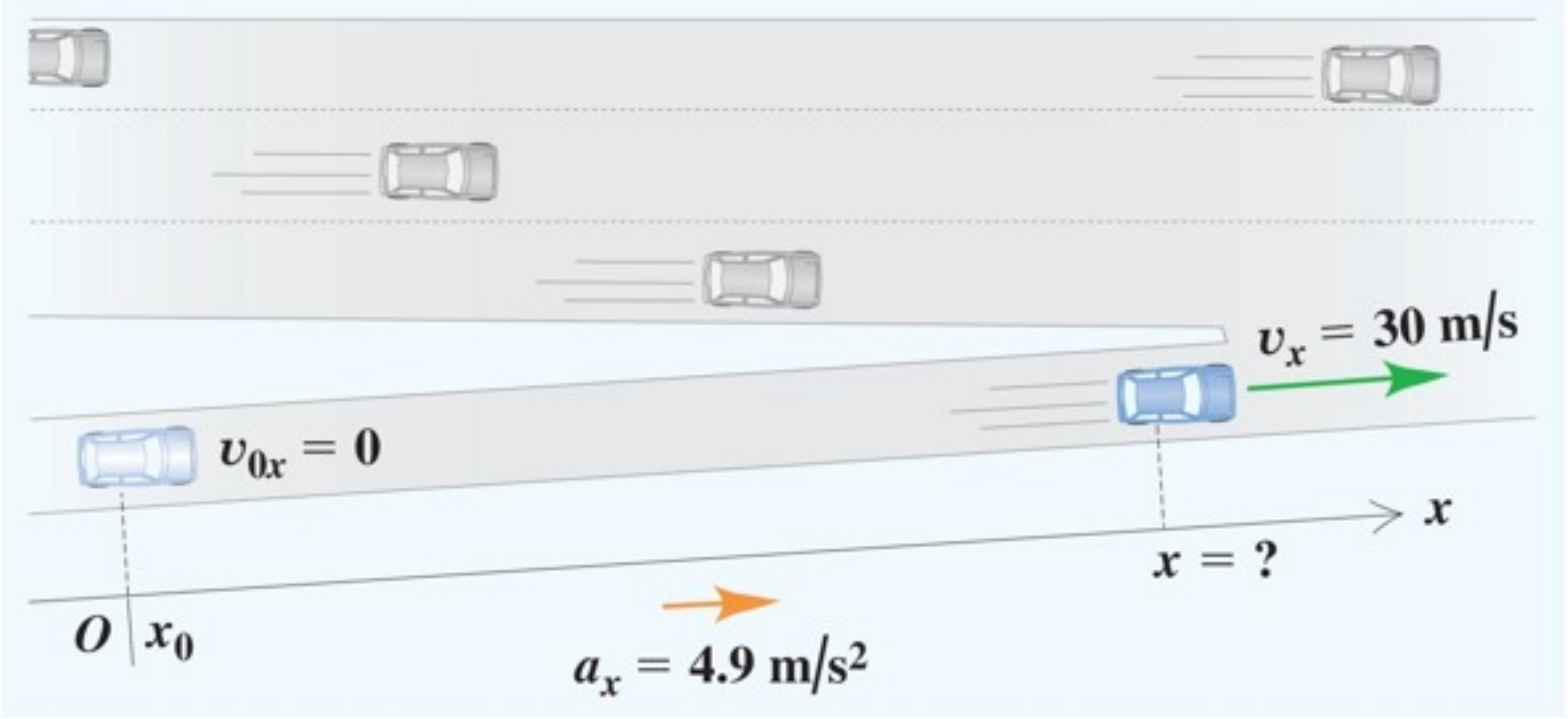
formulas for constant acceleration

➜ probably useful for you to remember the following equations

$$\boxed{\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \tfrac{1}{2} a t^2 \\ v^2 &= v_0^2 + 2a(x - x_0) \\ x &= x_0 + \tfrac{1}{2}(v_0 + v)t \end{aligned}}$$

entering the freeway

A sports car is sitting at rest on a freeway entrance ramp.The driver sees a break in traffic and floors the gas pedal, so that the car accelerates at a constant 4.9 m/s^2 as it moves in a straight line onto the freeway. What distance does the car travel in reaching a freeway speed of 30 m/s ?



$$v_0 = 0$$
$$v = 30\,\mathrm{m/s}$$
$$a = 4.9\,\mathrm{m/s^2}$$

define $x_0 = 0$

$x = ?$ ←solve for this

$t = ?$

$$\cancel{v = v_0 + at}$$

$$\cancel{x = x_0 + v_0 t + \tfrac{1}{2}at^2}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

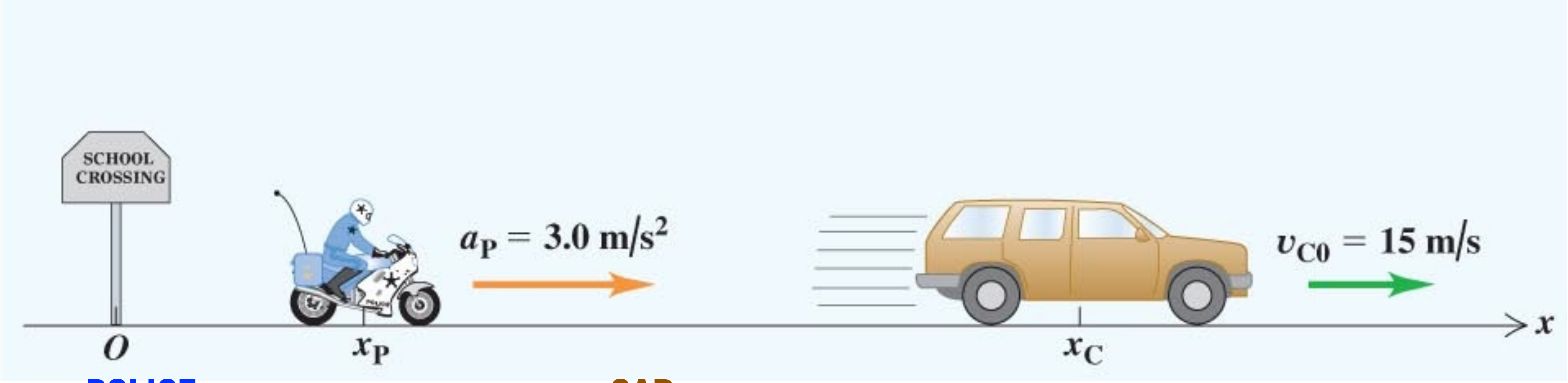
$$\cancel{x = x_0 + \tfrac{1}{2}(v_0 + v)t}$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

$$x = \frac{(30\,\mathrm{m/s})^2}{2 \times 4.9\,\mathrm{m/s^2}} = 92\,\mathrm{m}$$

pursuit

A motorist traveling at a constant velocity of 15 m/s passes a school-crossing corner where the speed limit is 10 m/s. A police office on a motorcycle stopped at the corner starts off in pursuit with constant acceleration of 3.0 m/s^2.
How much time elapses before the officer catches up with the car ?



POLICE

$$x_{P0} = 0$$

$$v_{P0} = 0$$

$$a_P = 3.0\,\mathrm{m/s^2}$$

CAR

$$x_{C0} = 0$$

$$v_{C0} = 15\,\mathrm{m/s}$$

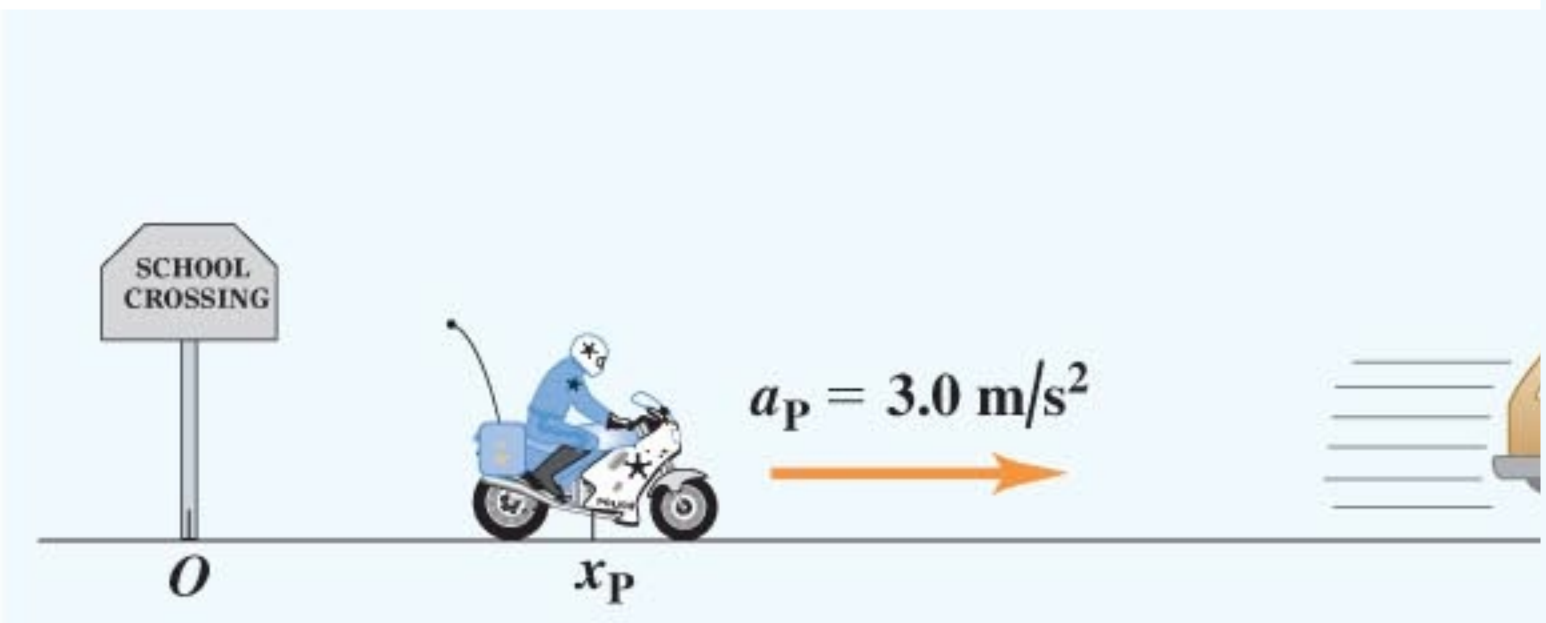
$$a_C = 0$$

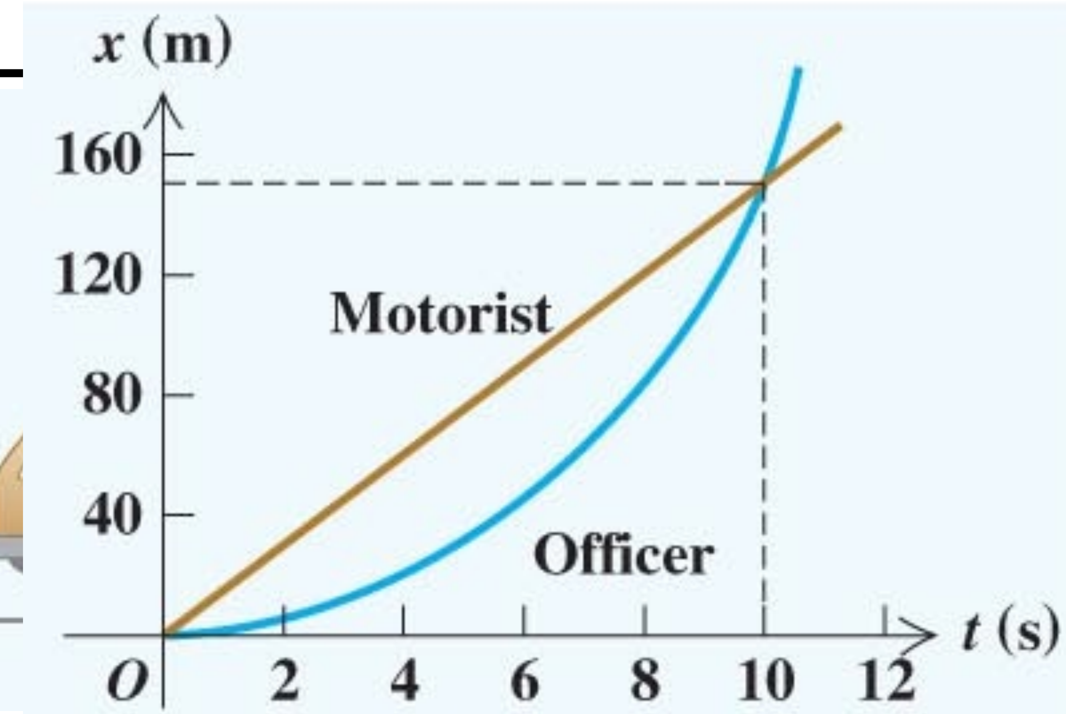
we want to know **when** the **POLICE** and the **CAR** are at the same location

$$\Longrightarrow \quad x_P = x_C$$

$$t = ?$$

pursuit





POLICE

$$x_{P0} = 0$$
$$v_{P0} = 0$$
$$a_P = 3.0\,\mathrm{m/s^2}$$

CAR

$$x_{C0} = 0$$
$$v_{C0} = 15\,\mathrm{m/s}$$
$$a_C = 0$$

we want to know **when** the **POLICE** and the **CAR** are at the same location

$$\implies \quad x_P = x_C$$
$$t = ?$$

$$x_P = x_{P0} + v_{P0}t + \tfrac{1}{2}a_P t^2$$
$$x_P = \tfrac{1}{2}a_P t^2$$

$$x_C = x_{C0} + v_{C0}t + \tfrac{1}{2}a_C t^2$$
$$x_C = v_{C0}t$$

$$x_P = x_C$$
$$\tfrac{1}{2}a_P t^2 = v_{C0}t$$
$$t = 0,\ \frac{2v_{C0}}{a_P}$$

$$t = 10\,\mathrm{s}$$

pursuit 2 - the panicking motorist

A motorist traveling at a constant velocity of 15 m/s passes a school-crossing corner where the speed limit is 10 m/s. A police office on a motorcycle stopped at the corner starts off in pursuit with constant acceleration of 3.0 m/s^2.
This time the motorist sees the cop and applies the brakes as he passes the corner, causing a constant acceleration of -2.0 m/s^2.
How much time elapses before the officer catches up with the car ?

POLICE

$$x_{P0} = 0$$

$$v_{P0} = 0$$

$$a_P = 3.0\,\mathrm{m/s^2}$$

CAR

$$x_{C0} = 0$$

$$v_{C0} = 15\,\mathrm{m/s}$$

$$a_C = -2.0\,\mathrm{m/s^2}$$

we want to know **when** the **POLICE** and the **CAR** are at the same location

$$\implies \quad x_P = x_C$$

$$t = ?$$

$$x_P = \tfrac{1}{2} a_P t^2$$

$$x_C = v_{C0} t + \tfrac{1}{2} a_C t^2$$

$$\tfrac{1}{2} a_P t^2 = v_{C0} t + \tfrac{1}{2} a_C t^2$$

$$0 = v_{C0} t + \tfrac{1}{2}(a_C - a_P) t^2$$

$$t = 0, \frac{2 v_{C0}}{a_P - a_C}$$

$$t = 3\,\mathrm{s}$$

free fall

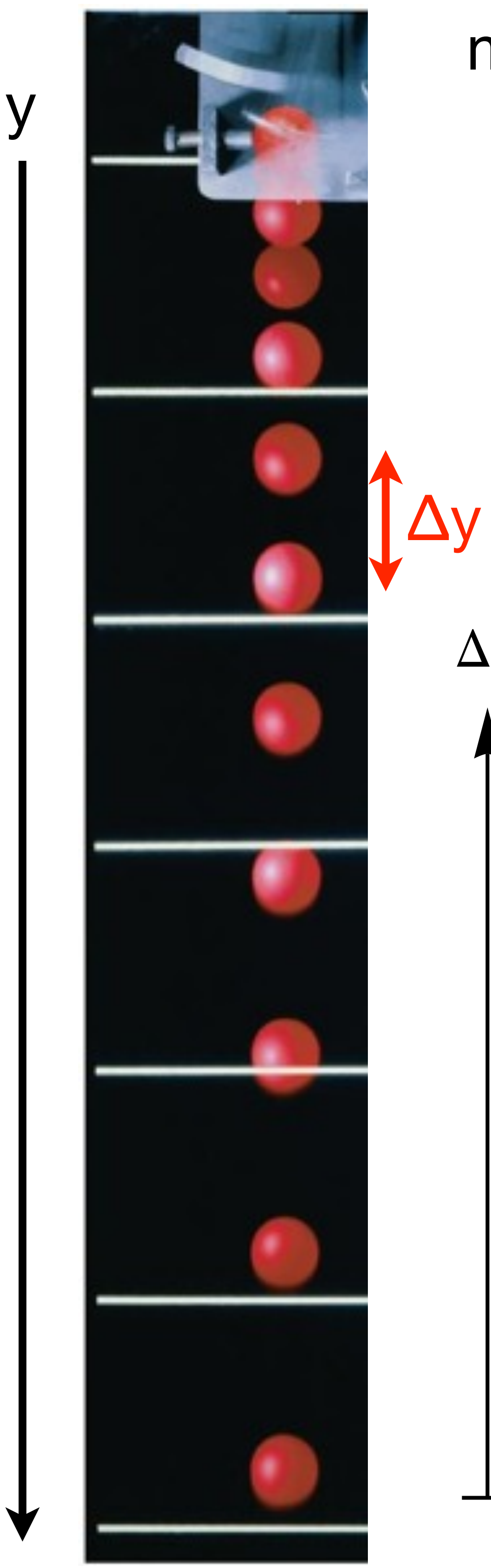
if we neglect the effect of air, objects dropped or thrown vertically up or down **accelerate at a constant rate**

objects accelerate toward the center of the Earth due to gravity, which we'll explore later in this course

now just because I tell you this, doesn't mean it is true !
- it's a **theory** that needs to be tested by doing **experiments**

free fall experiment - is the acceleration constant ?

high-speed photography of a ball falling in a vacuum chamber - a shot every Δt seconds



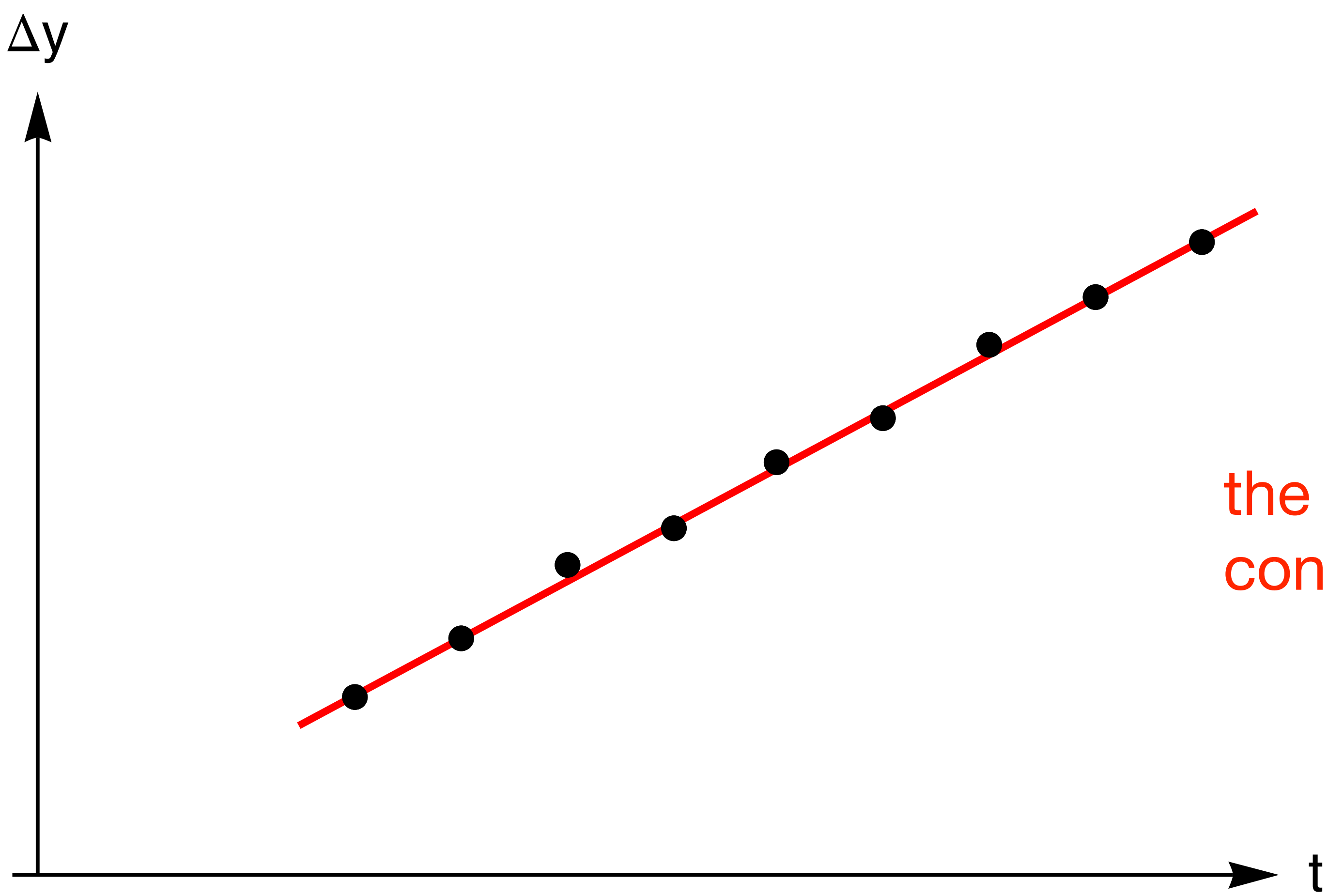
measure how far the ball has travelled in each Δt seconds

for constant acceleration, should increase linearly with *t*

$$y(t) = y_0 + \tfrac{1}{2}at^2$$

$$y(t+\Delta t) = y_0 + \tfrac{1}{2}a(t+\Delta t)^2$$

$$\begin{aligned}\Delta y &= y(t+\Delta t) - y(t) \\ &= \left(\tfrac{1}{2}a\Delta t^2\right) + \left(\tfrac{1}{2}a\Delta t\right)t\end{aligned}$$



the data suggests constant acceleration

with precise measurements, we find $|a| = 9.80\ \mathrm{m/s^2}$

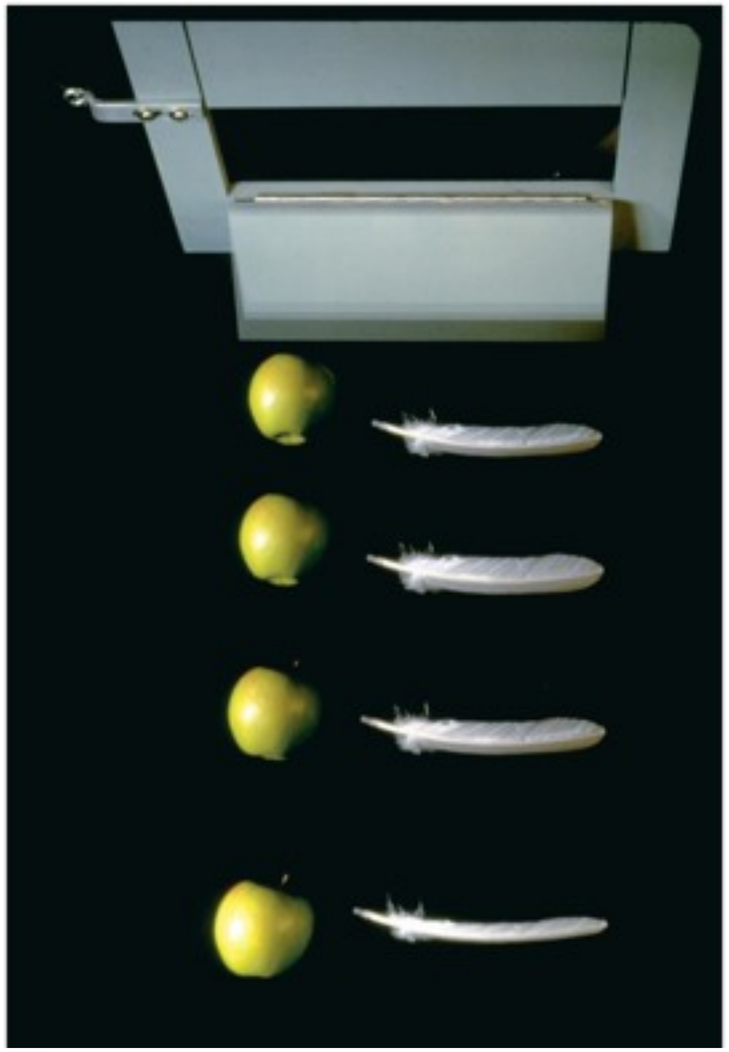
free fall

turns out all objects accelerate at the same rate

e.g. drop an apple versus drop a feather

“no way!”, you’d say, “a feather will float downwards, an apple will drop”

true, but this is a property of the air surrounding the feather
remove the issue of air resistance - do the experiment in vacuum

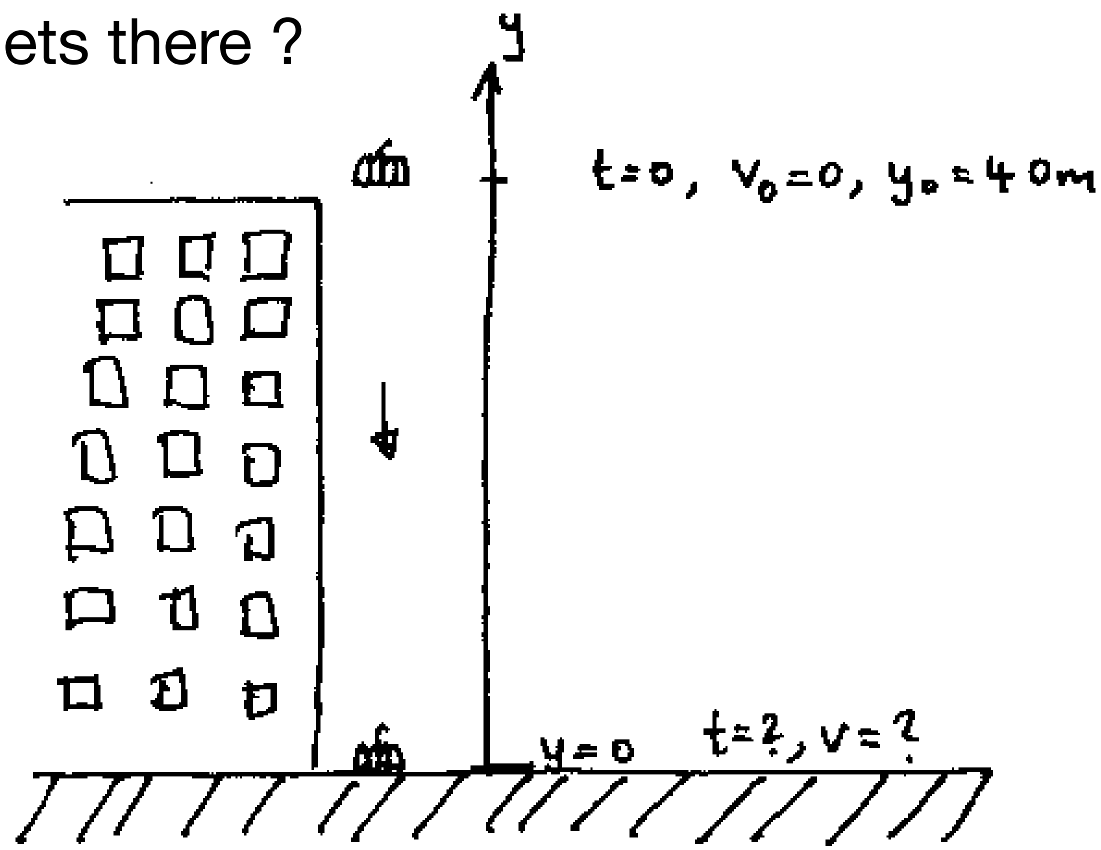


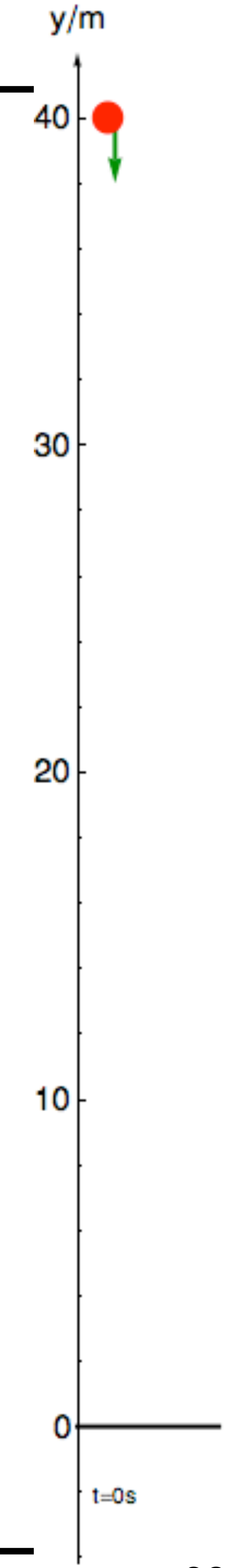
or a somewhat more expensive experiment ...

free fall - hammer & feather on the moon

pumpkin drop

Suppose you were to drop a pumpkin from the top of a 40m high building. Neglecting air resistance, how long does it take for the pumpkin to reach the ground and how fast is it moving when it gets there ?





moment of release

$$t = 0$$

$$y_0 = 40\,\mathrm{m}$$

$$v_0 = 0\,\mathrm{m/s}$$

reaches ground

$$y = 0\,\mathrm{m}$$

$$t = ?$$

$$v = ?$$

at all times

$$a = -9.80\,\mathrm{m/s^2}$$

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$0 = y_0 + \tfrac{1}{2} a t^2$$

$$t = \sqrt{-\frac{2y_0}{a}}$$

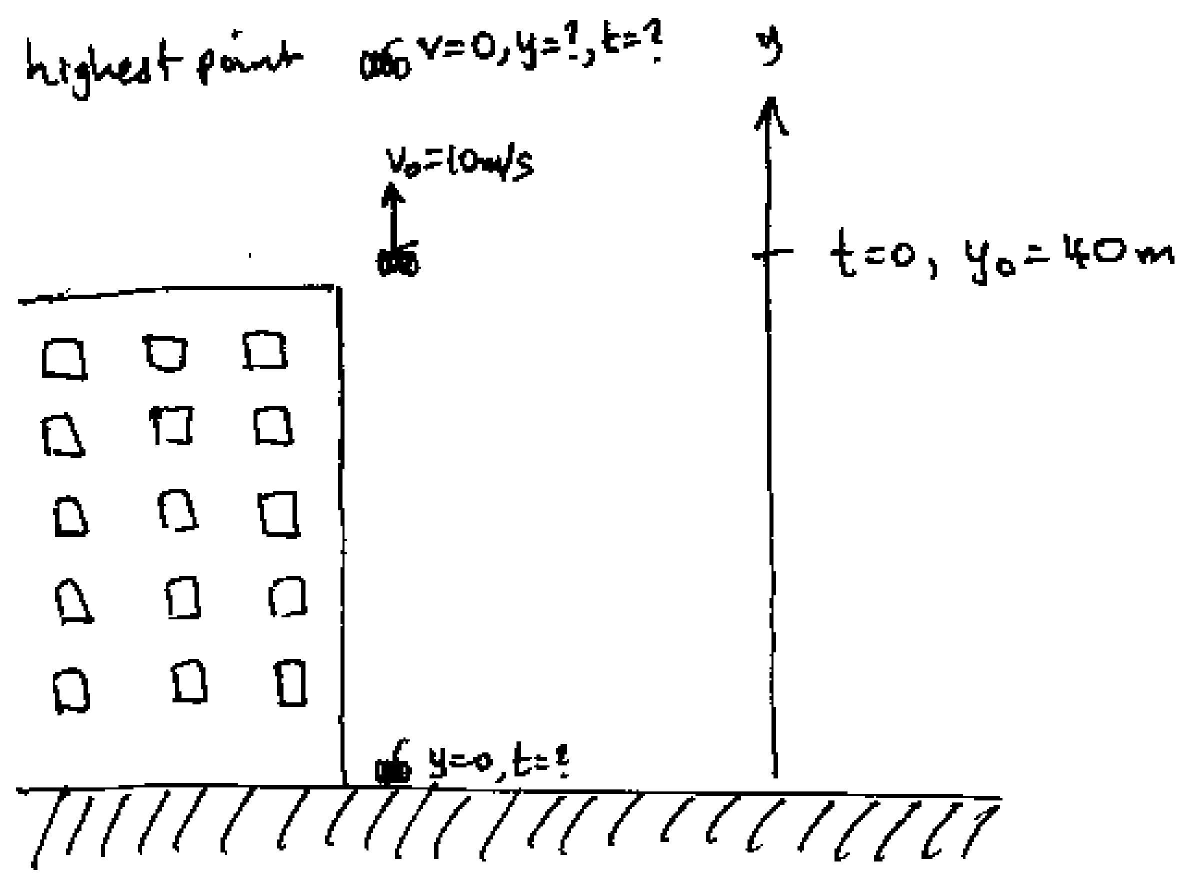
$$t = 2.9\,\mathrm{s}$$

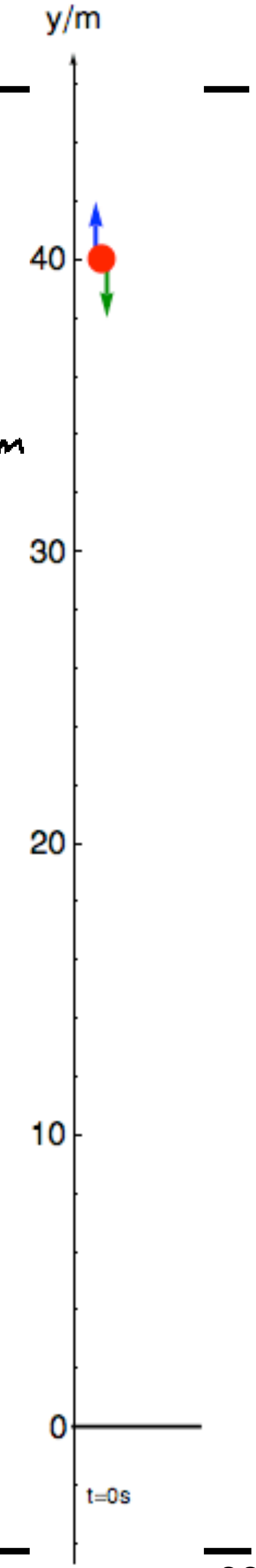
pumpkin throw

Suppose you were to throw a pumpkin vertically upward from the top of a 40m high building at 10 m/s.

Neglecting air resistance, what is the maximum height above the ground reached by the pumpkin and how long after release does it reach this point ?

When does the pumpkin reach the ground ?





moment of release

$$t = 0$$
$$y_0 = 40\,\mathrm{m}$$
$$v_0 = 10\,\mathrm{m/s}$$

highest point

$$v = 0\,\mathrm{m/s}$$
$$t = ?$$
$$y = ?$$

at all times

$$a = -9.80\,\mathrm{m/s^2}$$

$$v^2 = v_0^2 + 2a(y - y_0)$$
$$0 = v_0^2 + 2a(y - y_0)$$
$$y = y_0 - \frac{v_0^2}{2a}$$
$$y = 45\,\mathrm{m}$$

pumpkin throw

Suppose you were to throw a pumpkin vertically upward from the top of a 40m high building at 10 m/s.
Neglecting air resistance, what is the maximum height above the ground reached by the pumpkin and how long after release does it reach this point ?
When does the pumpkin reach the ground ?

moment of release

$$t = 0$$

$$y_0 = 40\,\mathrm{m}$$

$$v_0 = 10\,\mathrm{m/s}$$

highest point

$$v = 0\,\mathrm{m/s}$$

$$t = ?$$

$$y = ?$$

$$v = v_0 + at$$

$$0 = v_0 + at$$

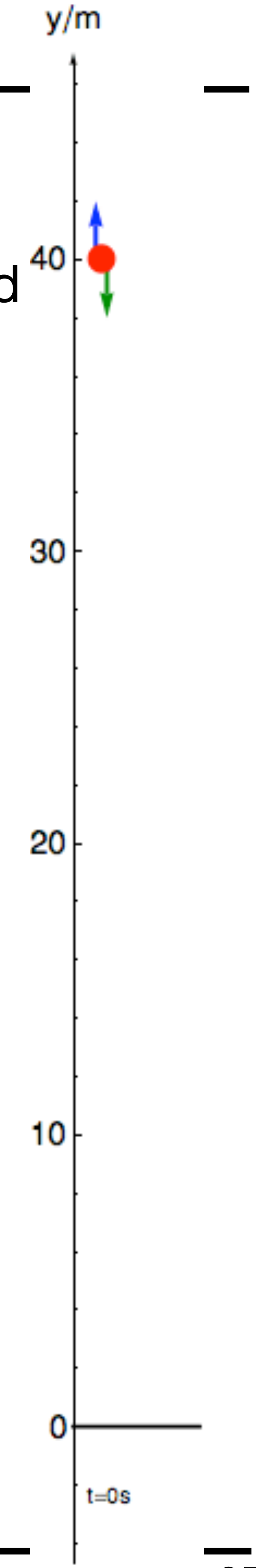
$$t = -\frac{v_0}{a}$$

$$t = 1.0\,\mathrm{s}$$

at all times

$$a = -9.80\,\mathrm{m/s^2}$$

pumpkin throw



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$$y = 0\,\mathrm{m}$$

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$$v = ?$$

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$0 = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

solve a quadratic !

$$t = \frac{1}{a}\left(-v_0 \pm \sqrt{v_0^2 - 2 a y_0}\right)$$

$$t = -2.0, \underline{4.1\,\mathrm{s}}$$

at all times

$$a = -9.80\,\mathrm{m/s^2}$$