hadron resonances from QCD (?)

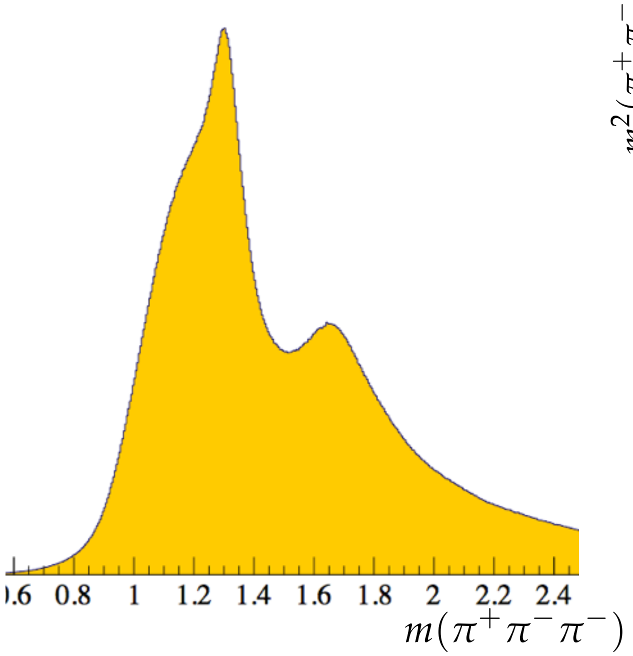
Jozef Dudek

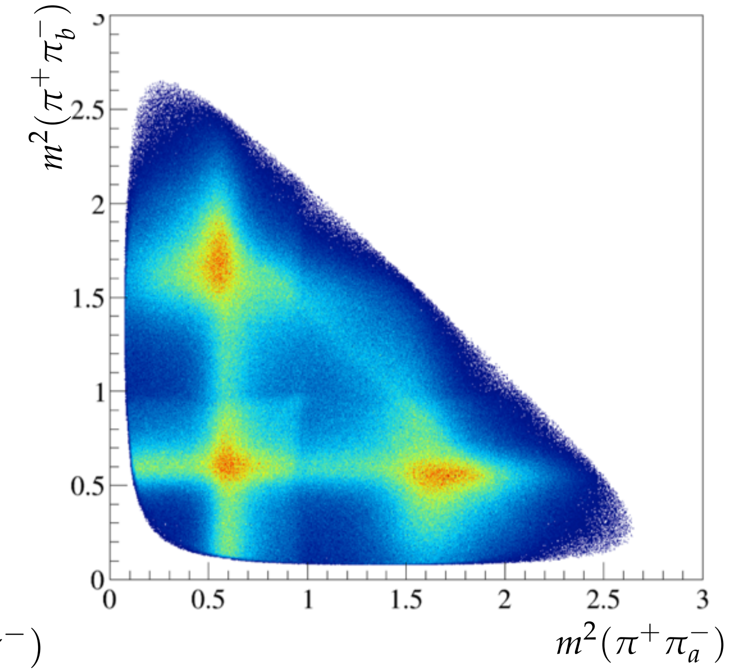
Old Dominion University

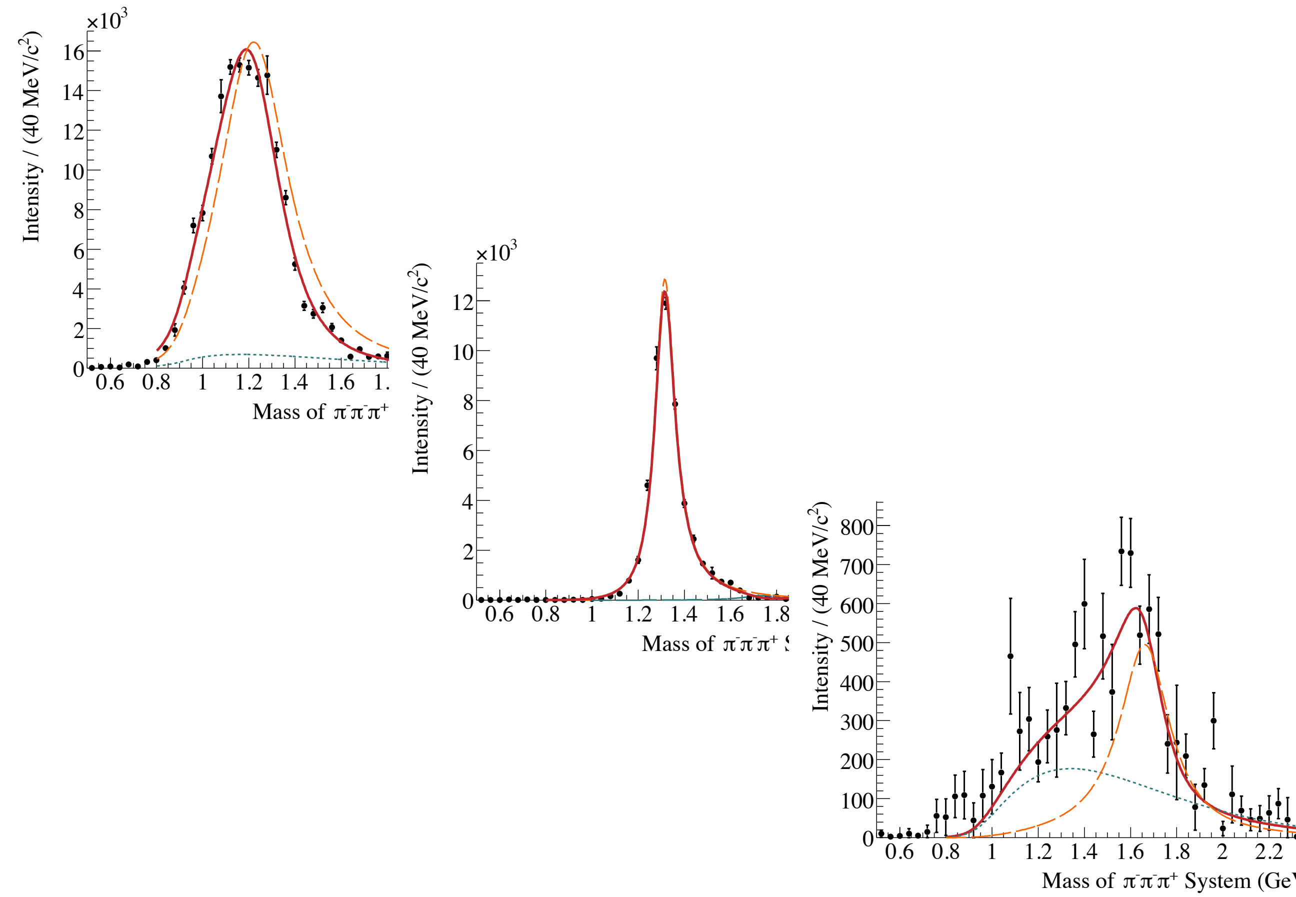
Jefferson Lab

what are we trying to do ?

how does 'complex' stuff like this ...







come from something as 'simple' as this ...

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

?

what are we trying to do ?

and is this stuff ...

... still relevant in the 'QCD age' ?

(it definitely is !)

contents

- path-integrals & quantum field theory on a lattice
- QCD (on a lattice)

i'll try to keep the formalism down to a minimum

- extracting a spectrum of QCD eigenstates

again, i'll avoid the details

- 'scattering' in a finite volume
    - elastic $\pi\pi$ scattering
    - coupled-channel scattering, the case of $\pi K, \eta K$

illustrate the idea with quantum mechanics

can we extract resonance info?

- extensions: other coupled systems, external currents, many-body decays …

(lattice) quantum field theories

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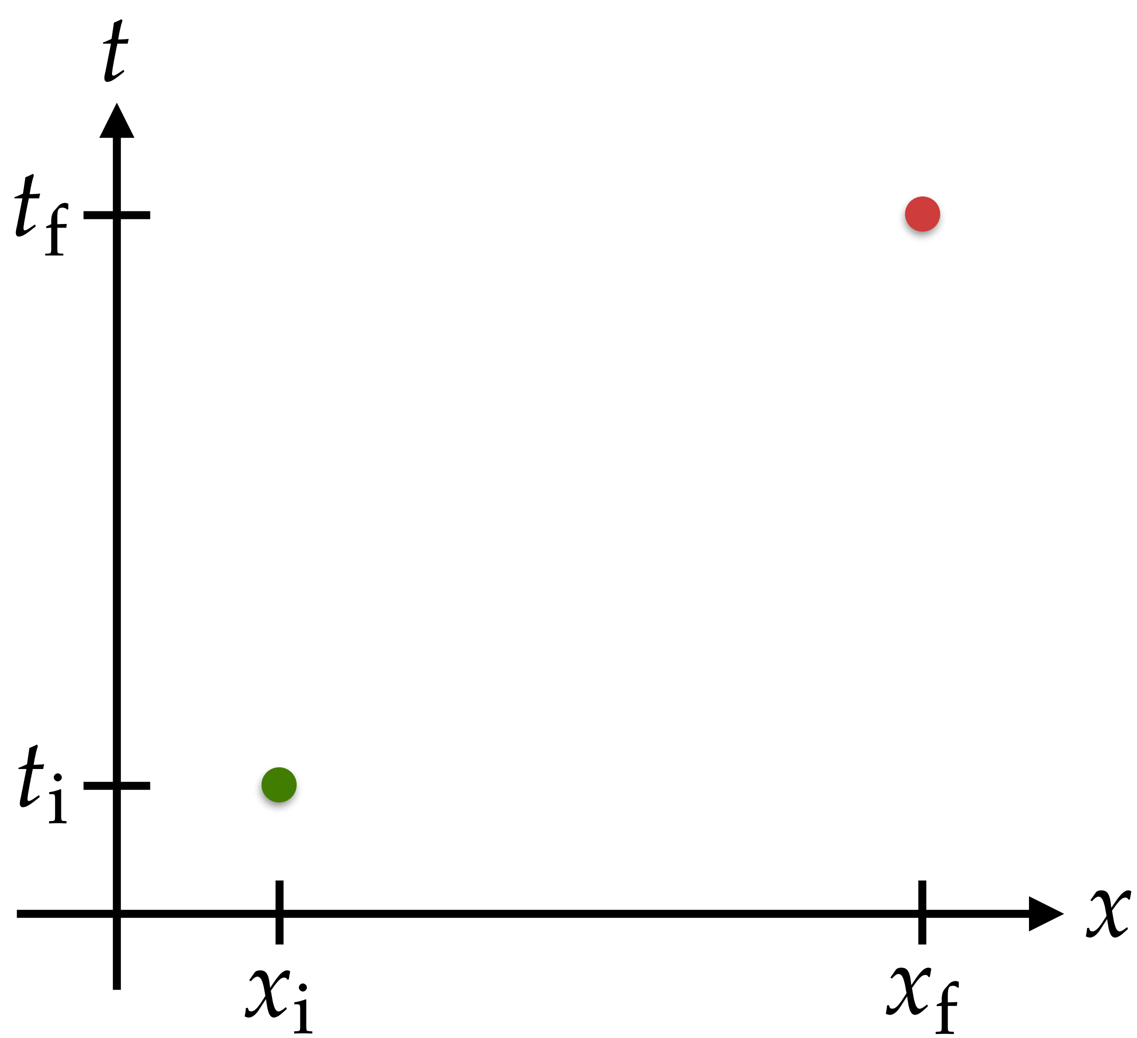
path-integrals in quantum mechanics

e.g. a free particle moving between a
fixed initial position (x_i,t_i)
and a
fixed final position (x_f,t_f)

SPACE-TIME DIAGRAM



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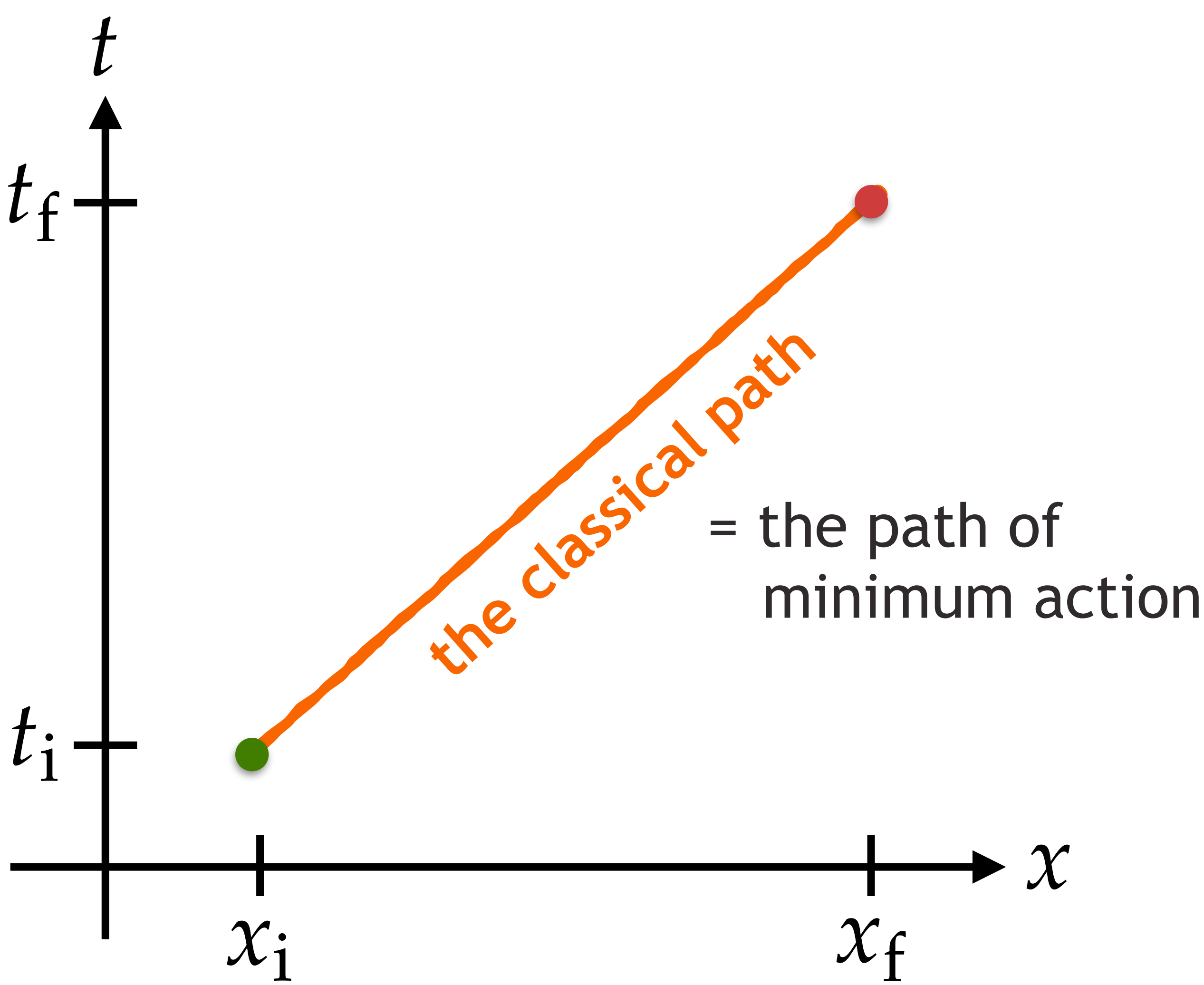

the action

$$S[x(t)] = \int_{t_i}^{t_f} dt\, L(x, \dot{x})$$

$$L = \frac{1}{2} m \dot{x}^2$$

path-integrals in quantum mechanics

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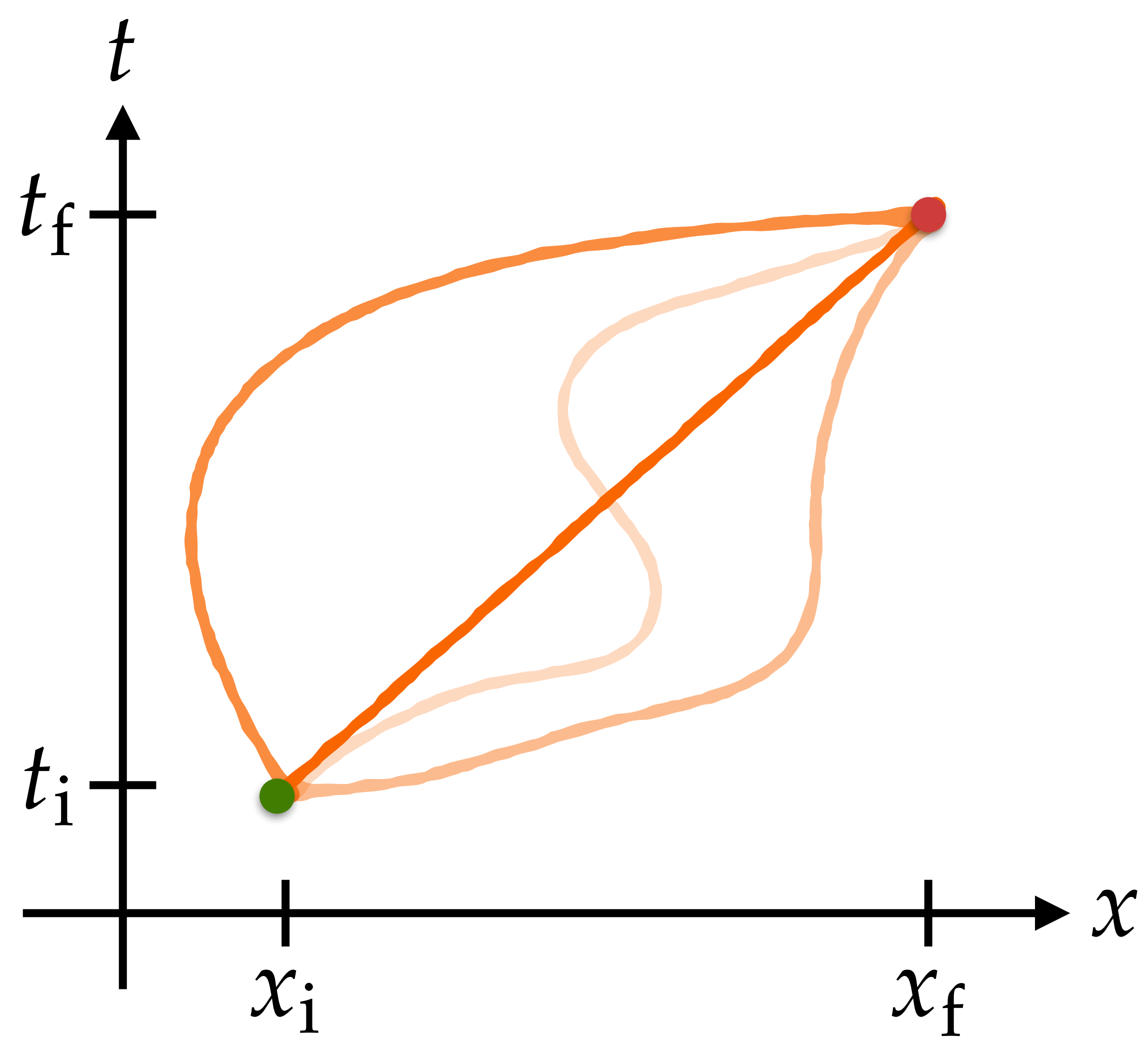


quantum mechanical amplitude

$$\langle x_f | e^{-i\hat{H}(t_f - t_i)} | x_i \rangle$$

$$= \int \mathcal{D}x \, e^{-iS[x(t)]}$$

'sum' over all paths

… the usual rules of quantum mechanics follow …

path-integrals in quantum field theory

consider a scalar field theory

REAL SCALAR FIELD LAGRANGIAN

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tfrac{1}{2}m^2\varphi^2 + V[\varphi]$$

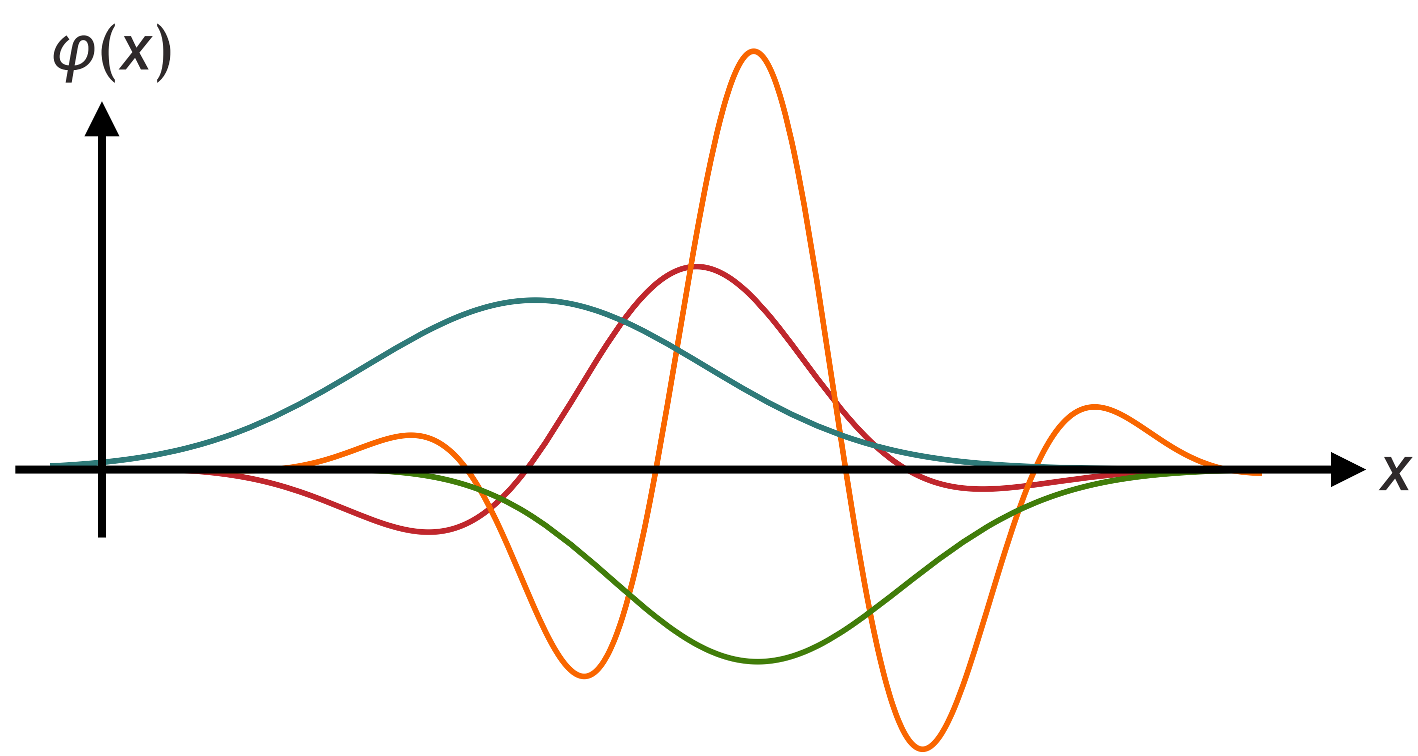
can define a path integral

$$Z = \int \mathcal{D}\varphi(x)\, e^{-iS[\varphi(x)]}$$

'sum' over all field configurations

with action $S[\varphi] = \int d^4x\, \mathcal{L}[\varphi(x)]$

e.g. in one-dimension



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'sum' over all field configurations

with action $S[\varphi] = \int d^4x\, \mathcal{L}[\varphi(x)]$

and correspondingly **correlation functions**

$$\langle 0 | \hat{\varphi}(x'')\hat{\varphi}(x') | 0 \rangle = \frac{1}{Z}\int \mathcal{D}\varphi(x)\; \varphi(x'')\, \varphi(x')\, e^{-iS[\varphi(x)]}$$

lattice quantum field theory

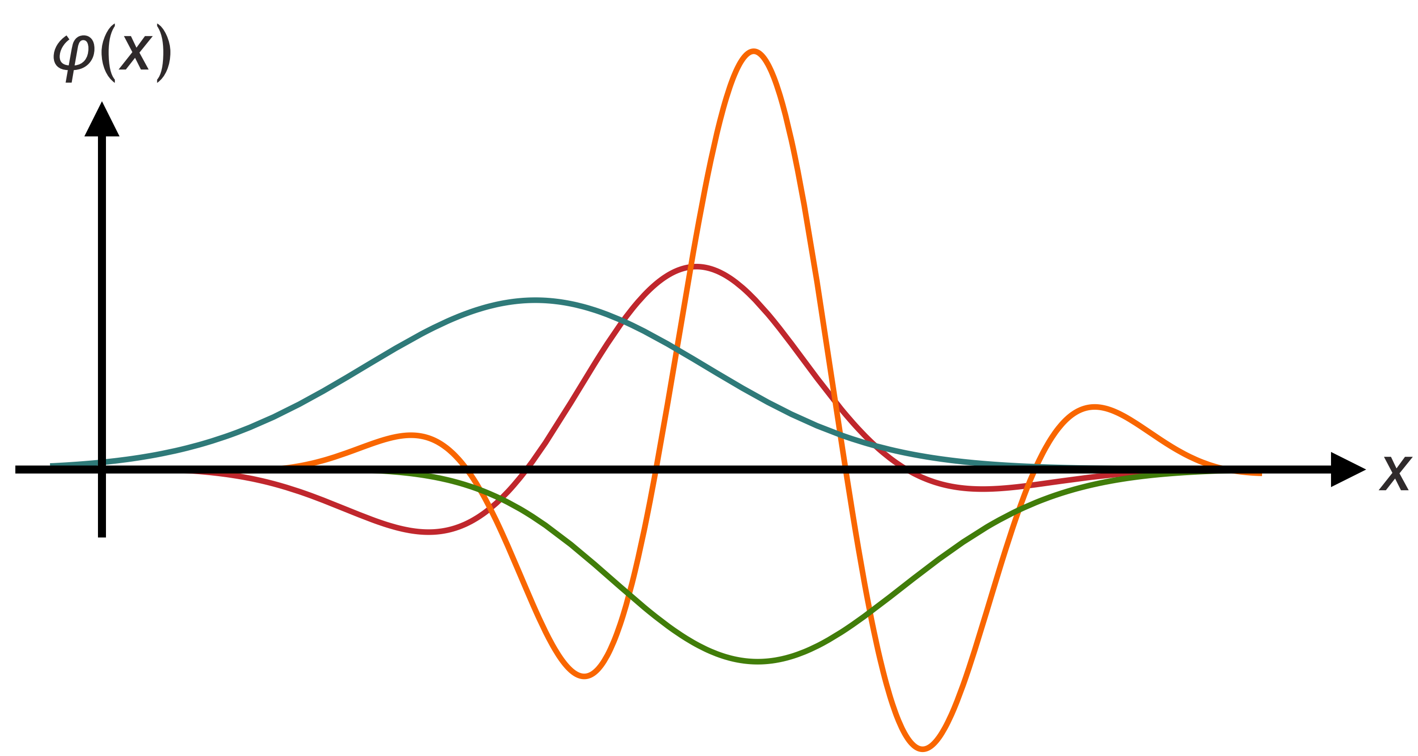
a concrete meaning for $\int \mathcal{D}\varphi(x)$

comes from considering the fields on a space-time grid

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

do an integral over all values the field can take at each point on the grid

e.g. in one-dimension



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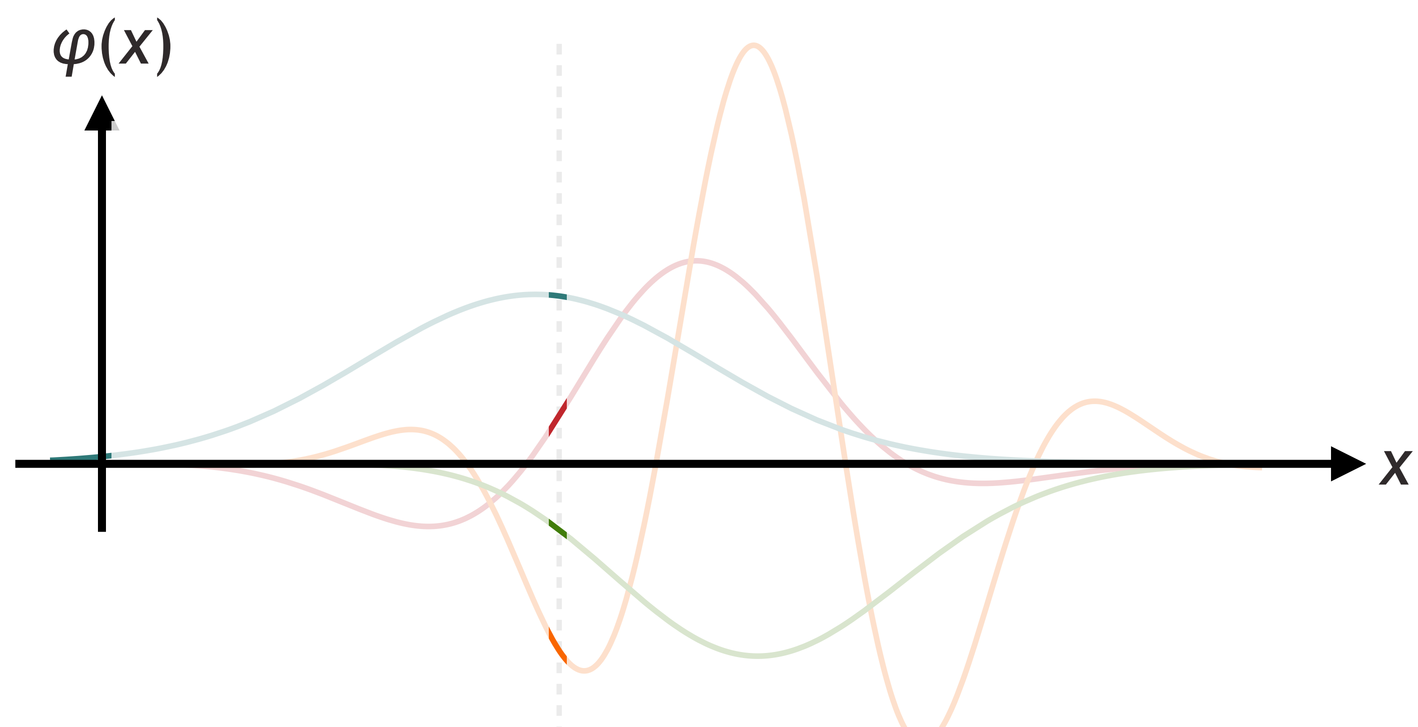

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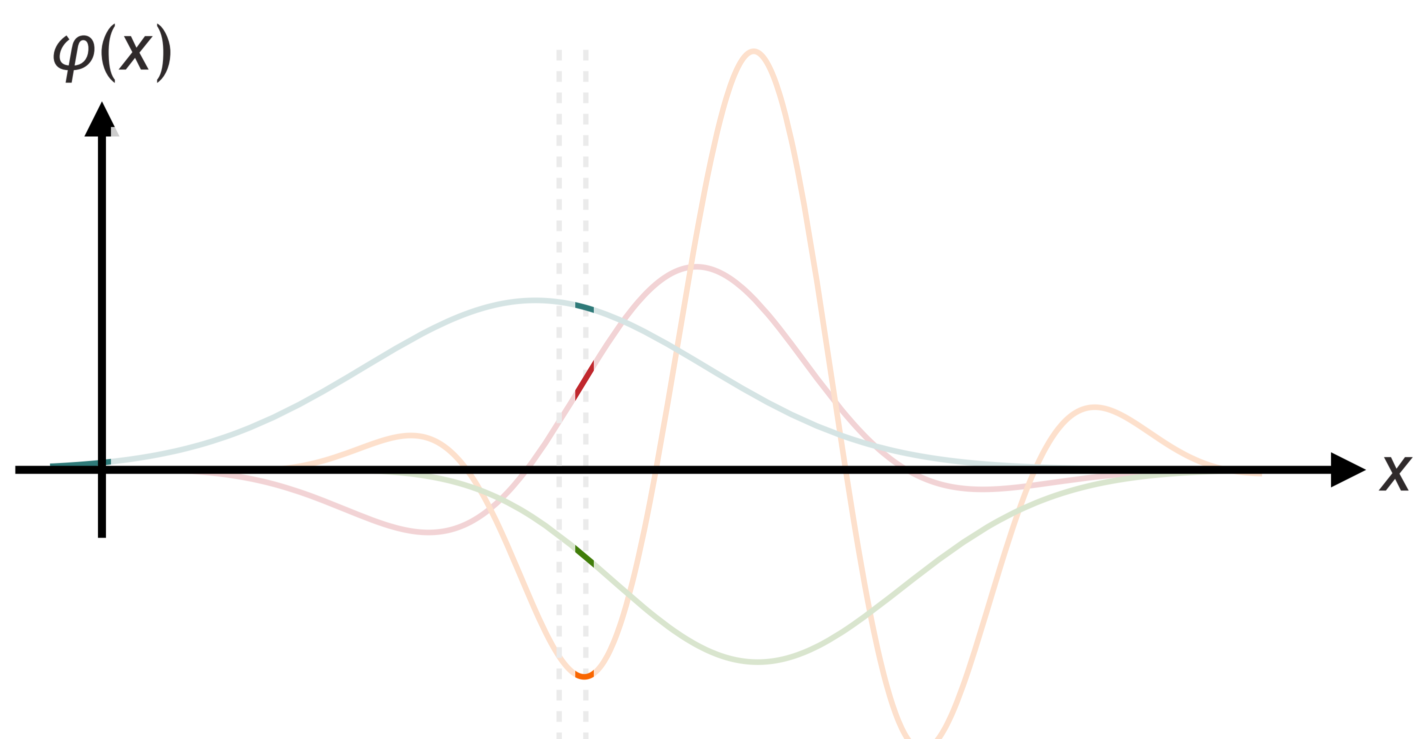

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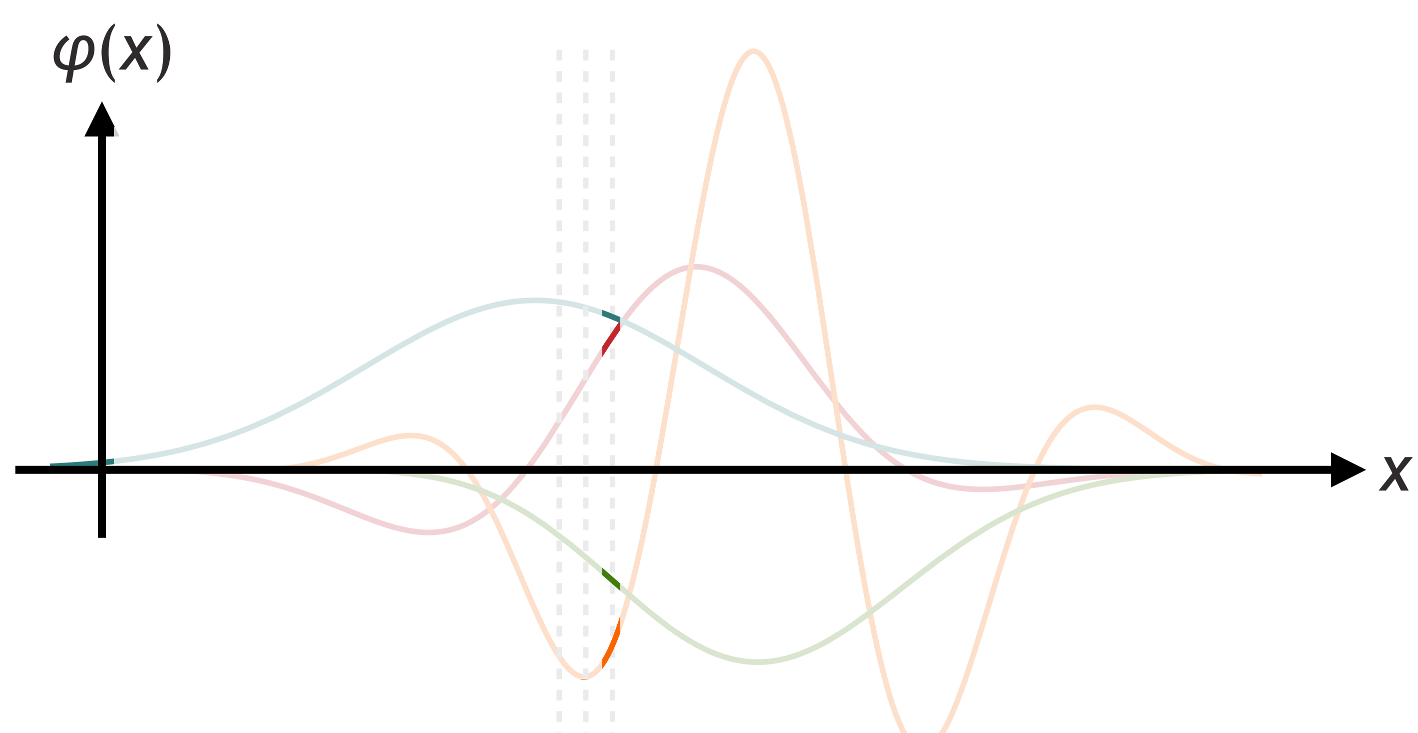

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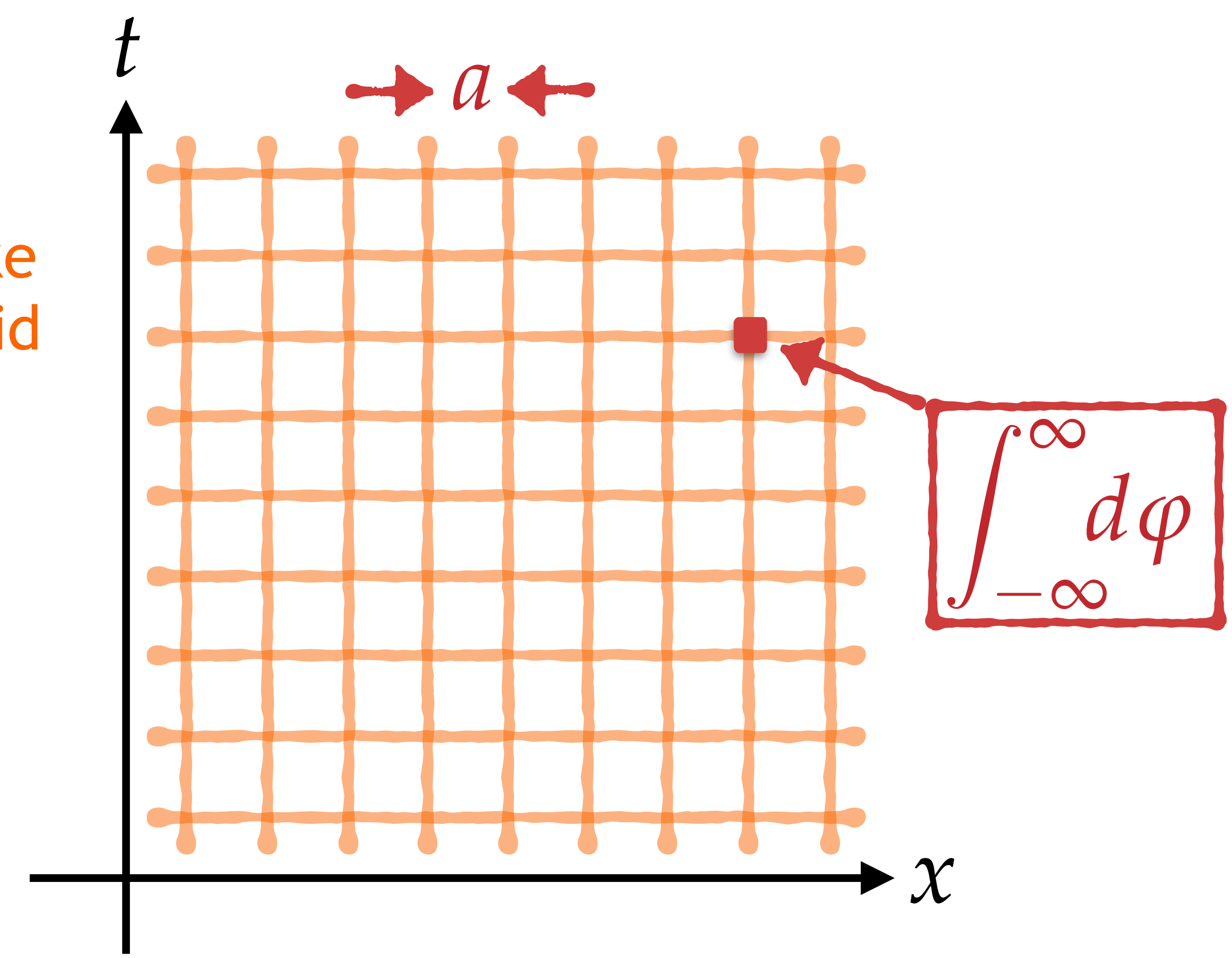
SPACE-TIME GRID



euclidean quantum field theory

$$Z = \int \mathcal{D}\varphi(x)\; e^{-iS[\varphi(x)]}$$

now make a transform to an *imaginary time variable* $t \to -i\tau$

then the argument of the exponential becomes

$$-iS = -i\int d^3x\, dt\, \mathcal{L} = -\int d^3x\, d\tau\, \mathcal{L}_\mathrm{E} = -S_\mathrm{E}$$

and the integrand transforms

$$e^{-iS} \to e^{-S_\mathrm{E}}$$

EUCLIDEAN PATH INTEGRAL

$$Z_\mathrm{E} = \int \mathcal{D}\varphi(x)\, e^{-S_\mathrm{E}[\varphi]}$$

computational approach

EUCLIDEAN PATH INTEGRAL

$$Z_{\mathrm{E}} = \int \mathcal{D}\varphi(x)\, \boxed{e^{-S_{\mathrm{E}}[\varphi]}}$$

probability for a field configuration $\varphi(x)$

importance sampled Monte Carlo

generate field configurations (on the space-time grid) according to the probability above

obtain an ensemble of configurations $\{\varphi_x\}^{i=1\ldots N}$

computing observables

an observable function of the field (e.g. a correlation function)

$$\langle 0 | O[\hat{\varphi}] | 0 \rangle = \int \mathcal{D}\varphi \, O[\varphi] \, e^{-S_{\mathrm{E}}[\varphi]}$$

can now be estimated as an average over the ensemble

$$\langle 0 | O[\hat{\varphi}] | 0 \rangle \approx \overline{O} = \frac{1}{N} \sum_{i=1}^{N} O[\varphi^{(i)}]$$

and the uncertainty due to the finite ensemble can be estimated via the variance on the mean

$$\epsilon(O) = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} \left(O[\varphi^{(i)}] - \overline{O} \right)^2}$$

ENSEMBLE MEAN & ERROR

$$\langle 0 | O[\hat{\varphi}] | 0 \rangle \approx \overline{O} \pm \epsilon(O)$$

two-point correlation functions

consider $\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle$

and since time evolution in Euclidean time is $\mathcal{O}(t) = e^{\hat{H}t}\mathcal{O}(0)e^{-\hat{H}t}$

we have $\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle = \langle 0 | \mathcal{O}_f(0)\, e^{-\hat{H}t}\, \mathcal{O}_i^\dagger(0) | 0 \rangle$

now let's assume the Hamiltonian has a complete set of discrete eigenstates

$$\hat{H}|\mathfrak{n}\rangle = E_\mathfrak{n}|\mathfrak{n}\rangle$$

$$1 = \sum_\mathfrak{n} |\mathfrak{n}\rangle\langle\mathfrak{n}|$$

and thus $\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle = \sum_\mathfrak{n} e^{-E_\mathfrak{n} t} \langle 0 | \mathcal{O}_f(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_i^\dagger(0) | 0 \rangle$

what about QCD ?

quantum chromodynamics

gauge theory with SU(3) ‘color’ symmetry

QCD LAGRANGIAN

$$\mathcal{L} = \overline{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi - \tfrac{1}{2}\mathrm{tr}\big(F_{\mu\nu}F^{\mu\nu}\big)$$

gauge covariant derivative $D_\mu = \partial_\mu + igA_\mu$

field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\big[A_\mu, A_\nu\big]$

QCD FIELDS

quark field $\psi^i_\alpha(x)$

- color index $i = 1\ldots3$
- Dirac spin index $\alpha = 1\ldots4$

gluon field $A^{ij}_\mu(x)$

- traceless matrix in color space
- Lorentz vector

$$= \sum_{a=1}^{8} A^a_\mu(x)\, t^a$$

quantum chromodynamics

gauge theory with SU(3) 'color' symmetry

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relativistic fermions

$$\overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi$$

color vector current

$$g\ \left(\overline{\psi}\gamma^\mu t^a \psi\right)\ A_\mu^a$$

massless gluons

$$\left(\partial_\mu A_\nu - \partial_\mu A_\nu\right)^2$$

gluon self interactions

$$g[A,A]\,\partial A\ ,\ g^2\left([A,A]\right)^2$$

QCD on a lattice

the QCD action in Euclidean space-time reads

$$\mathcal{S}_\mathrm{E} = \int d^4x_\mathrm{E}\, \overline{\psi}(\gamma_\mu D_\mu + m)\psi + \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

and we'd like to discretize this on a hypercubic grid

quark fields take (spinor) values on the sites of the grid $\psi^i_\alpha(x_\mu = a\, n_\mu)$

derivatives can be constructed as finite differences

e.g. $\partial f(x) \to \frac{1}{2a}\big(f(x+a) - f(x-a)\big)$

but what shall we do with the gluon fields … ?

‘parallel transporters’ & gauge invariance

in the continuum theory - consider a quark-antiquark field pair separated by some distance

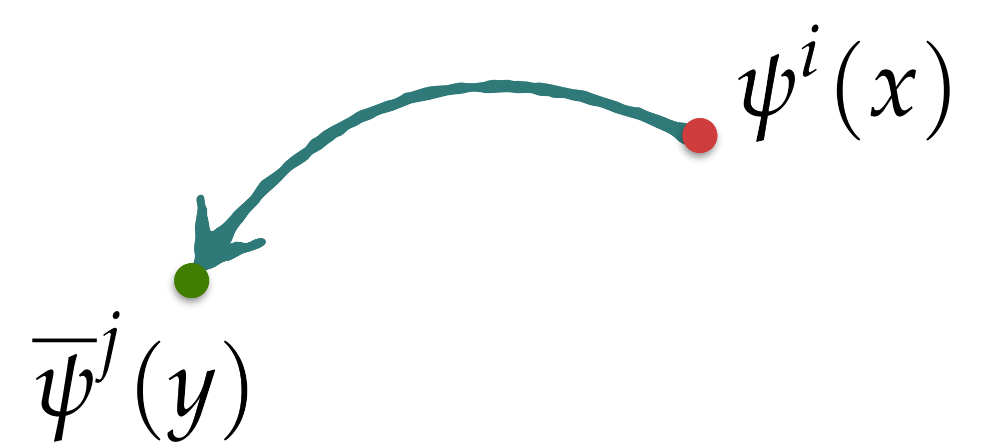
the combination $\overline{\psi}^j(y)\,\delta_{ji}\,\psi^i(x)$ is not gauge-invariant

can perform **different** local gauge transformations at *x* and *y*

a gauge-invariant combination is

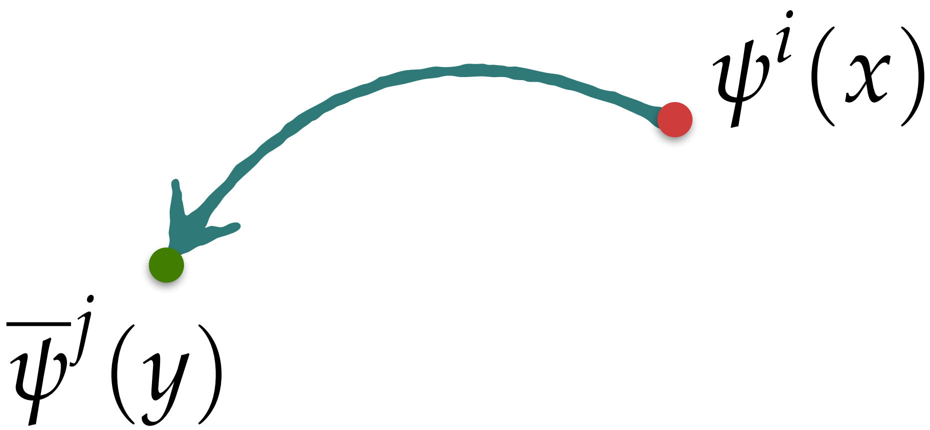
$$\overline{\psi}^j(y)\left[e^{ig\int_x^y dz_\mu A^\mu(z)}\right]_{ji}\psi^i(x)$$

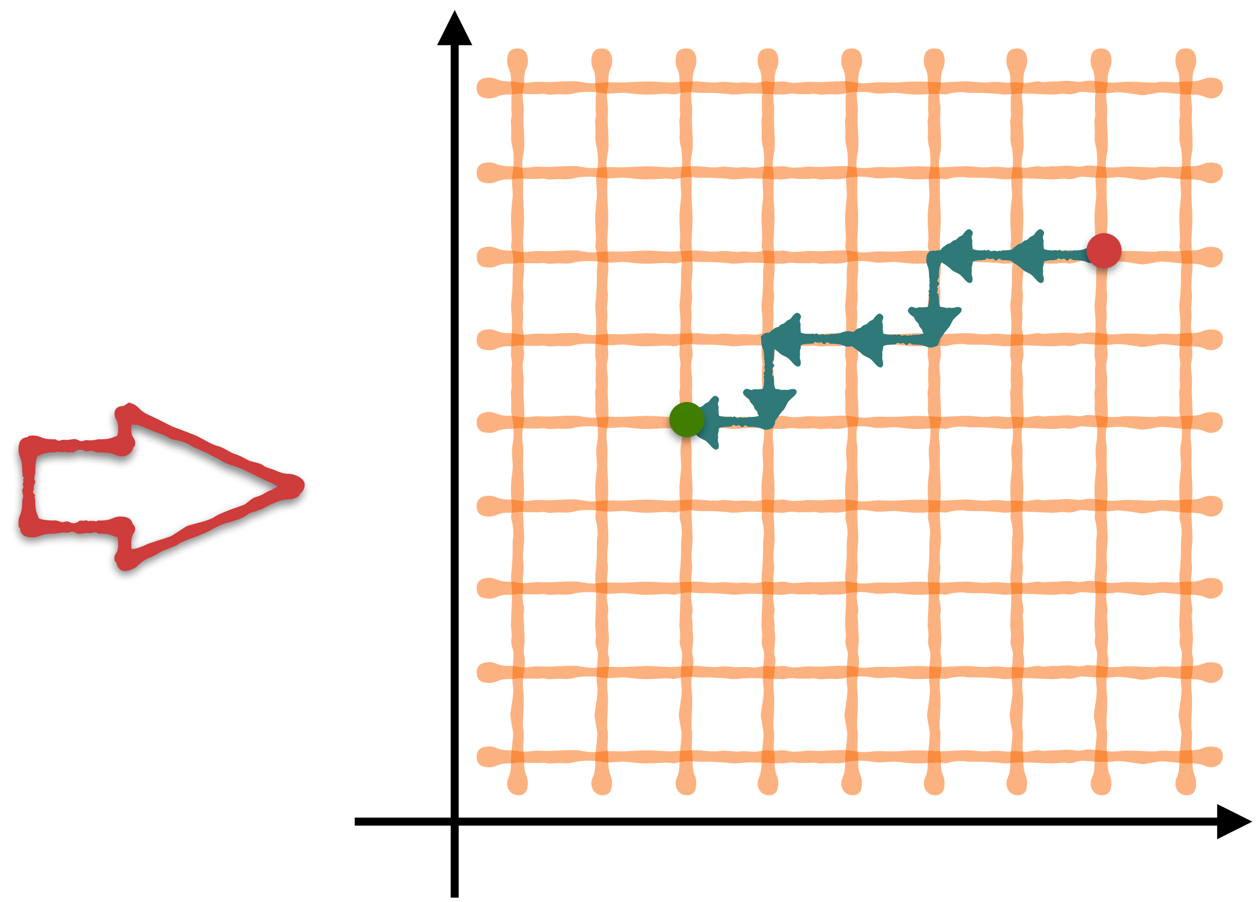
‘Wilson line’
transports the color



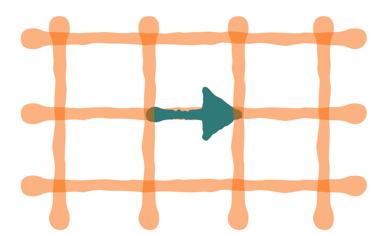
gauge links

on the lattice - can only make hops to neighboring sites





shortest path between neighboring sites = a 'link'



$$\left[e^{igaA_\mu(x)} \right]_{ji}$$

$$U_\mu(x) = e^{igaA_\mu(x)}$$

SU(3) matrix on each **link** of the lattice

lattice QCD action

gauge invariant version of a finite difference:

$$\overline{\psi}(x)\,\gamma_\mu\,U_\mu(x)\,\psi(x+\hat{\mu}a) - \overline{\psi}(x)\,\gamma_\mu\,U^\dagger_\mu(x-\hat{\mu}a)\,\psi(x-\hat{\mu}a)$$

$$\xrightarrow[a\,\to\,0]{} \; 2a\,\overline{\psi}\gamma_\mu\big(\partial_\mu + igA_\mu\big)\psi$$

… using constructions like these can build discretized actions …

$$S_\mathrm{E}^\mathrm{ferm} = \overline{\psi}_x^{i\alpha}\, M_{x,y}^{i\alpha,j\beta}[U]\,\psi_y^{j\beta}$$

integrating out the quarks

it's possible to perform **exactly** the fermion integration in the path integral

$$S_{\mathrm{E}} = S_{\mathrm{E}}^{\mathrm{ferm}} + S_{\mathrm{E}}^{\mathrm{gauge}} = \overline{\psi} M[U] \psi + S_{\mathrm{E}}^{\mathrm{gauge}}[U]$$

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, e^{-S_{\mathrm{E}}} = \int \mathcal{D}U \, e^{-S_{\mathrm{E}}^{\mathrm{gauge}}[U]} \boxed{\int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, e^{-\overline{\psi} M[U] \psi}}$$

$$= \det M[U]$$

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, e^{-S_{\mathrm{E}}} = \int \mathcal{D}U \boxed{\det M[U] \, e^{-S_{\mathrm{E}}^{\mathrm{gauge}}[U]}}$$

can treat this as the probability for configuration $U_\mu(x)$

integrating out the quarks

what happens to correlation functions ?

$$\langle 0 | \hat{\psi}^{i\alpha}(x)\, \hat{\overline{\psi}}^{j\beta}(y) | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \; \psi^{i\alpha}(x)\, \overline{\psi}^{j\beta}(y)\, e^{-S_{\mathrm{E}}}$$

correlation between
a quark field at *x* of color *i* and spin α
and
a quark field at *y* of color *j* and spin β

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \; \psi^{i\alpha}(x)\, \overline{\psi}^{j\beta}(y)\, e^{-S_{\mathrm{E}}} = \int \mathcal{D}U \, e^{-S_{\mathrm{E}}^{\mathrm{gauge}}[U]} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, \psi^{i\alpha}(x)\, \overline{\psi}^{j\beta}(y)\, e^{-\overline{\psi}\, M[U]\, \psi}$$

$$= \int \mathcal{D}U \, \Big[M^{-1}[U] \Big]^{i\alpha, j\beta}_{x,y} \det M[U] \, e^{-S_{\mathrm{E}}^{\mathrm{gauge}}[U]}$$

the probability distribution

$$= \sum_{\{U\}} \Big[M^{-1}[U] \Big]^{i\alpha, j\beta}_{x,y}$$

will need to compute
this on every configuration

a simple meson correlation function

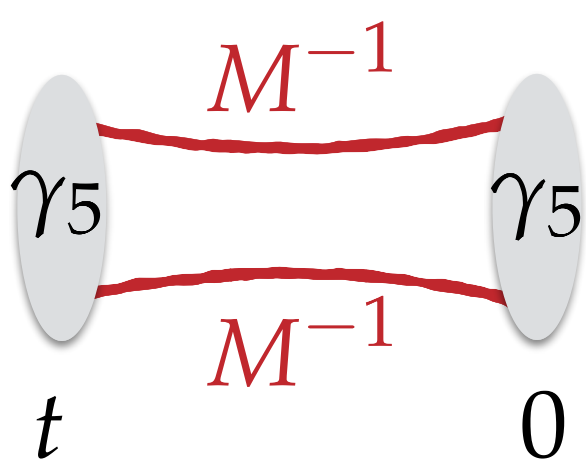
consider an actually useful correlation function

$$\langle 0 | \sum_{\vec{x}} \overline{\psi}\gamma_5\psi(\vec{x},t) \, \overline{\psi}\gamma_5\psi(\vec{0},0) | 0 \rangle$$

projected into zero momentum

pseudoscalar quantum numbers

$$= -\sum_{\{U\}} \mathrm{tr}\left[M^{-1}[U]\right]_{\vec{0}0,\vec{x}t} \gamma_5 \left[M^{-1}[U]\right]_{\vec{x}t,\vec{0}0} \gamma_5$$



$$[M[U]]_{\vec{y}t',\vec{x}t} \, \psi_{\vec{x}t} = \delta_{\vec{y},\vec{0}} \, \delta_{t',0}$$

$$\psi_{\vec{x}t} = \left[M[U]^{-1}\right]_{\vec{x}t,\vec{0}0}$$

linear system
of the form **A**x=b

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in fact there are much better ways to compute hadron correlation functions
… smearing the quark fields …
… *distillation* … PRD80 054506 (2009)

the approximations

of course we are making approximations in order to make this practical

$a > 0$

the lattice spacing plays multiple roles:

it's a momentum/energy cutoff $\Lambda \sim \frac{1}{a}$

it appears as a scale when computing dimensionfull quantities $\hat{m} = a\,m$

its size controls discretization errors

$$X(a) = X(0) + a\,\Delta X_1 + a^2\,\Delta X_2 + \ldots$$

$L < \infty$

we calculate in a finite volume

provided $L \gg \frac{1}{m_\pi}$ the effects are manageable

in fact we'll use the finite volume as a tool

the approximations

of course we are making approximations in order to make this practical

$m_q > m_q^{\text{phys}}$

calculating $\det M[U]$

or $M^{-1}[U]$ takes a lot of computer power

and the amount increases dramatically as the quark mass reduces

most current calculations use heavier than physical quarks

in principal all these are controlled approximations
that can be overcome

e.g. compute at multiple *a* values and extrapolate

extracting a spectrum

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two-point correlation functions

consider $\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle$

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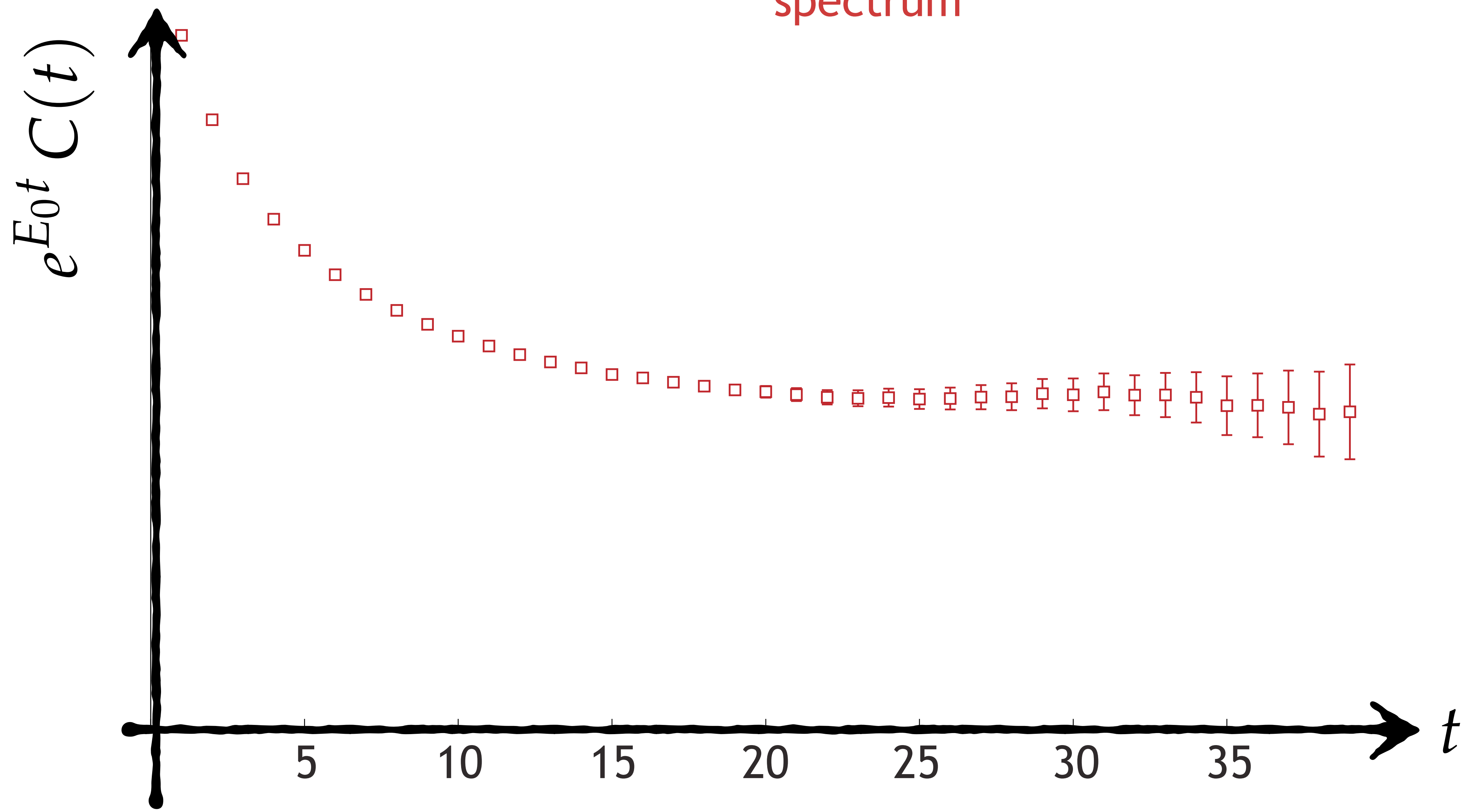
time dependence determined by the energy spectrum

amplitude to produce state *n*



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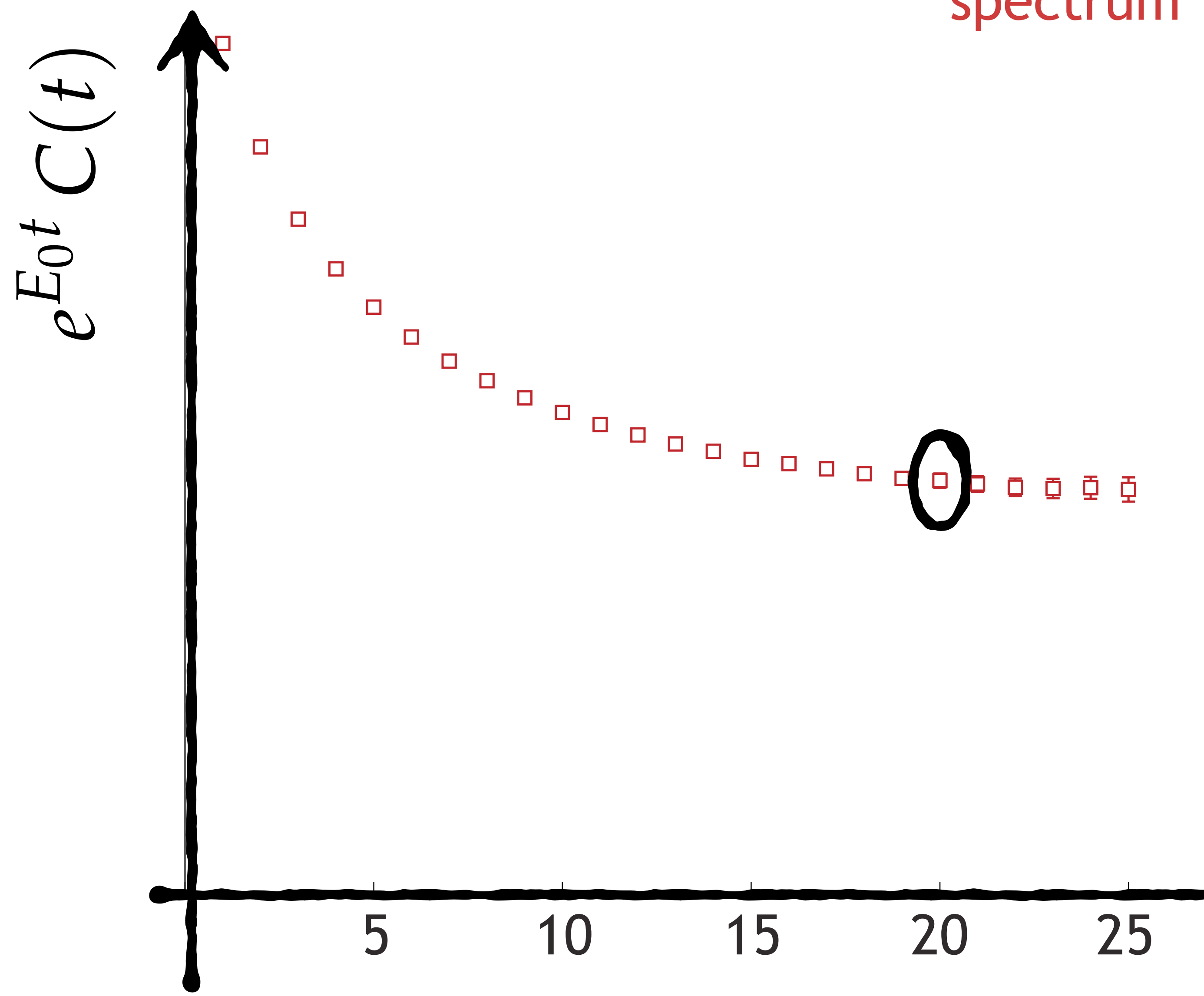
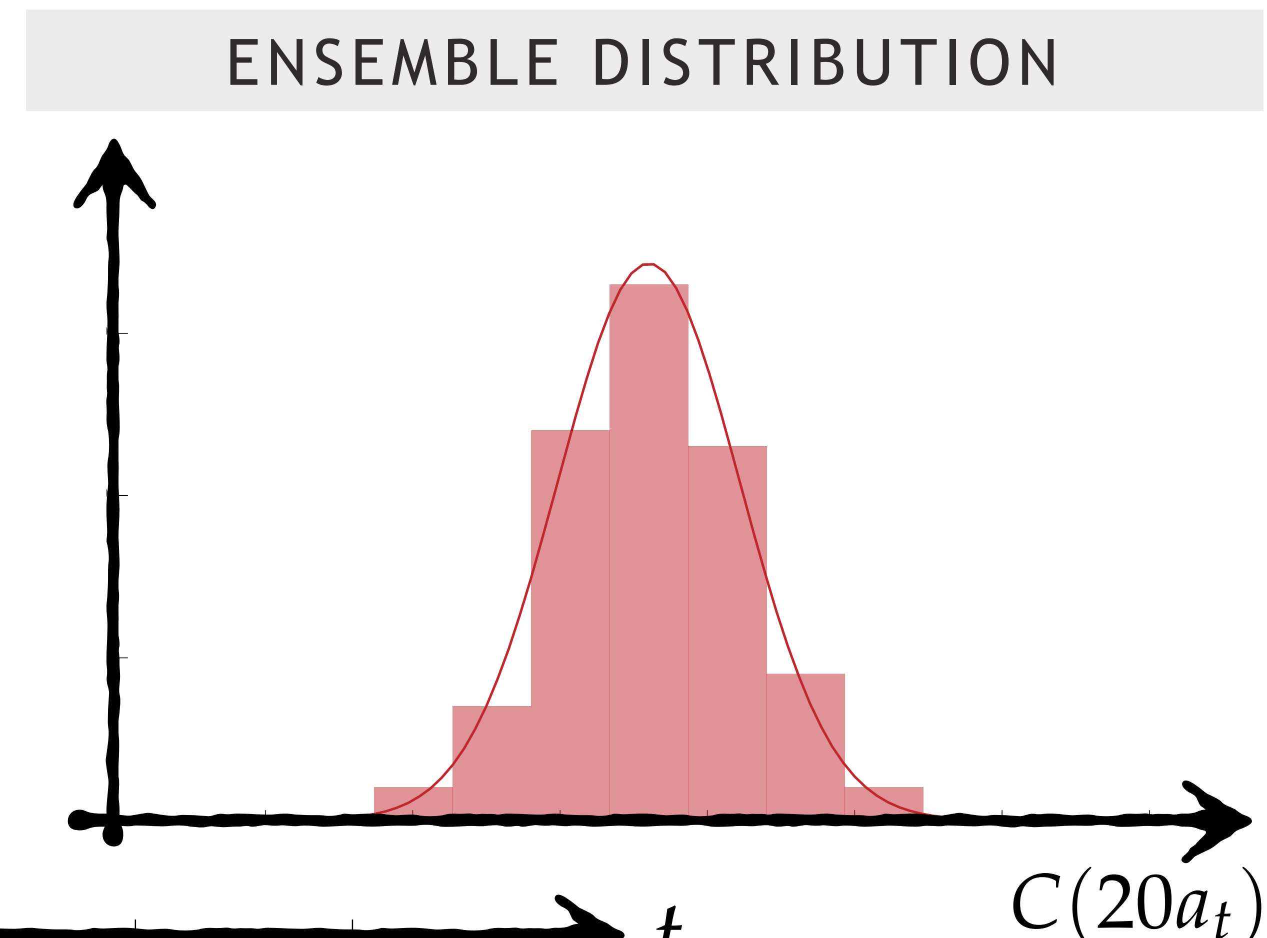




extracting the spectrum

$$\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \mathcal{O}_f(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_i^\dagger(0) | 0 \rangle$$

doing χ^2 fits to single correlator to get the excited spectrum doesn't work well

e.g. suppose two states are nearly degenerate

there is a powerful approach which uses a **basis of operators**

extracting the spectrum

$$\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \mathcal{O}_f(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_i^\dagger(0) | 0 \rangle$$

there is a powerful approach which uses a **basis of operators**

e.g. the following operators all have the quantum number of the pion

$$\bar{\psi}\gamma_5\psi$$
$$\bar{\psi}\gamma_0\gamma_5\,\vec{\gamma}\cdot\overleftrightarrow{D}\,\psi$$
$$\bar{\psi}\vec{\gamma}t_a\psi\cdot\vec{B}^a$$

and each has a different amplitude to interpolate the pion from the vacuum

$$\langle \pi | \mathcal{O}^\dagger | 0 \rangle$$

presumably some linear combination is optimal to interpolate the pion

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

and some other linear combination is optimal for the first excited state, and so on ...

‘generalized eigenvalue problem’

it turns out that this can be cast as a **variational problem** with solution

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

with $\boldsymbol{C}(t)$ a **matrix of correlation functions**

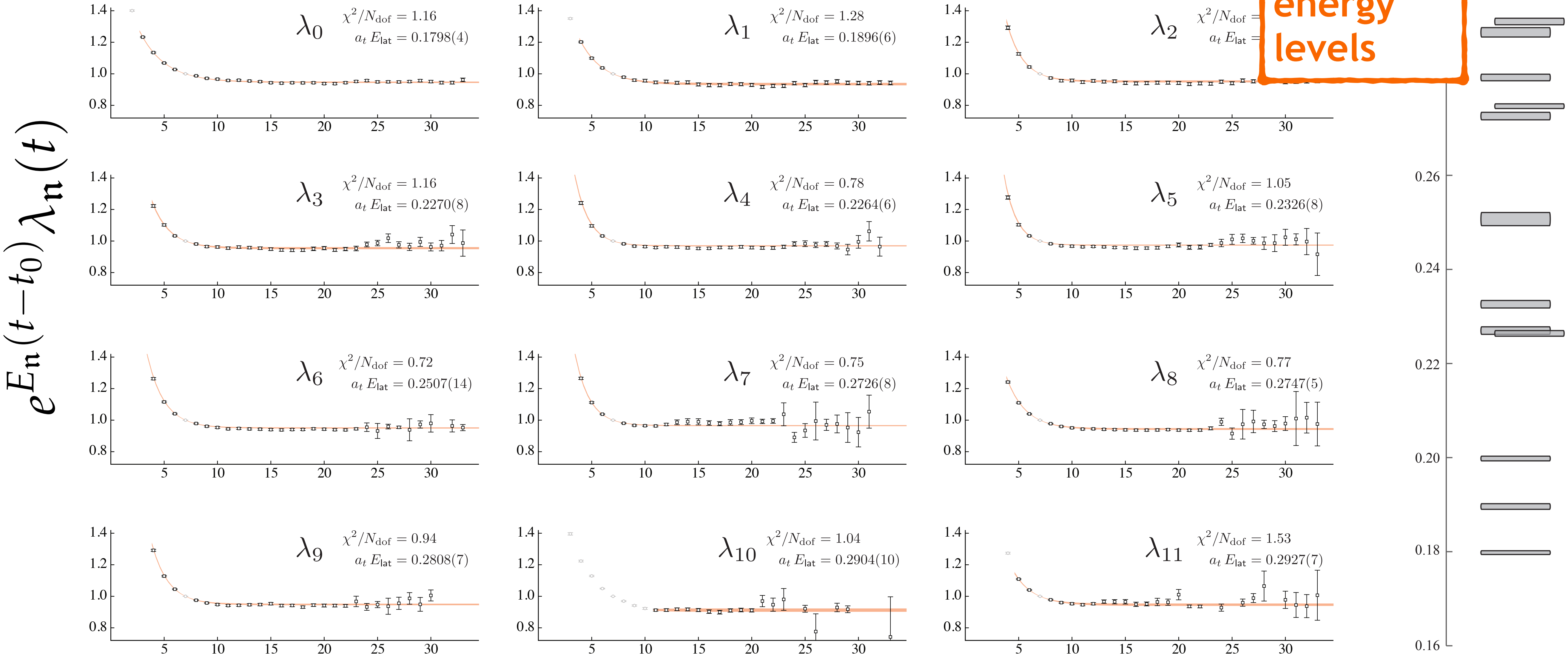
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) | 0 \rangle$$

the **eigenvectors** provide the optimal operator weights $\Omega^{\mathfrak{n}} = \sum_i v_i^{\mathfrak{n}} \mathcal{O}_i$

and the **eigenvalues** are related to the energies $\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

it works really well !

twelve (!) energy levels



i'm not telling you what quantum numbers this is ...
or what operators I used (will get to that later) ...

what operators ?

consider charmonium and ignore (for now) decays of the charmonium states

since conventional wisdom suggests charmonium states are $c\bar{c}$ bound states

fermion bilinears would seem to be sensible operators

e.g.

	J^{PC}
$\bar{\psi}\gamma_5\psi$	0^{-+}
$\bar{\psi}\psi$	0^{++}
$\bar{\psi}\gamma_i\psi,\ \bar{\psi}\gamma_0\gamma_i\psi$	1^{--}
$\bar{\psi}\gamma_5\gamma_i\psi$	1^{++}
$\bar{\psi}\gamma_0\gamma_5\gamma_i\psi$	1^{+-}

but this rather limited - can we make a larger basis ... ?

fermion bilinears

could include gauge-covariant derivatives $\overleftrightarrow{D}_i = \overleftarrow{\partial}_i - \overrightarrow{\partial}_i - 2igA_i$

e.g. $\overline{\psi}\, \overleftrightarrow{D}_i\, \psi$ transforms like $J^{PC} = 1^{--}$

e.g. $\overline{\psi}\, \gamma_i\, \overleftrightarrow{D}_j\, \psi$ transforms like ?

has 3×3=9 elements
- it's reducible

can construct irreducible operators by projecting into circular basis

$$\gamma_m = \sum_i \epsilon_i(m)\gamma_i$$
$$\overleftrightarrow{D}_m = \sum_i \epsilon_i(m)\, \overleftrightarrow{D}_i$$

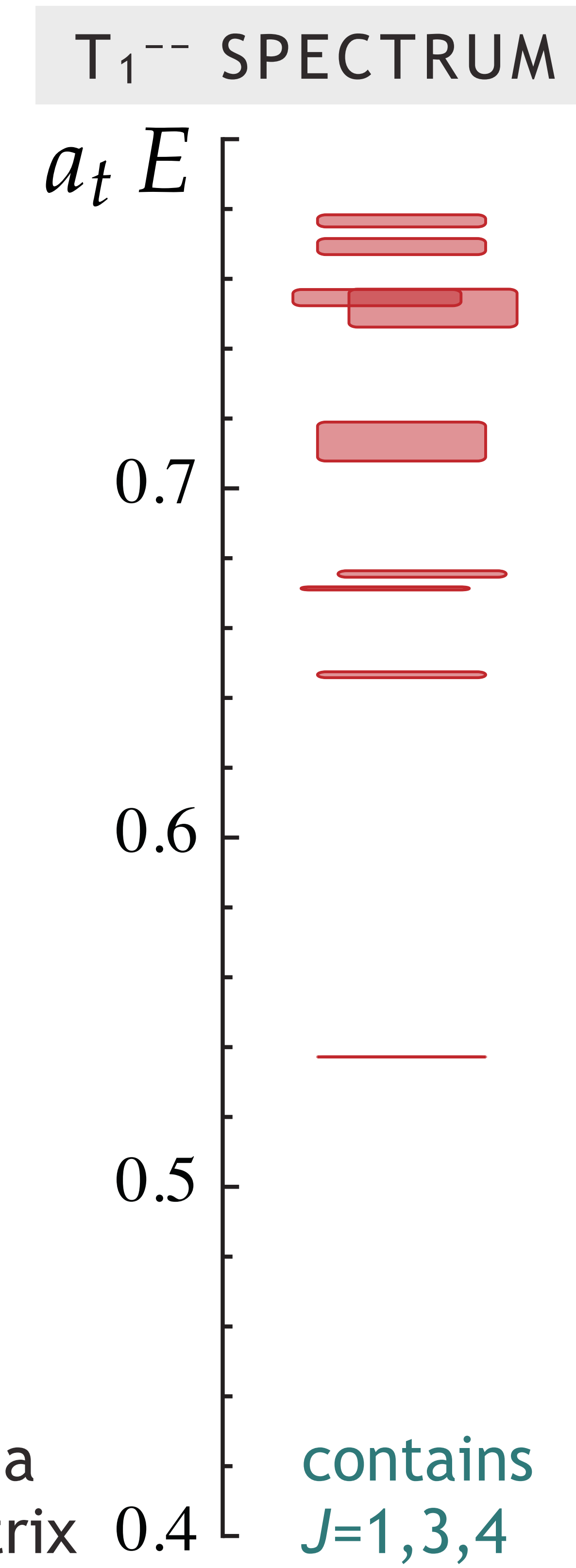
$$\vec{\epsilon}(m = \pm 1) = \mp \tfrac{1}{\sqrt{2}}\left[1, \pm i, 0\right]$$
$$\vec{\epsilon}(m = 0) = \left[0, 0, 1\right]$$

then e.g. $\left\langle 1m_1; 1m_2 \middle| 2M \right\rangle \overline{\psi}\, \gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi$ is definitely a *J*=2 operator

PRL103 262001 (2009)
PRD82 034508 (2010)

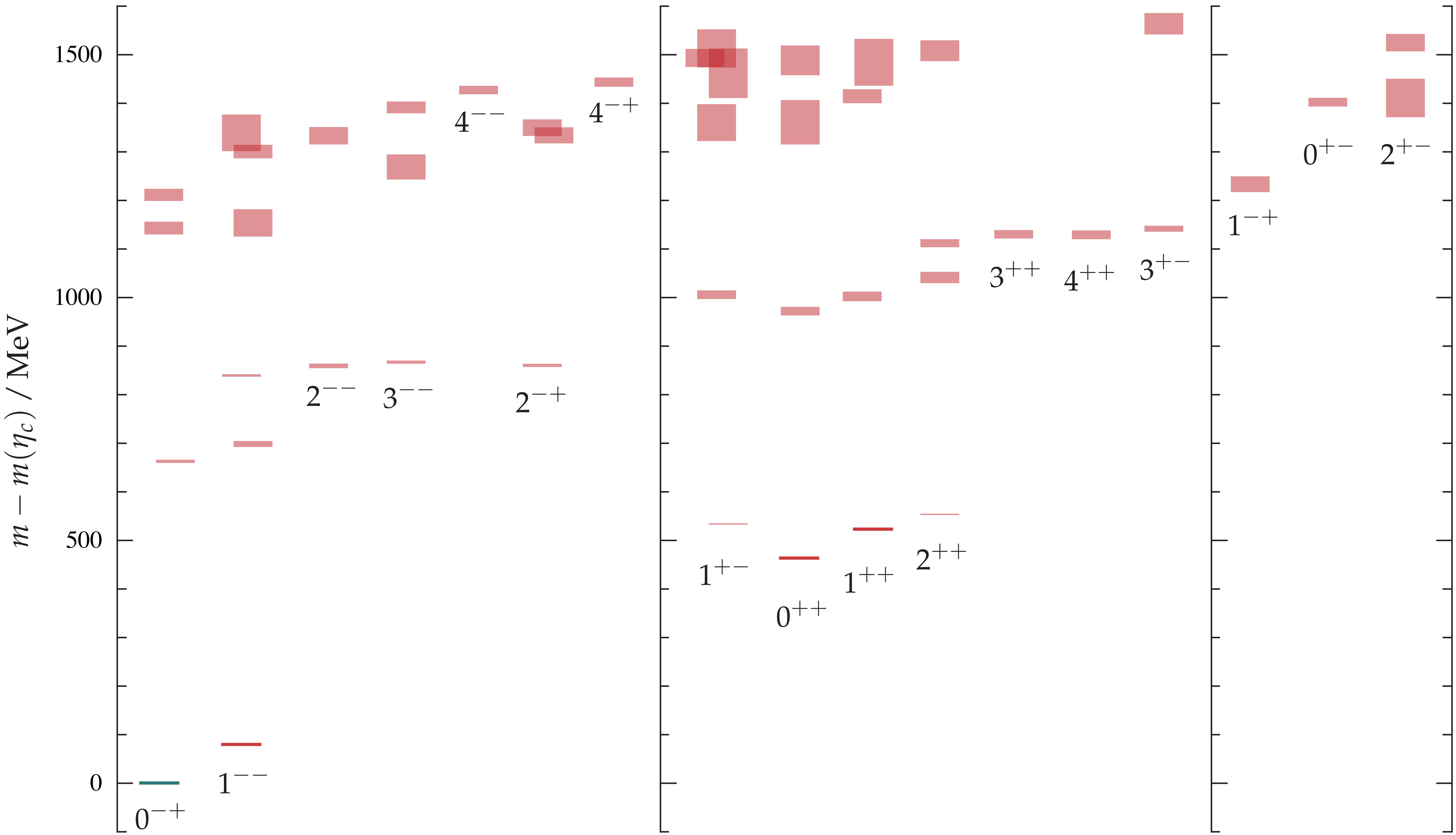
fermion bilinears

the **Hadron Spectrum Collaboration** has performed calculations with up to three derivatives



lowest 9 states from a 26×26 correlator matrix

charmonium



JHEP07 126 (2011)

... lots of interesting physics here, but I don't have the time ...

but what about *resonances* ?

I previously said:

now let's assume the Hamiltonian has a complete set of discrete eigenstates

$$\hat{H}\big|\mathfrak{n}\big\rangle = E_{\mathfrak{n}}\big|\mathfrak{n}\big\rangle$$

$$1 = \sum_{\mathfrak{n}} \big|\mathfrak{n}\big\rangle\big\langle\mathfrak{n}\big|$$

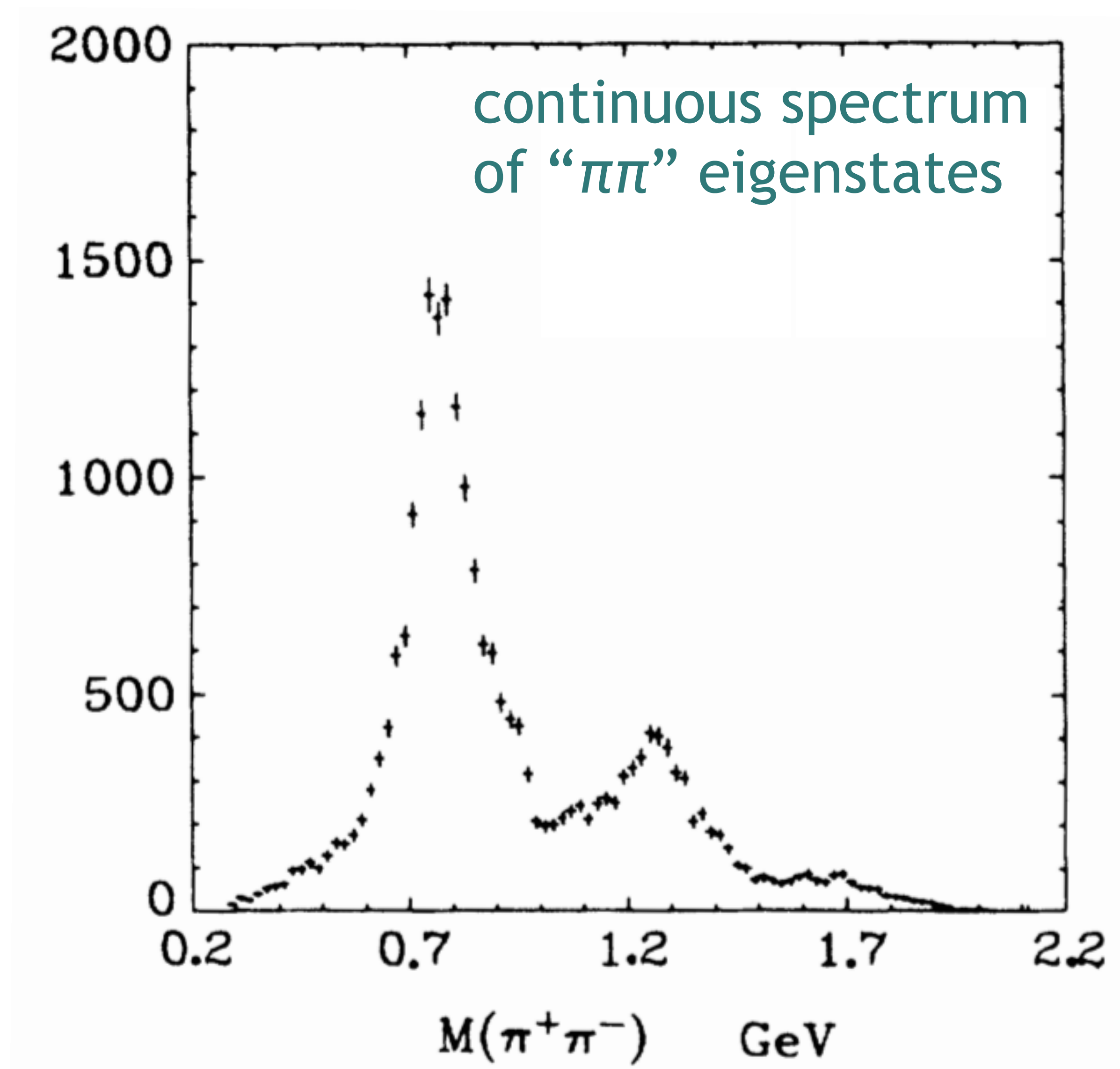
but the QCD spectrum should be continuous !

PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c†

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,‡
J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin,‖ and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 25 September 1972)



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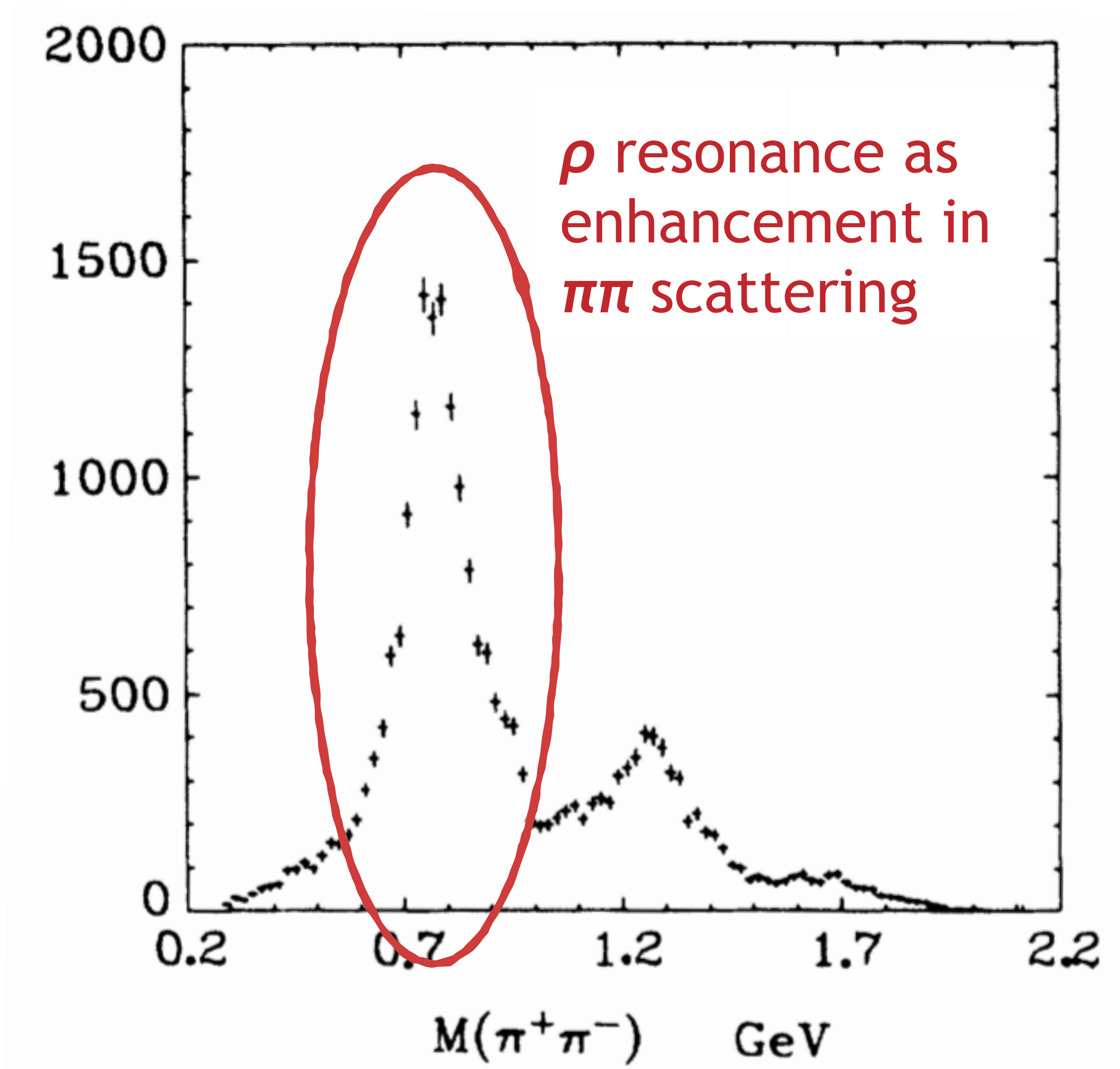
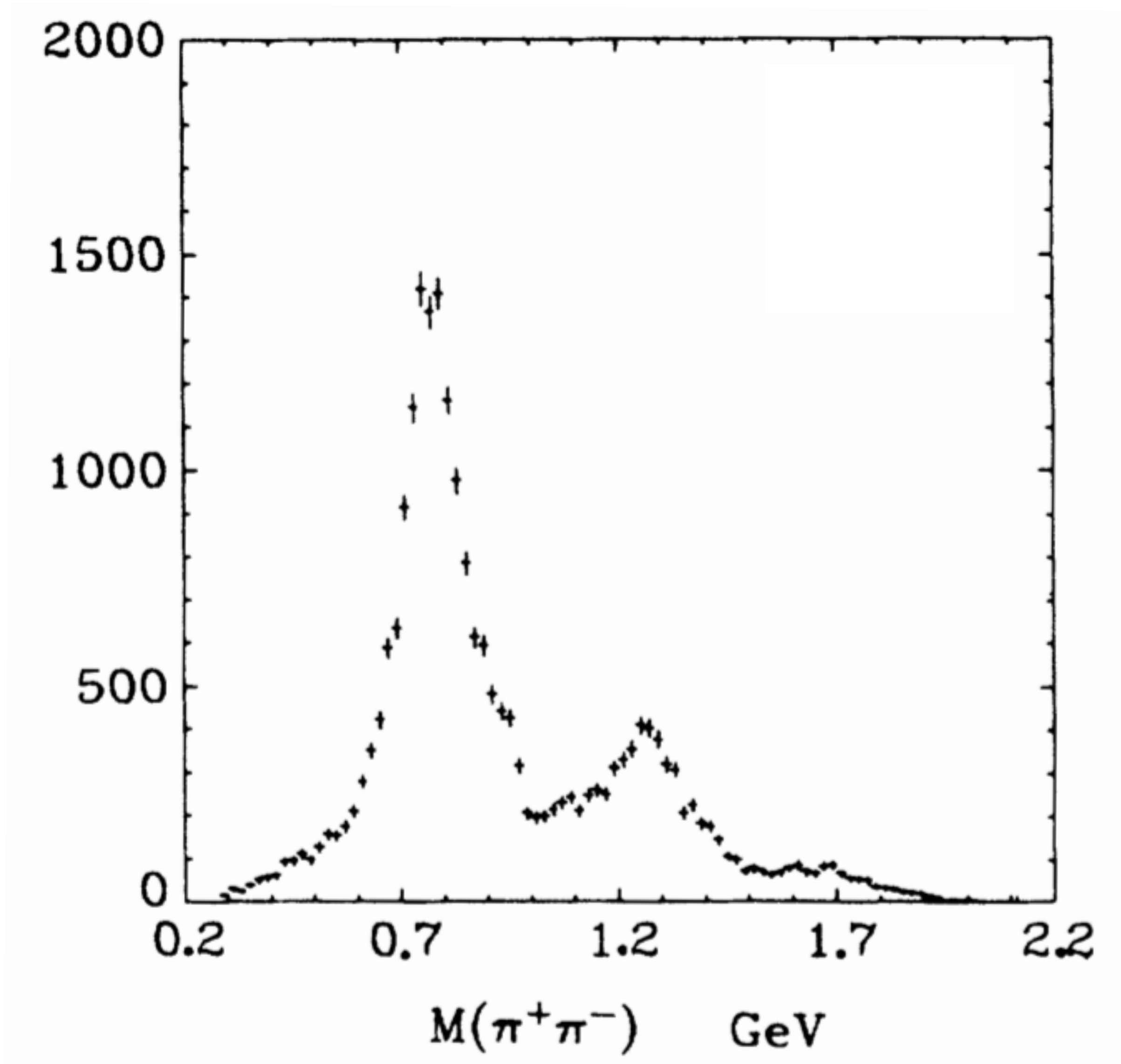


but what about *resonances* ?

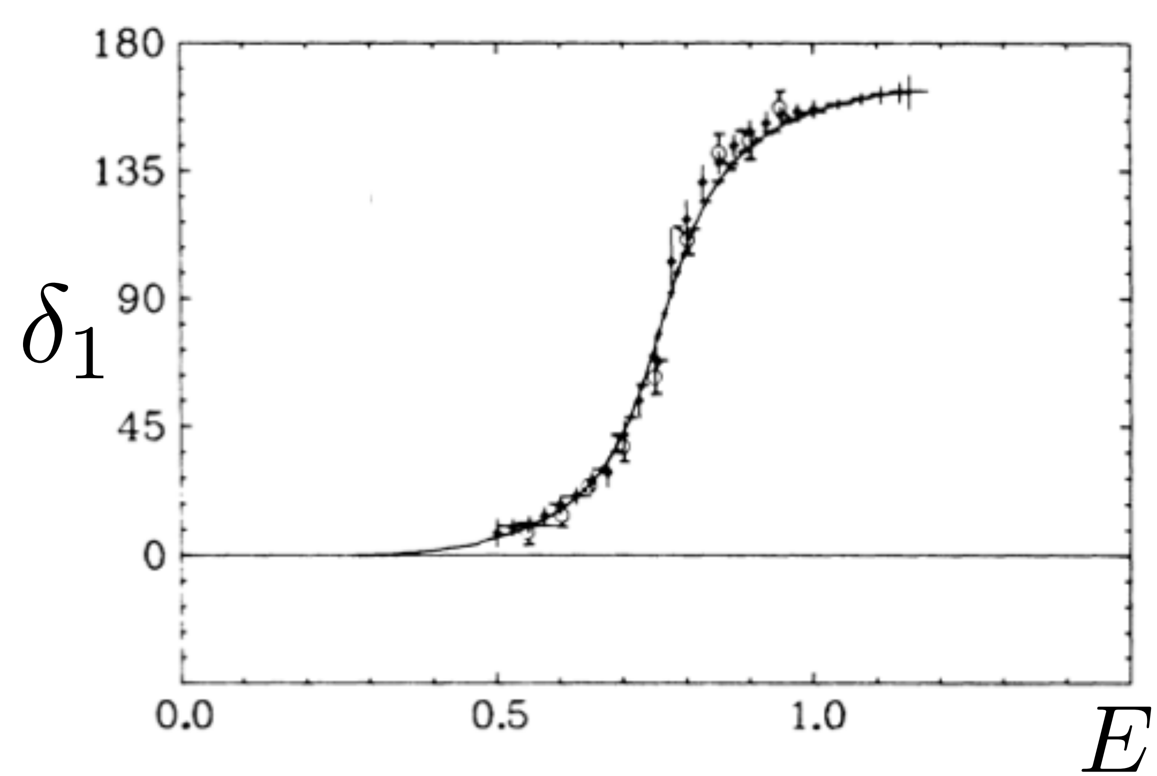
angular dependence

PARTIAL WAVE AMPLITUDE

$$f_\ell(E) = \frac{1}{2i}\left(e^{2i\,\delta_\ell(E)} - 1\right)$$



RESONANT PHASE SHIFT



scattering & physics in finite volume

Jozef Dudek

Old Dominion University

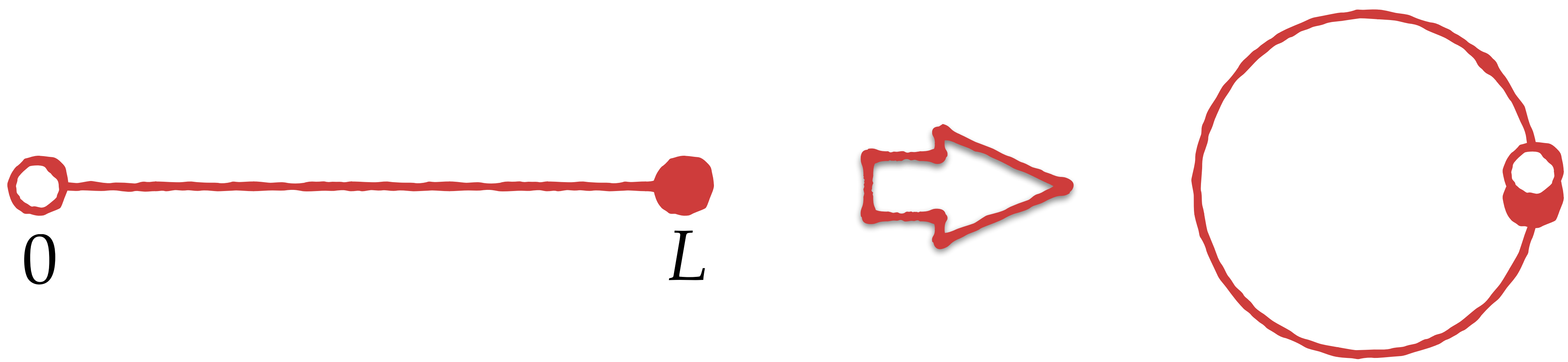
Jefferson Lab

one-dim quantum mechanics

free-particle momentum eigenstates $\psi_p(x) \sim e^{ipx}$

consider a system of length *L*

with periodic boundary conditions



$$\psi_p(0) = \psi_p(L)$$

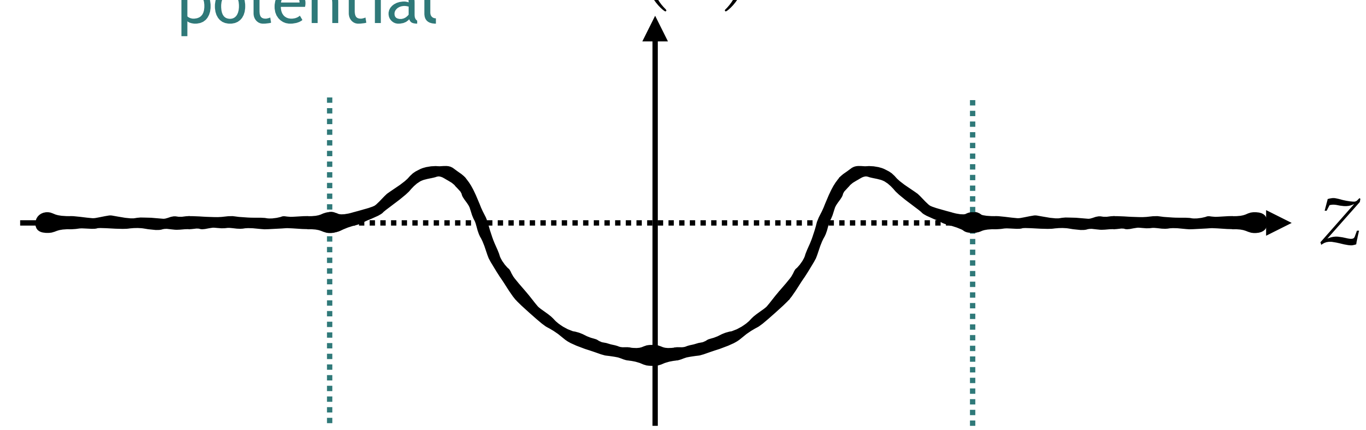
$$1 = e^{ipL}$$

$$p = \frac{2\pi}{L} n$$

quantized momentum

one-dim quantum mechanics

- consider scattering of two identical bosons separated by *z*



$$-\frac{1}{m}\frac{d^2\psi}{dz^2} + V(z)\psi(z) = E\psi(z)$$

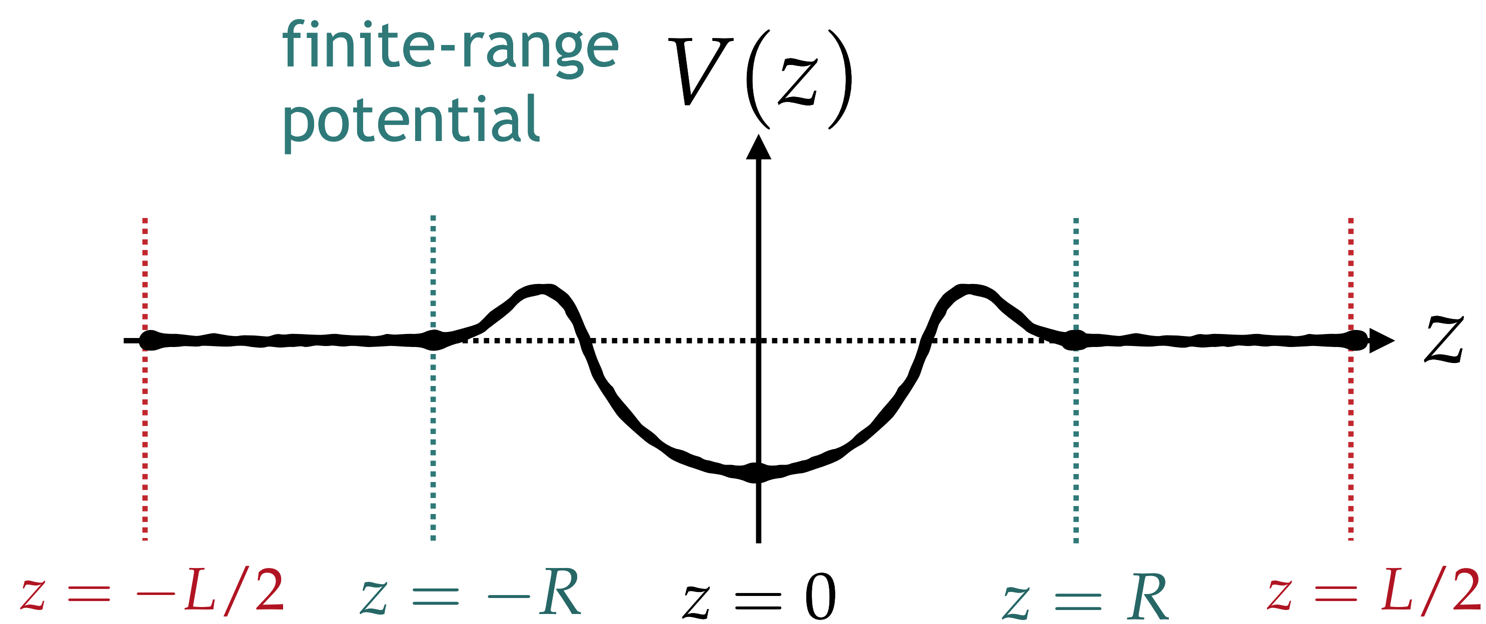
outside the well

$$\psi(|z|>R) \sim \cos\big(p|z| + \delta(p)\big)$$

$\delta(p)$ elastic scattering phase-shift

one-dim quantum mechanics

- consider scattering of two identical bosons separated by z



outside the well

$$\psi(|z| > R) \sim \cos\big(p|z| + \delta(p)\big)$$

- apply a periodic boundary condition

$$\left.\begin{aligned} \psi(-L/2) &= \psi(L/2) \\ \frac{d\psi}{dz}(-L/2) &= \frac{d\psi}{dz}(L/2) \end{aligned}\right\} \frac{pL}{2} + \delta(p) = n\pi$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

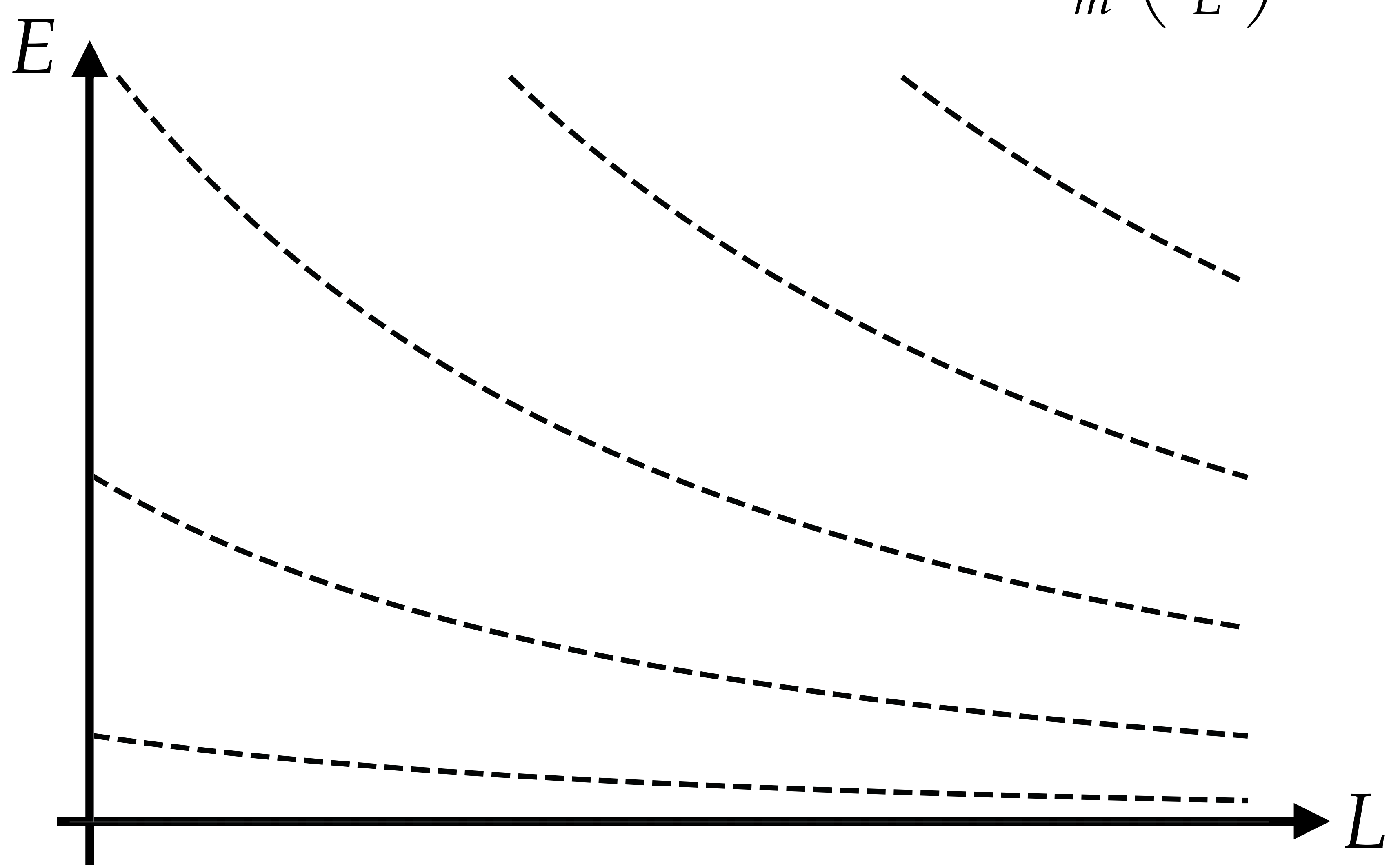
one-dim quantum mechanics

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. no interaction, $\delta(p) = 0$

$$E_n(L) = \frac{1}{m}\left(\frac{2\pi}{L}\right)^2 n^2$$

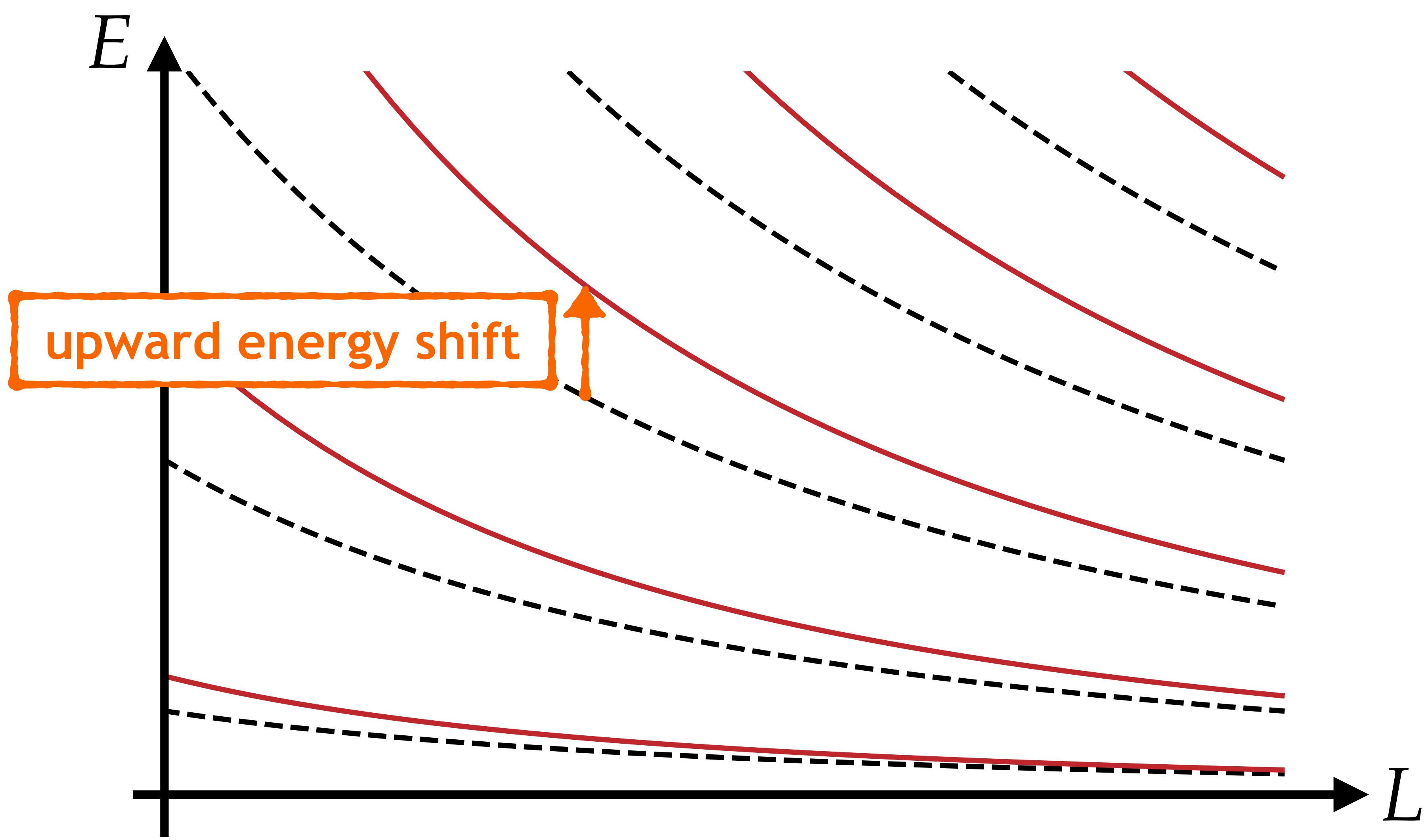


one-dim quantum mechanics

$$p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. a weak repulsive interaction, $\delta(p) = -ap$

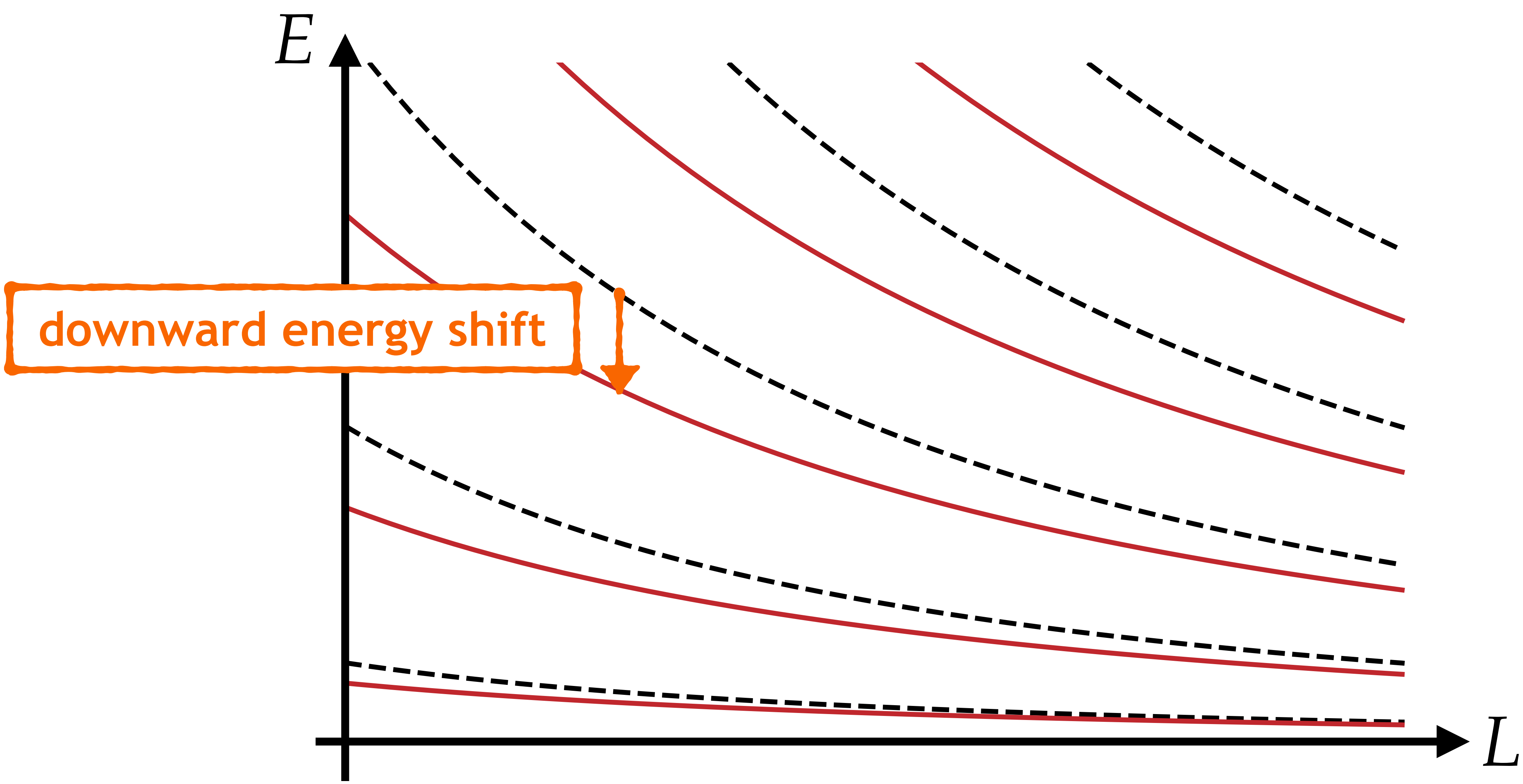


one-dim quantum mechanics

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. a weak attractive interaction, $\delta(p) = ap$

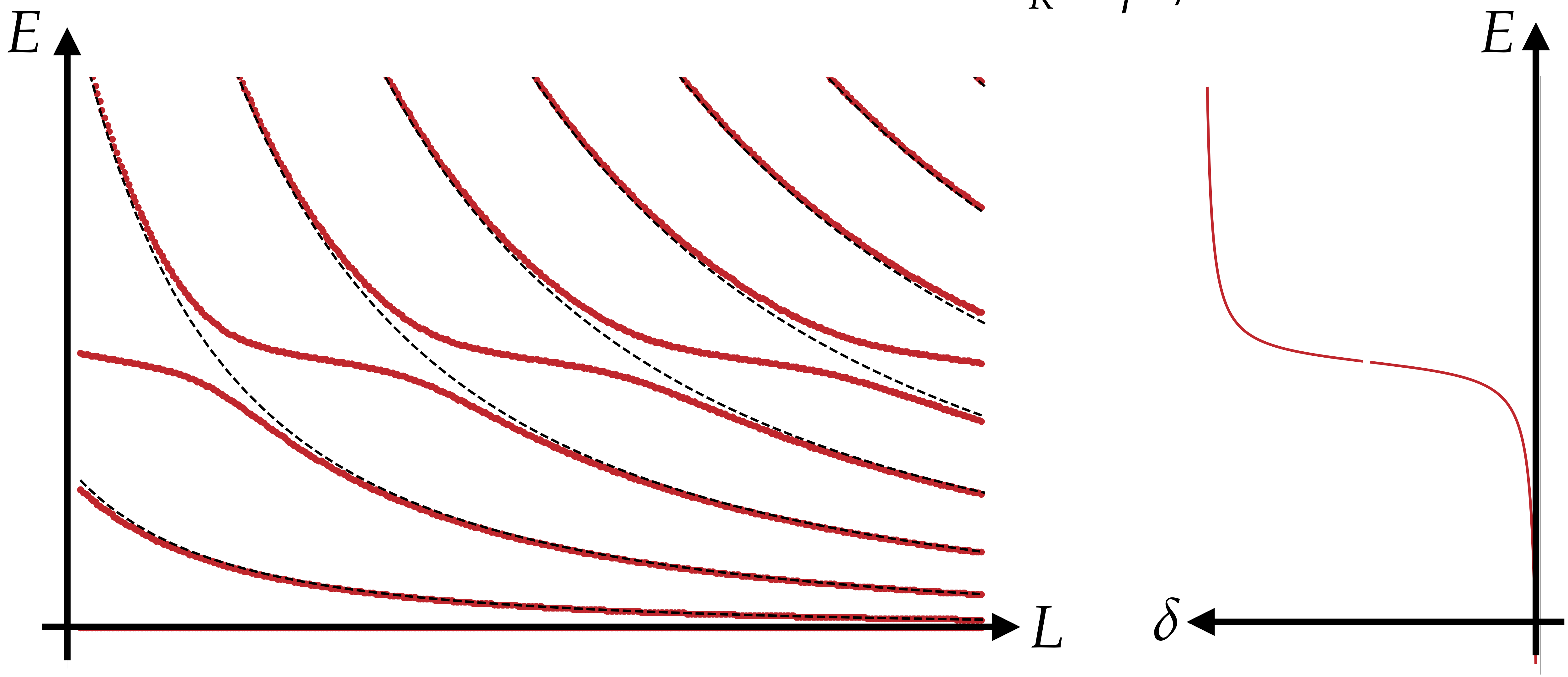


one-dim quantum mechanics

$$p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. a non-rel Breit-Wigner resonance $\tan\delta(p) = \frac{\Gamma/2}{E_R - p^2/m}$

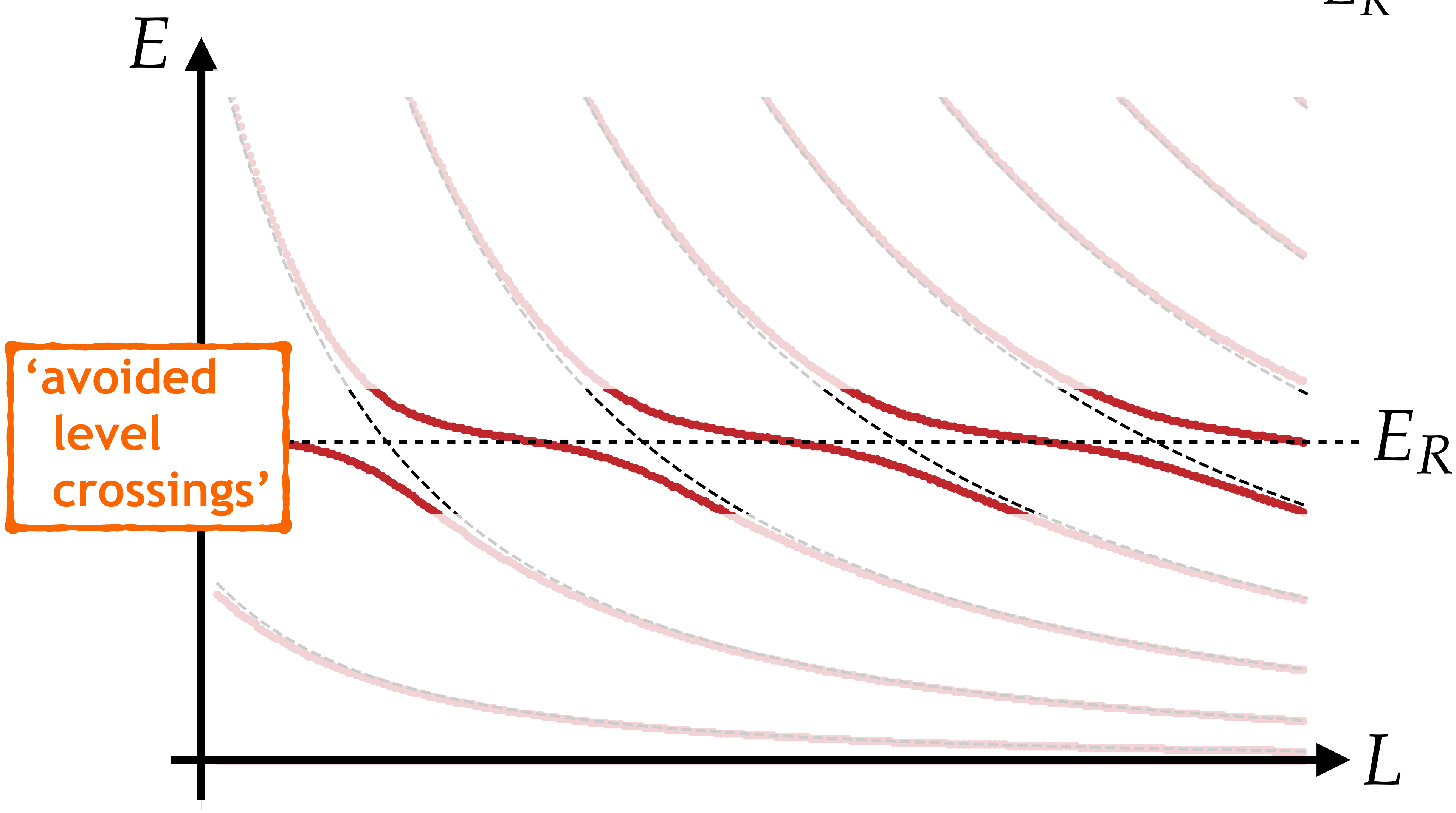


one-dim quantum mechanics

$$p = \frac{2\pi}{L} n - \frac{2}{L} \delta(p)$$

discrete energy spectrum

e.g. a non-rel Breit-Wigner resonance $\tan \delta(p) = \frac{\Gamma/2}{E_R - p^2/m}$



3+1 dim quantum field theory

periodic boundary conditions in (*x*,*y*,*z*) → periodic cube → hypertoroid

allowed free particle momenta $\vec{p} = \frac{2\pi}{L}\left[n_x, n_y, n_z\right]$

relationship between spectrum & elastic scattering phase-shift worked out by Lüscher

somewhat complicated by lack
of full rotational symmetry (cube ≠ sphere)

Lüscher NPB354 (1991) 531

ignoring the complications for a moment, we get

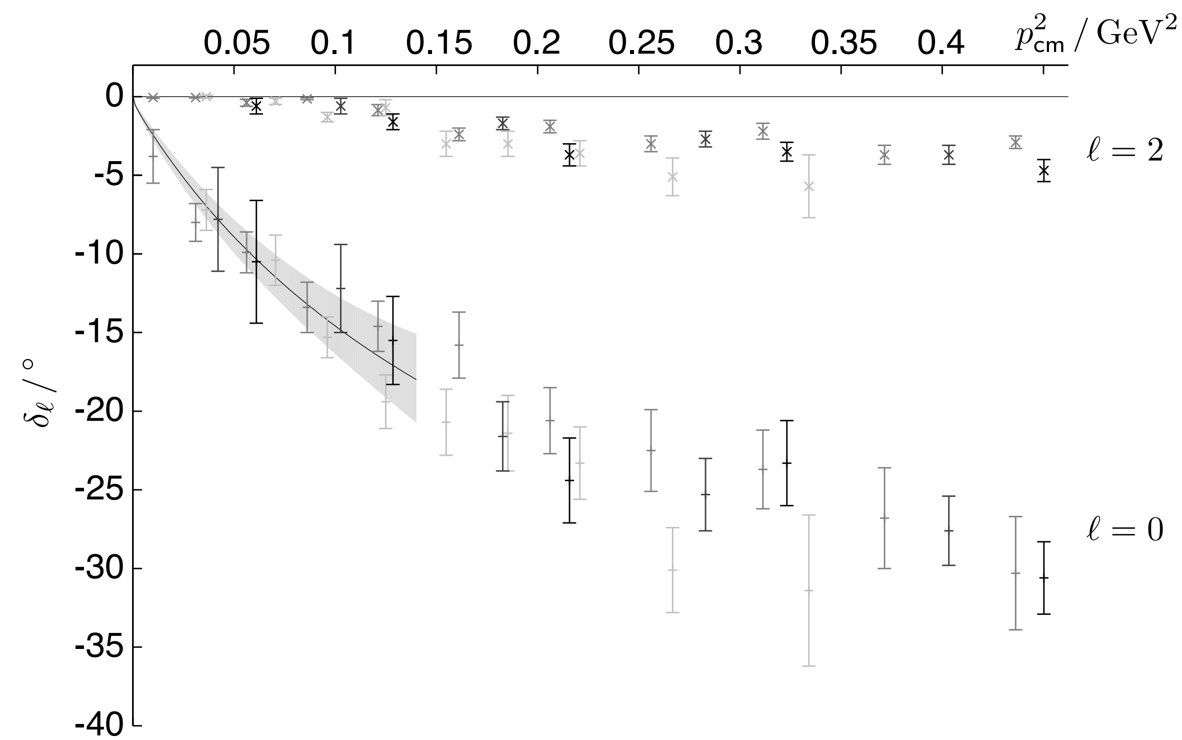
$$\delta_\ell(E) = \boxed{f_\ell(E, L)}$$

known function

$\pi\pi$ I=2 scattering

experimentally, weak and repulsive

$\pi\pi$ I=2 PHASE-SHIFT



$\pi\pi$ I=2 scattering from lattice QCD

compute the spectrum of I=2 eigenstates with $J^P = 0^+$

⇒ evaluate correlation functions with operators having these quantum numbers

what operators should we use ?

minimal quark content $\bar{u}\bar{u}dd$

since we expect the physics at low-energy to be $\pi\pi$ scattering,
how about **operators resembling a pair of pions** ?

$$\mathcal{O}_{\pi\pi}^{|\vec{p}|} = \sum_{\hat{p}} \pi^+(\vec{p})\pi^+(-\vec{p})$$

i.e. want large values of $\langle \mathfrak{n} | \mathcal{O} | 0 \rangle$

in $C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \left| \langle \mathfrak{n} | \mathcal{O} | 0 \rangle \right|^2$

ππ I=2 scattering from lattice QCD

compute the spectrum of *I*=2 eigenstates with $J^P = 0^+$

⇒ evaluate correlation functions with operators having these quantum numbers

what operators should we use ?

minimal quark content $\bar{u}\bar{u}dd$

since we expect the physics at low-energy to be ππ scattering,
how about operators resembling a pair of pions ?

$$\mathcal{O}_{\pi\pi}^{|\vec{p}|} = \sum_{\hat{p}} \pi^+(\vec{p})\pi^+(-\vec{p})$$

for pion operators use the 'variationally optimal' combinations $\pi^+ = \sum_i v_i \left(\bar{u}\mathbf{\Gamma}_i d\right)$

to make a basis, consider different relative momentum

$\pi_{[000]}\pi_{[000]}$

$\pi_{[100]}\pi_{[-100]}$

$\pi_{[110]}\pi_{[-1-10]}$

$\pi_{[111]}\pi_{[-1-1-1]}$

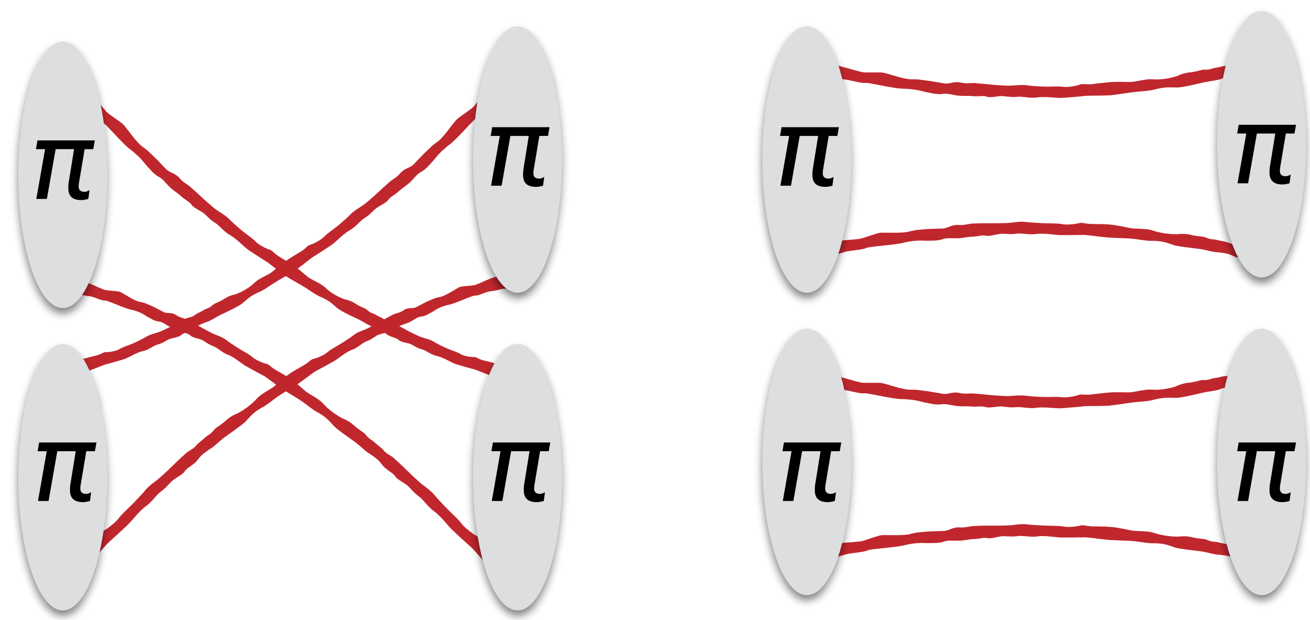
ππ I=2 scattering from lattice QCD

evaluate a matrix of correlation functions

$$C_{|\vec{p}|,|\vec{q}|} = \langle 0 | \mathcal{O}_{\pi\pi}^{|\vec{p}|}(t)\, \mathcal{O}_{\pi\pi}^{|\vec{q}|\dagger}(0) | 0 \rangle$$

formally integrate out the quark fields …

need to compute the following quark propagation diagrams, averaged over an ensemble of gauge configurations



an aside on the lattices

I'm going to present some results from a particular lattice QCD set-up *PRD79 034502 (2009)*

"anisotropic Clover lattices"

lattice spacing in space directions: $a_s \sim 0.12\,\mathrm{fm}$

lattice spacing in time direction: $a_t \sim a_s/3.5$ $\quad a_t^{-1} \sim 6\,\mathrm{GeV}$

three flavors of quark, two light & one strange

$$m_s \approx m_s^{\mathrm{phys}}$$

$$m_u = m_d > m_{u,d}^{\mathrm{phys}}$$

exact isospin symmetry

$m_\pi \sim 391\,\mathrm{MeV}$

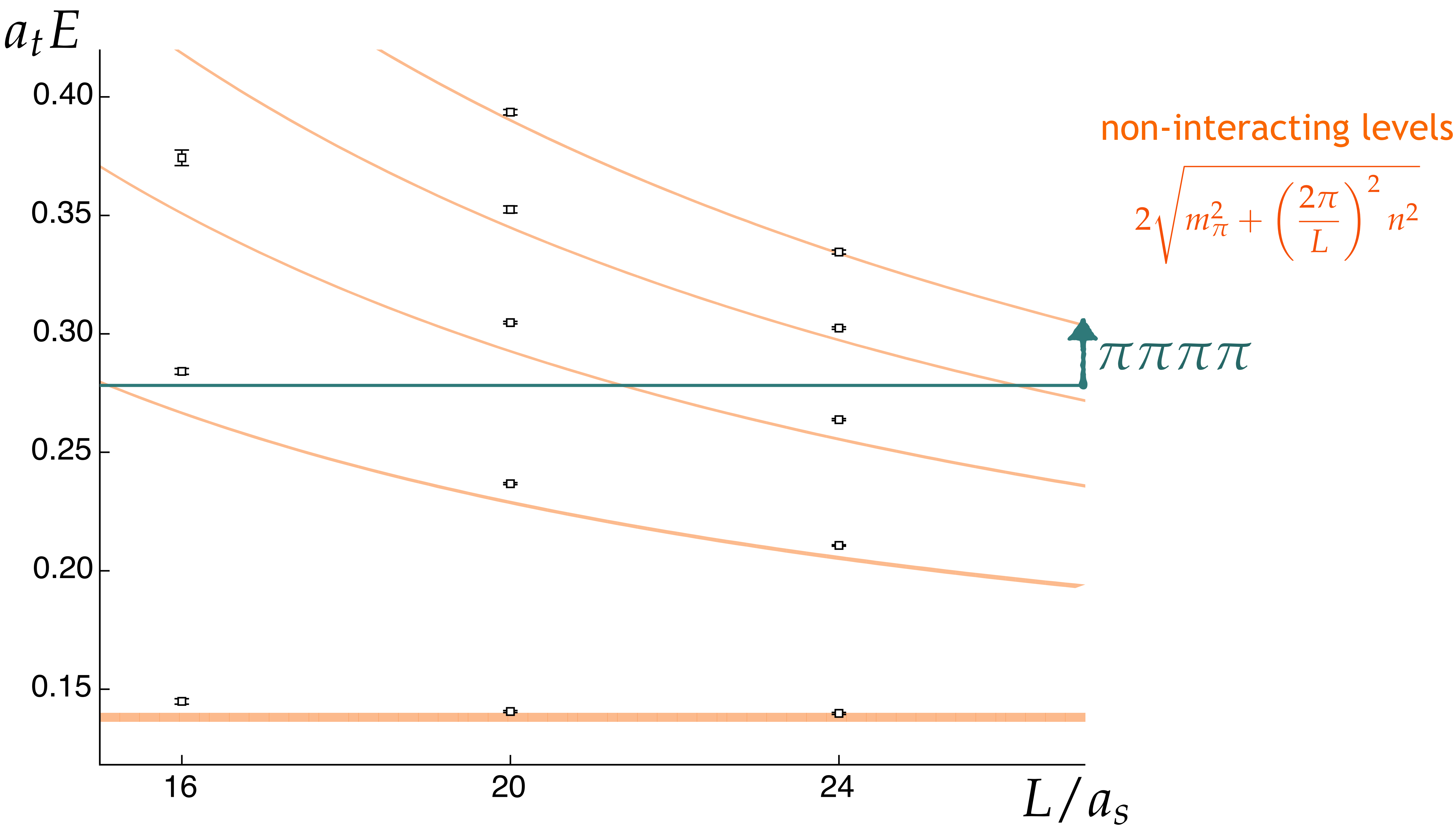
multiple volumes: 16^3, 20^3, 24^3

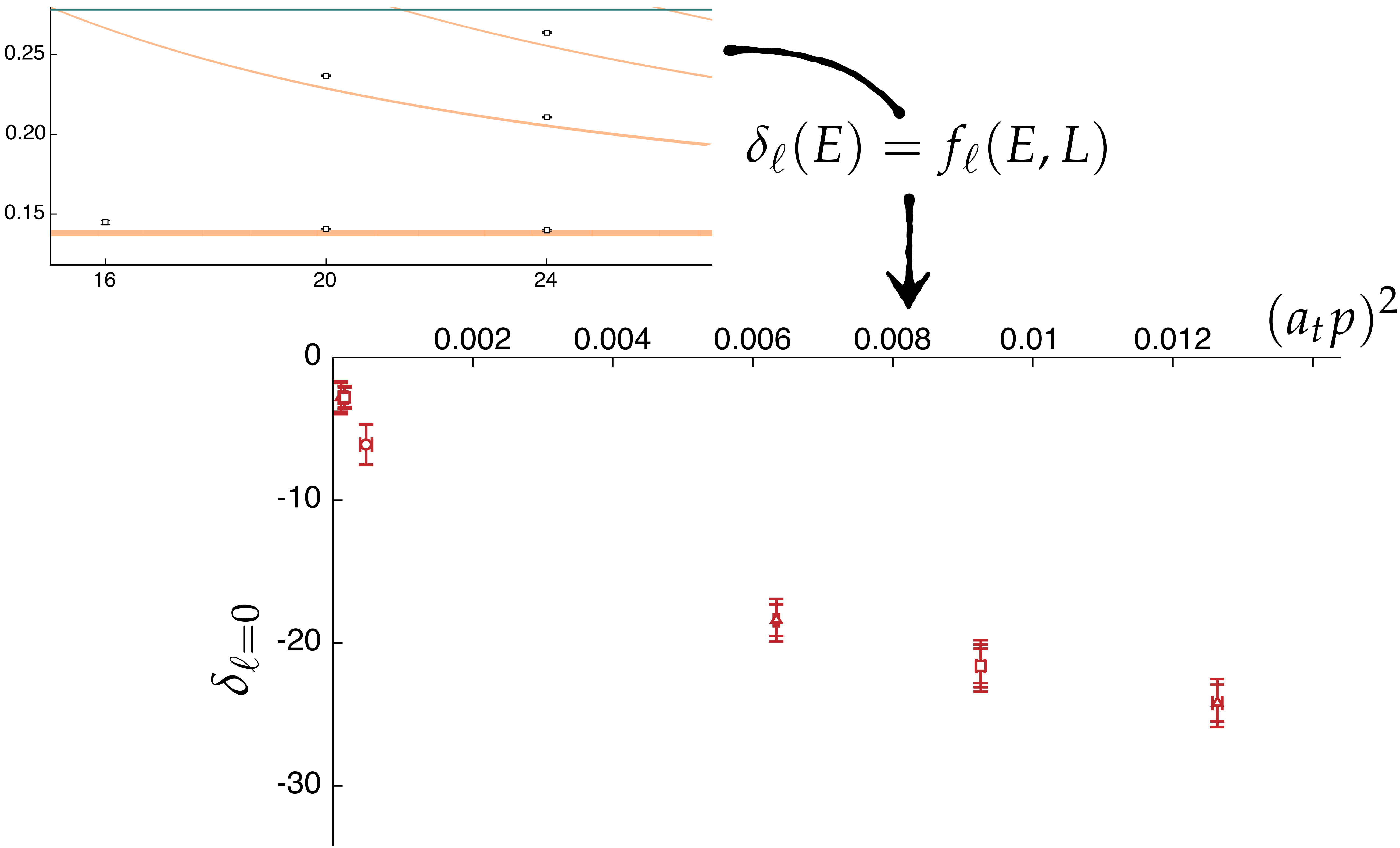
~ 2.0, 2.5, 3.0 fm

$m_\pi L$ ~ 4, 5, 6

ππ I=2 scattering from lattice QCD

$m_\pi \sim 391\,\mathrm{MeV}$





0.25
0.20
0.15
16
20
24
$\delta_\ell(E) = f_\ell(E, L)$
$(a_t p)^2$
0.002
0.004
0.006
0.008
0.01
0.012
0
-10
-20
-30
$\delta_{\ell=0}$

moving frames - more information

in a finite-volume considering a moving frame contains extra info

... surely some mistake ?

length contraction along the direction of motion changes the quantization condition

(and also reduces the symmetry group)

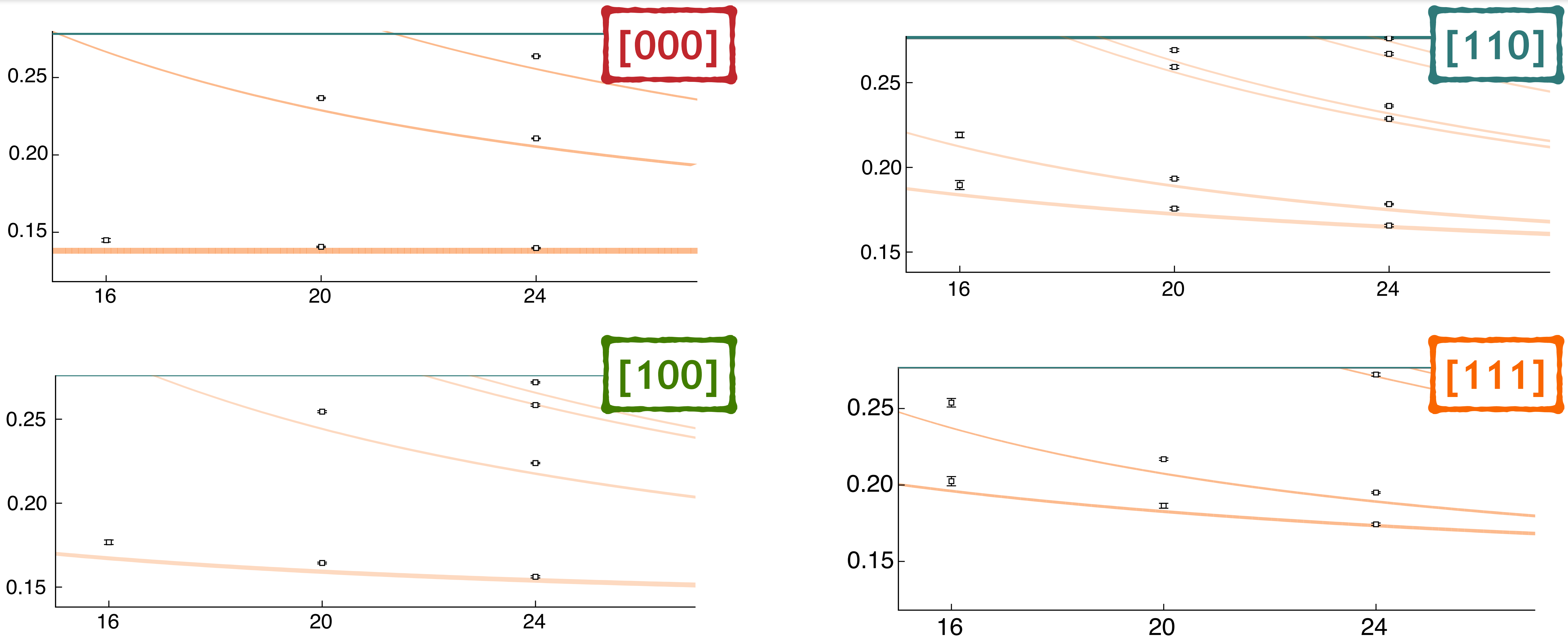
Gottlieb & Rumm. NPB450 (1995) 397
Kim et. al. NPB727 (2005) 218
& others

after a bit of group theory, it's quite easy to construct the relevant operators

$$\mathcal{O}_{\pi\pi}^{\vec{P},\Lambda;|\vec{p}|} = \sum_{\hat{p}} C(\vec{P},\Lambda;\vec{p},\vec{P}-\vec{p})\, \pi^+(\vec{p})\, \pi^+(\vec{P}-\vec{p})$$

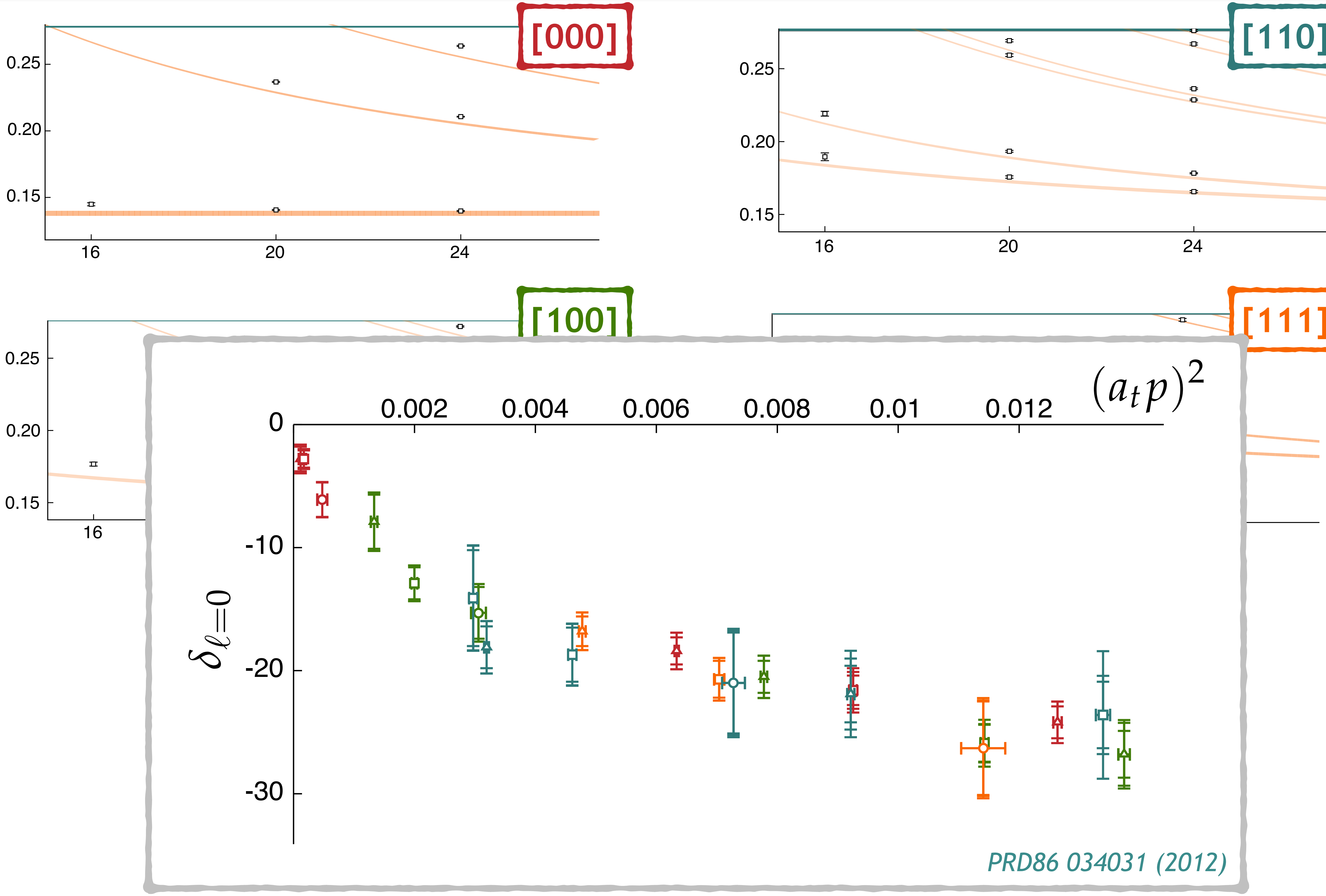
Clebsch-Gordan for irreducible representation Λ of the 'little-group'

ππ I=2 scattering from lattice QCD



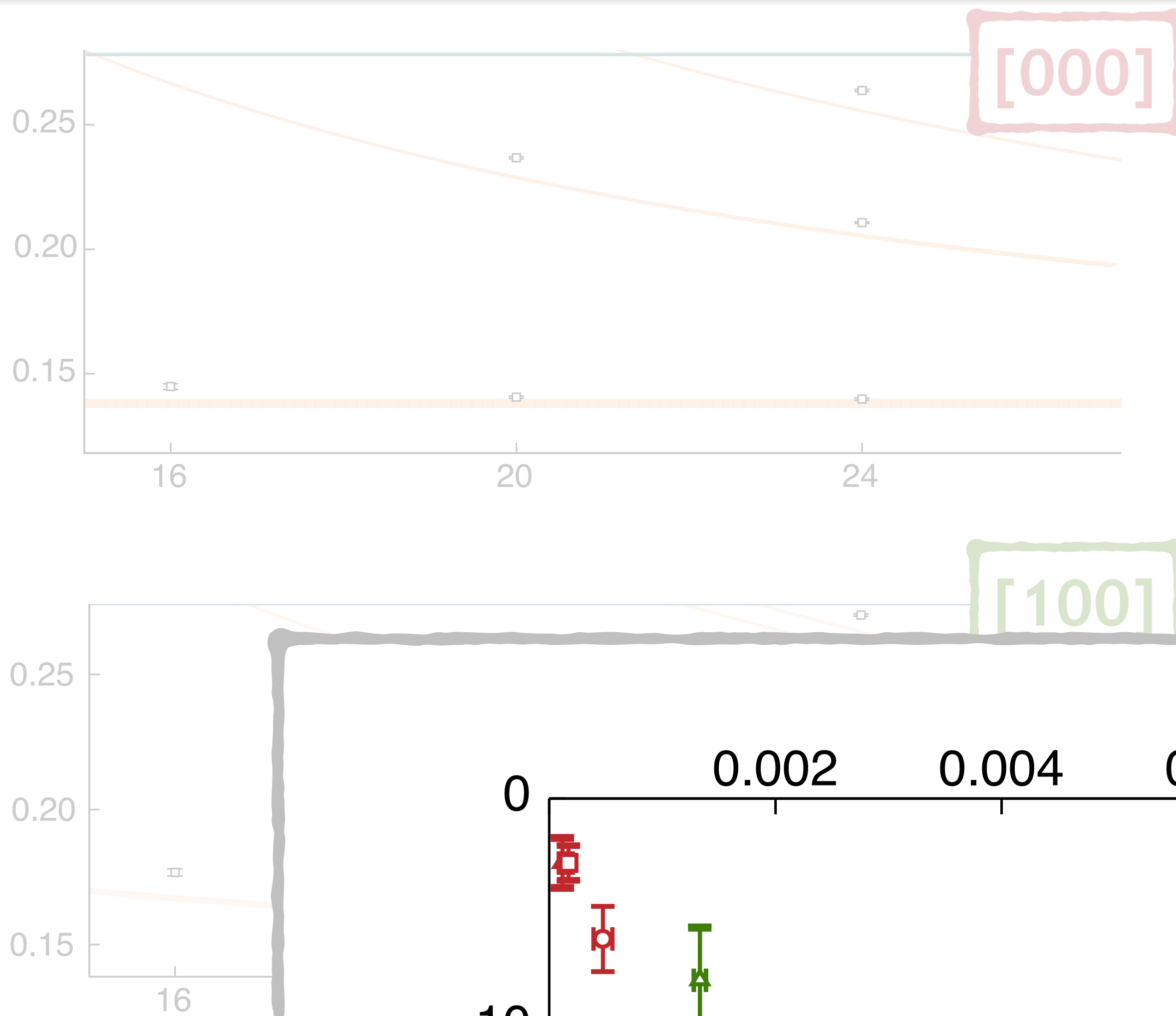
ππ I=2 scattering from lattice QCD

$m_\pi \sim 391\,\mathrm{MeV}$

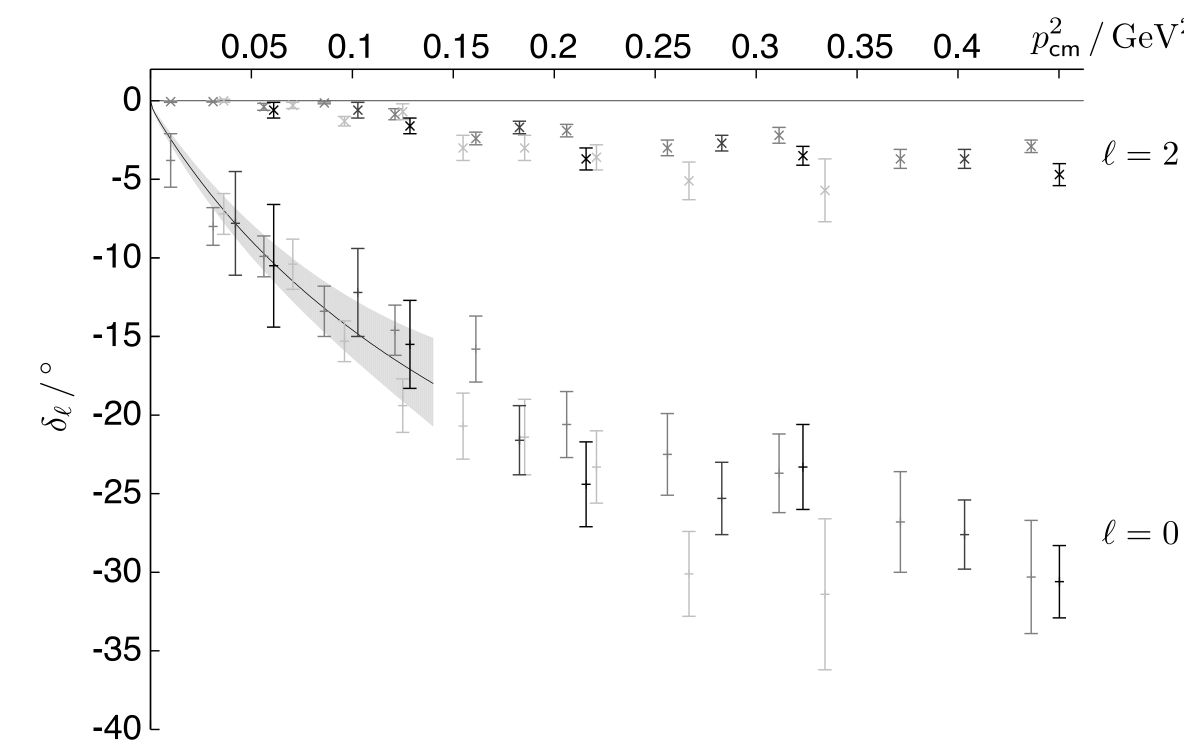


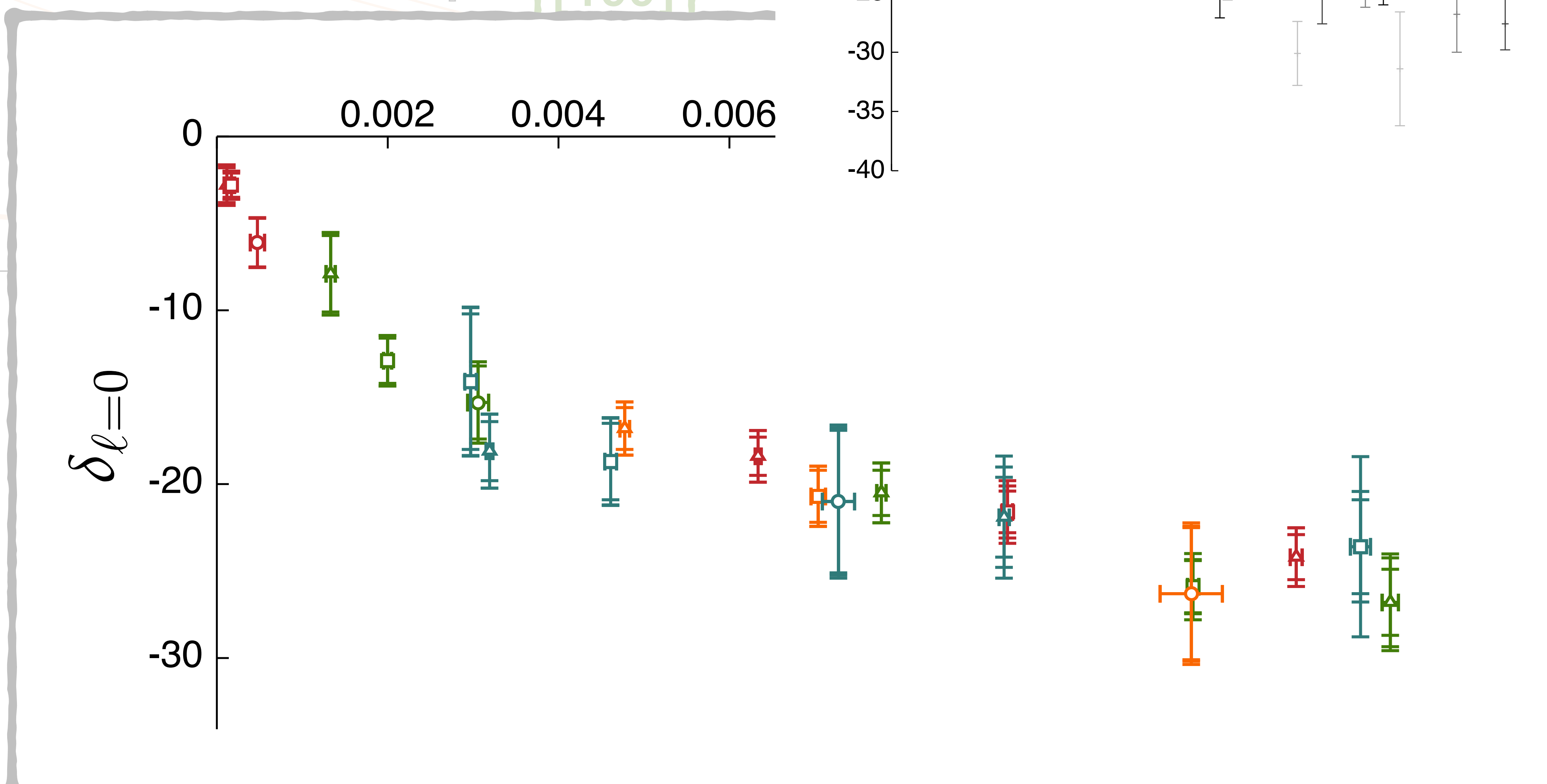
ππ I=2 scattering from lattice QCD

$m_\pi \sim 391\,\mathrm{MeV}$



ππ I=2 PHASE-SHIFT





$\pi\pi$ *I*=1 scattering from lattice QCD

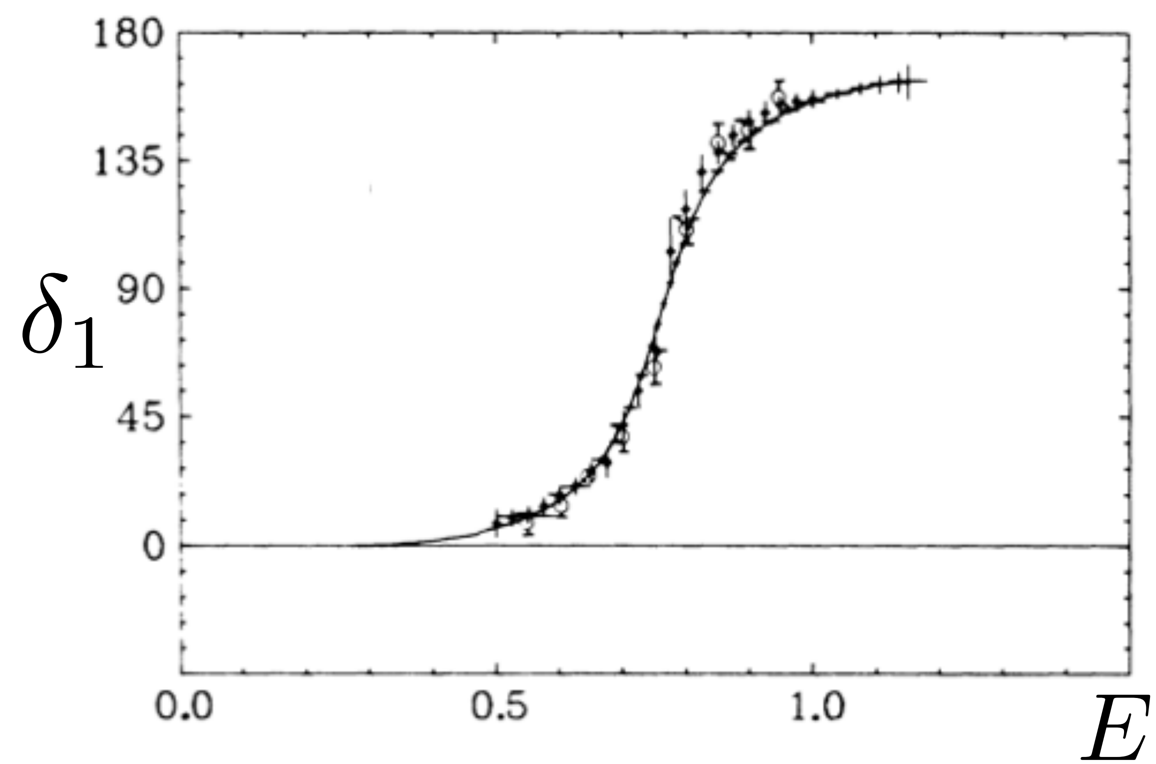
experimentally the *I*=1 *P*-wave is qualitatively different - there is a resonance

ρ(770) [h]

$I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV

RESONANT PHASE SHIFT



but we'll follow the same approach - first, compute the spectrum in finite volume …

$\pi\pi$ I=1 scattering from lattice QCD

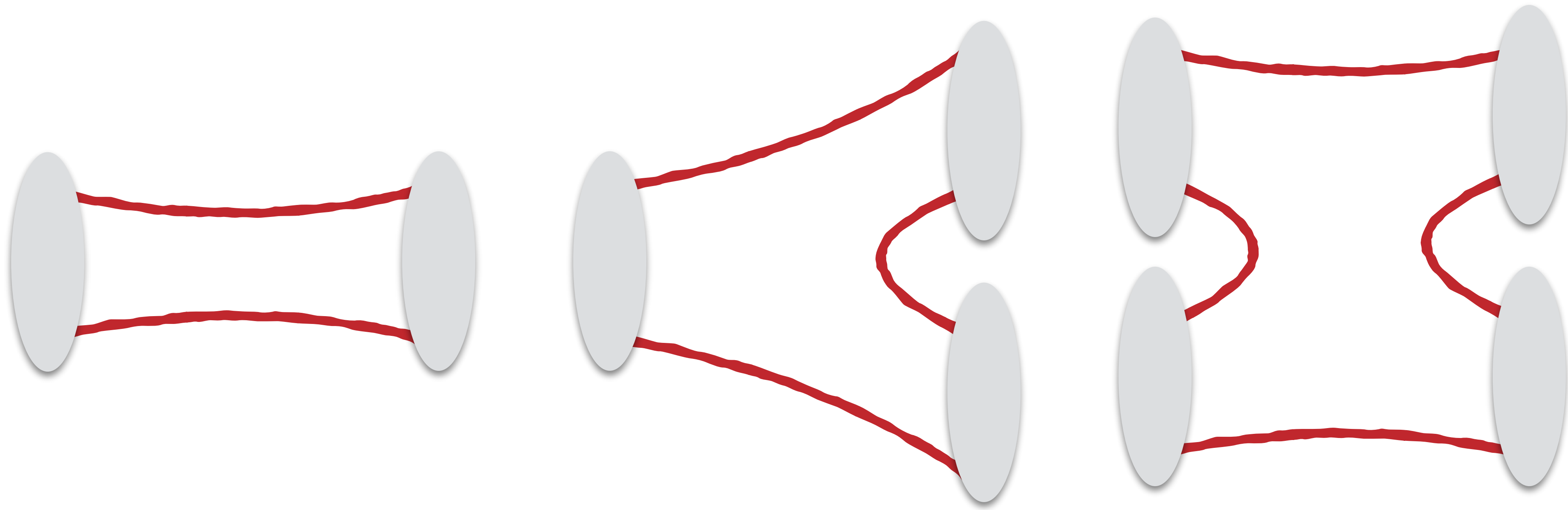
we can again consider a basis of $\pi\pi$-like operators

but in isospin=1, we can also have a smaller quark content, $\bar{u}d$

⇒ why not supplement with a basis of $\bar{u}\Gamma D \ldots D d$ constructions

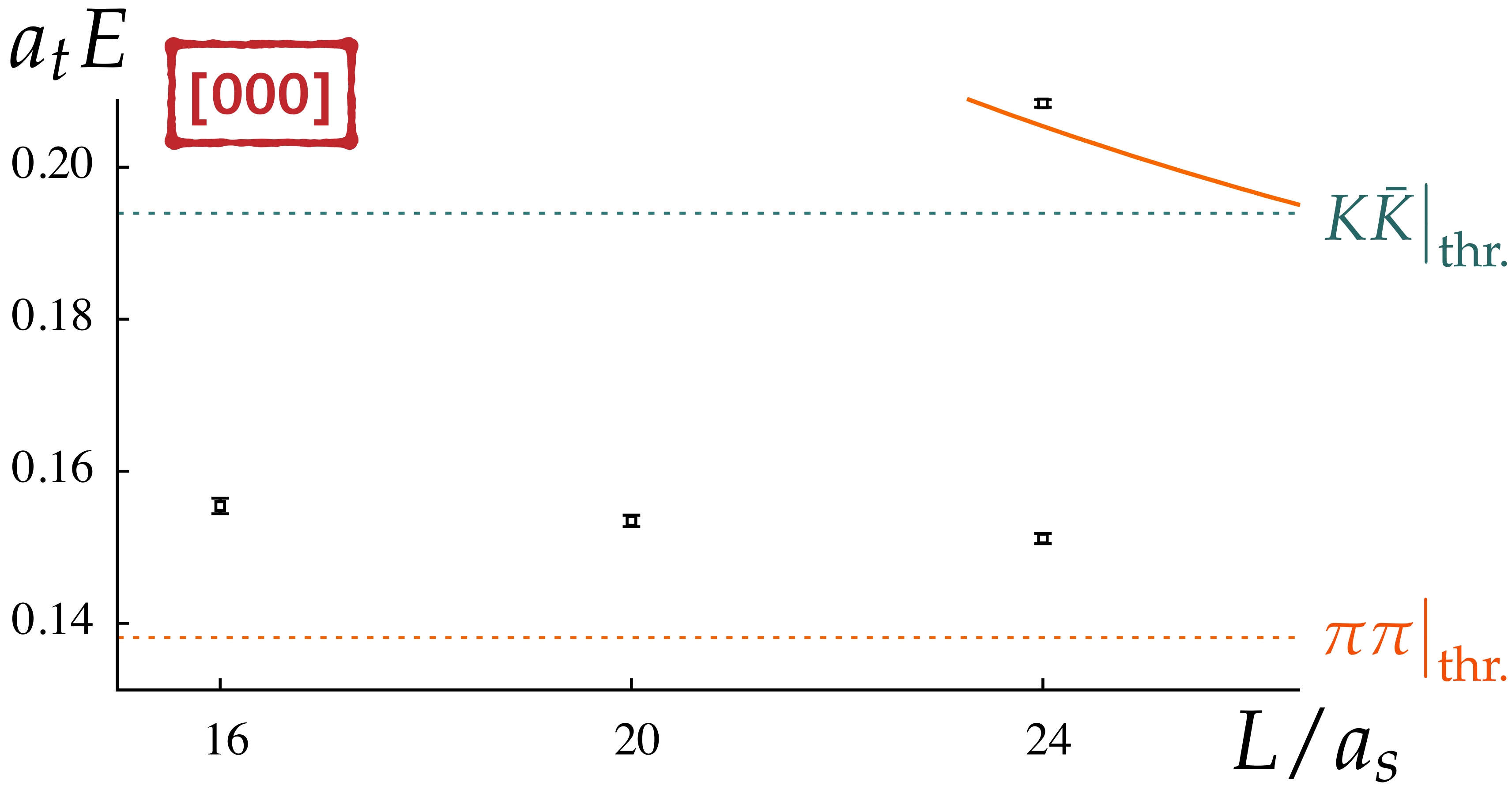
formally integrate out the quark fields ...

more involved set of quark propagations:

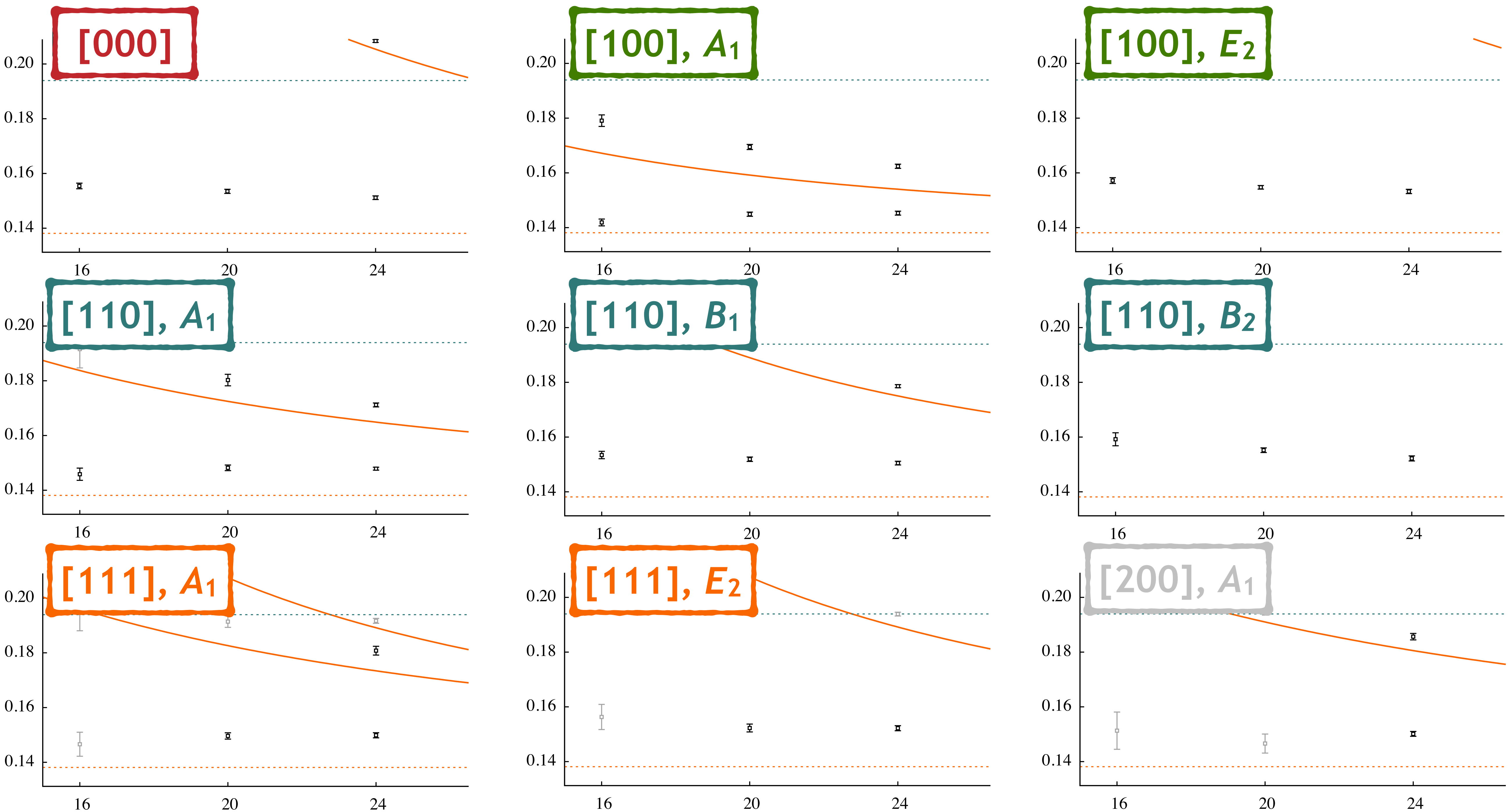


but we've got the technology to accomplish this ...

rest frame spectrum

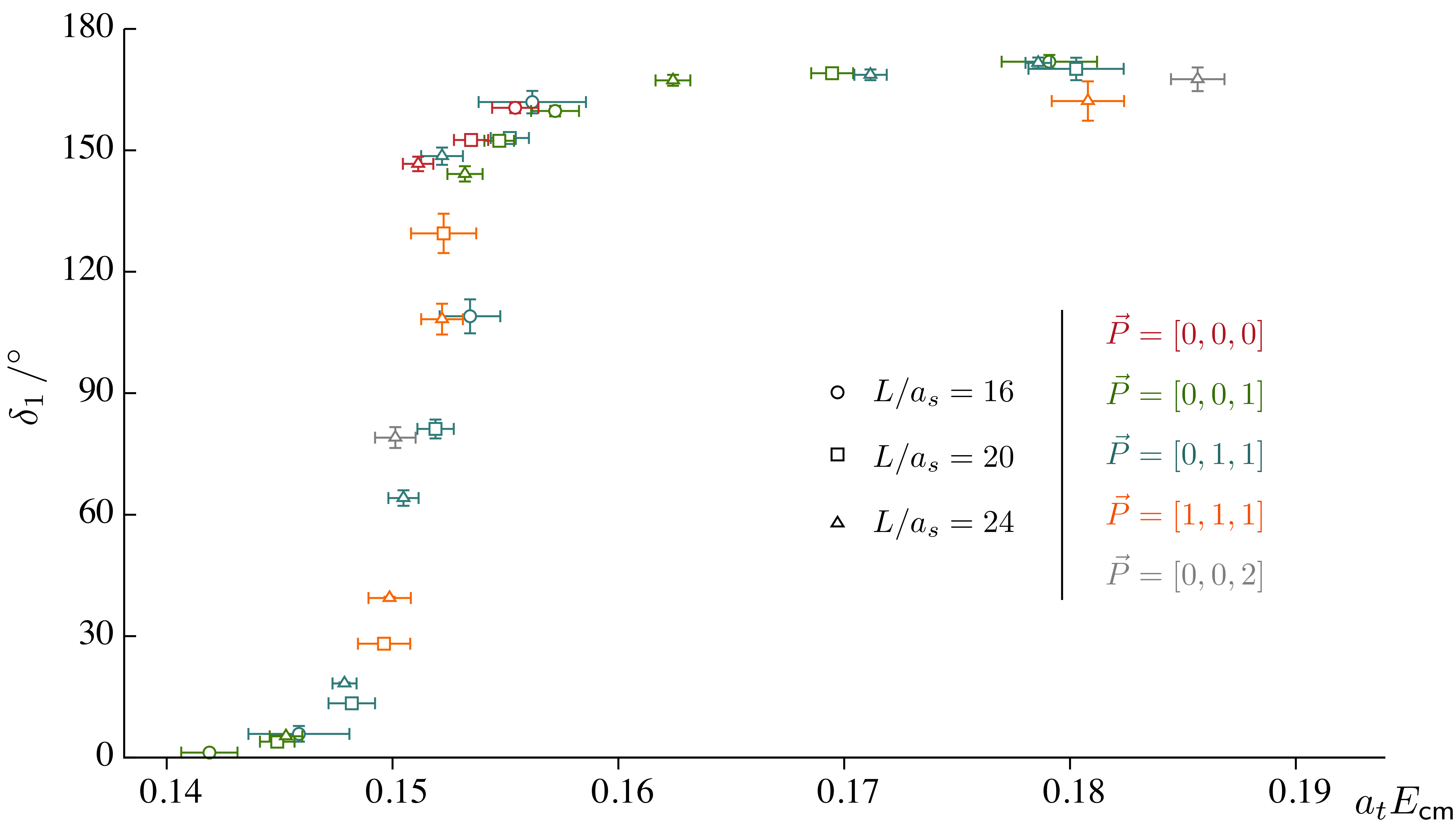


in-flight spectra



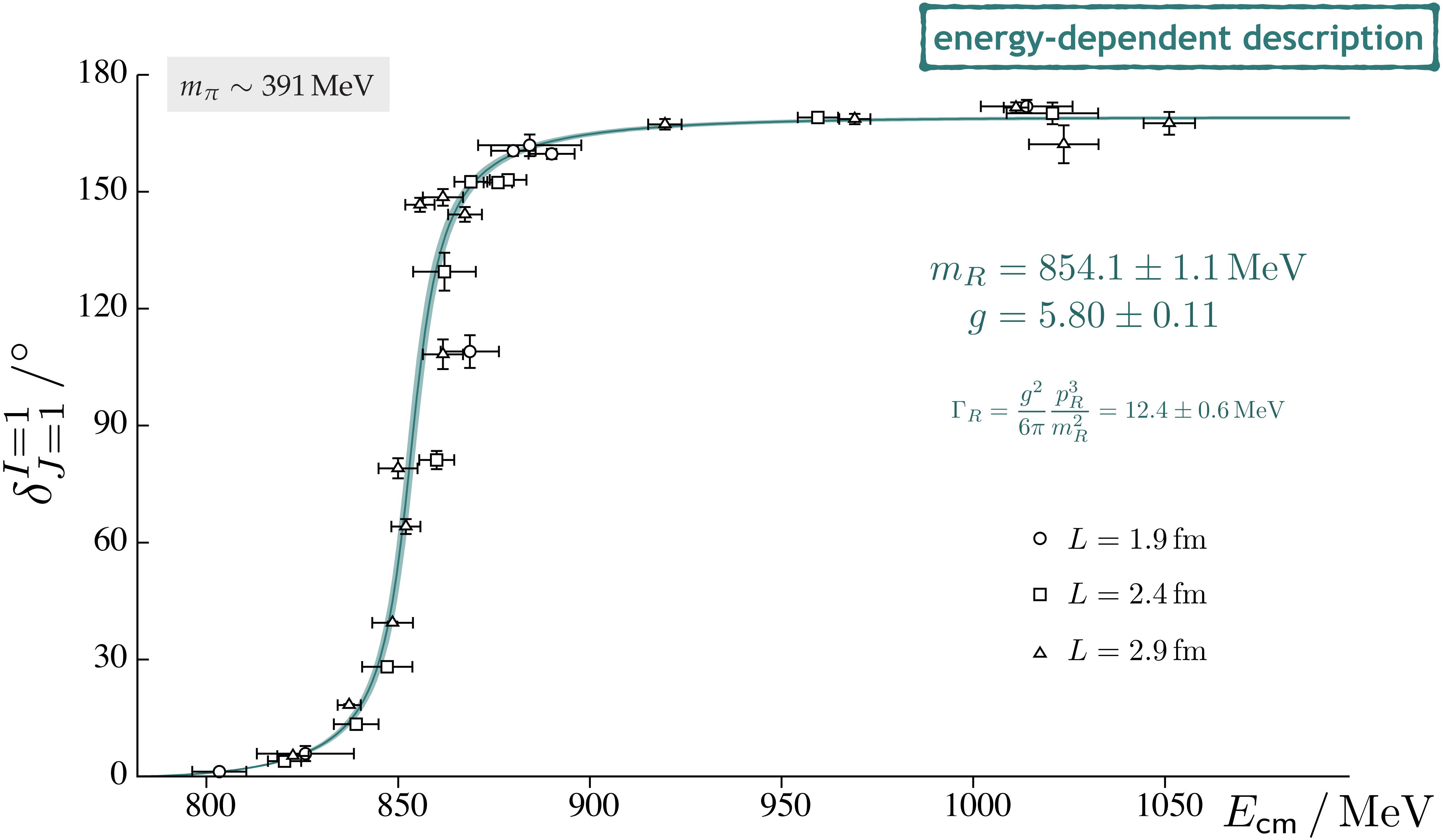
ππ I=1 scattering from lattice QCD

$m_\pi \sim 391\,\mathrm{MeV}$



PRD87 034505 (2013)

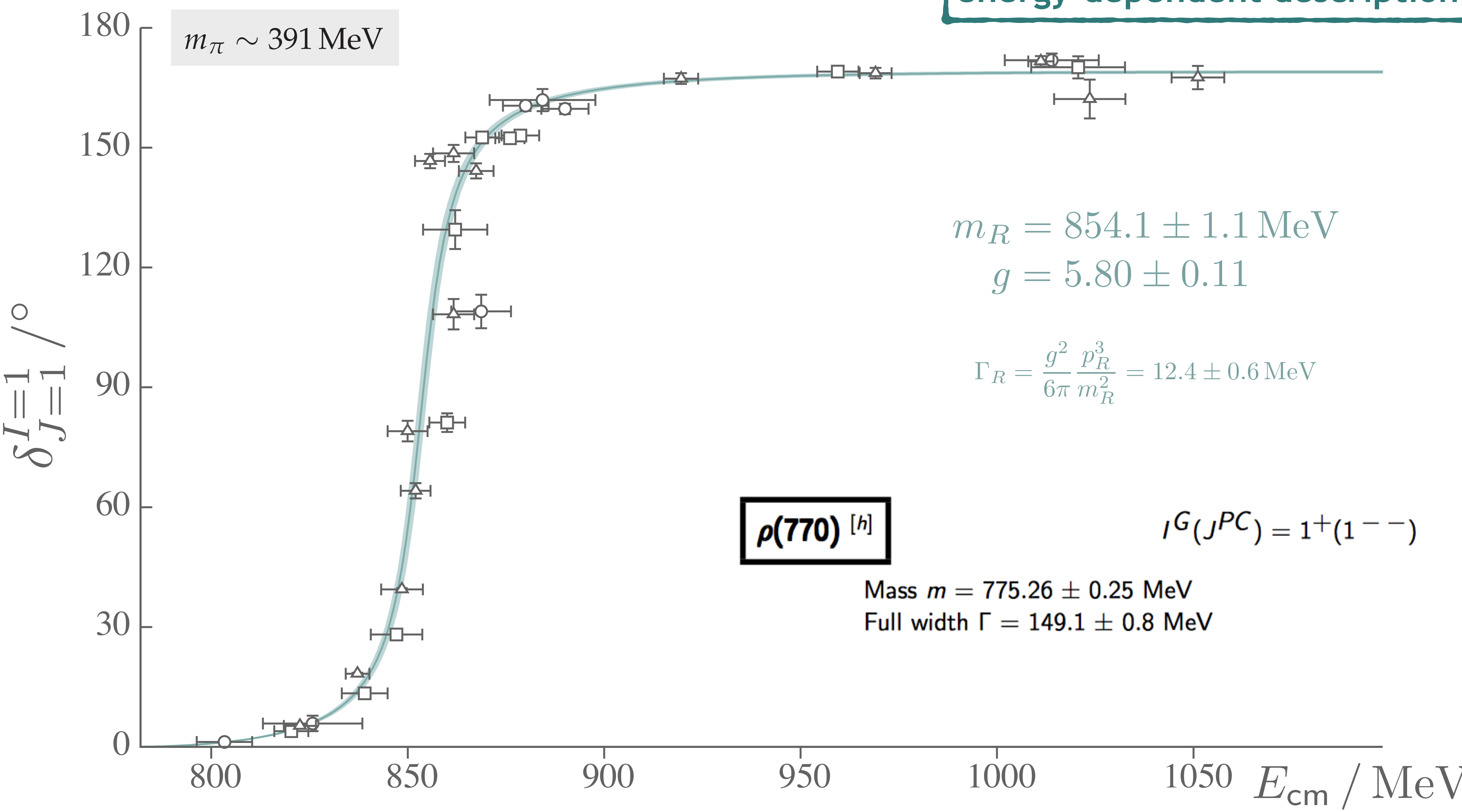
ππ *P*-wave phase-shift



PRD87 034505 (2013)

$\pi\pi$ *P*-wave phase-shift

energy-dependent description



PRD87 034505 (2013)

coupled-channel scattering

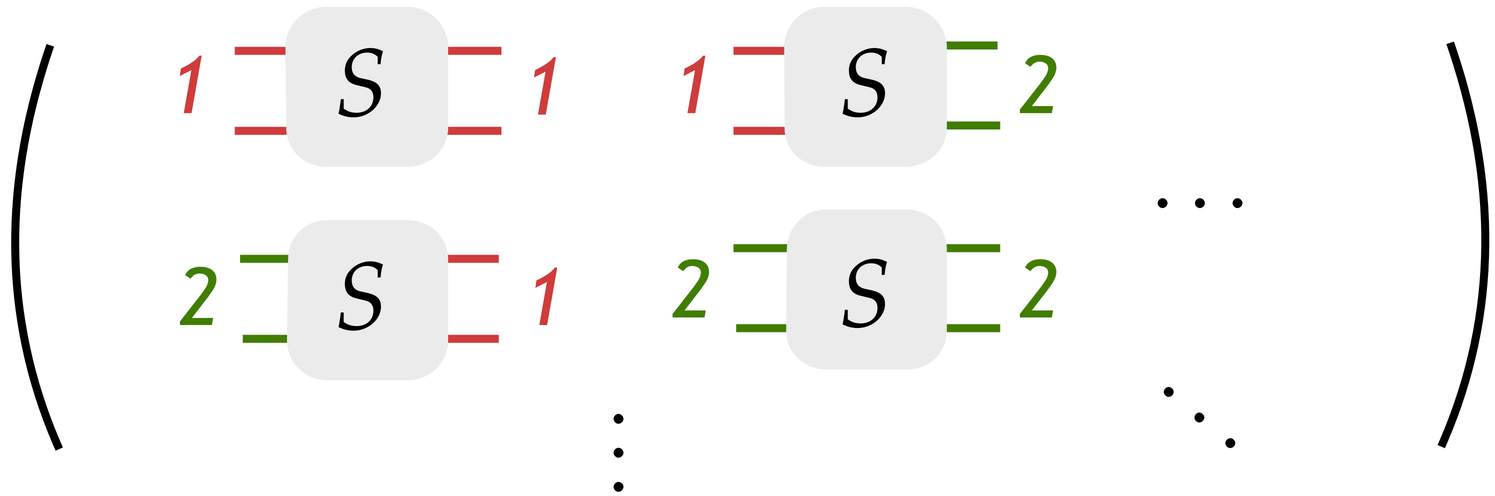
but most experimental resonances can decay to more than one channel

elastic scattering is not enough, need to consider coupled-channel scattering

coupled-channel scattering

scattering now described by an *S*-matrix

e.g. $\mathbf{S} =$



unitarity condition $\mathbf{S}^\dagger \mathbf{S} = \mathbf{1}$

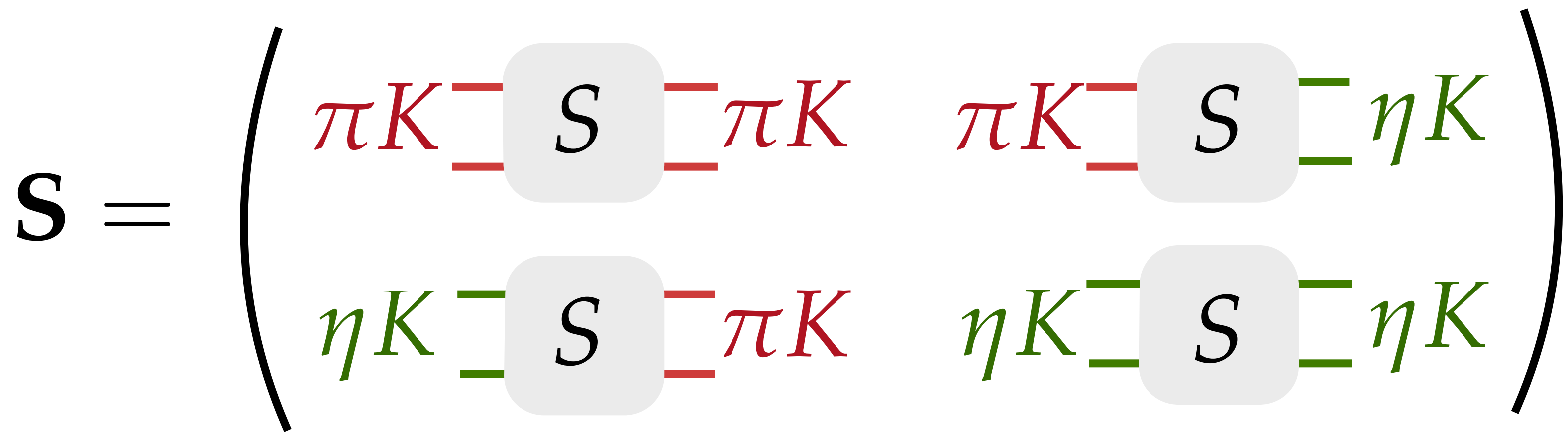
or equivalently a *t*-matrix $\mathbf{S} = \mathbf{1} + 2i\sqrt{\boldsymbol{\rho}}\,\mathbf{t}\sqrt{\boldsymbol{\rho}}$

phase space $\rho_{ij} = \delta_{ij} \frac{2k_i}{\sqrt{s}}$

unitarity condition $\mathrm{Im}\left[t^{-1}(s)\right]_{ij} = -\delta_{ij}\,\rho_i(s)\,\Theta(s - s_i^{\text{thr.}})$

coupled *πK*, *ηK* scattering

below *η′K* threshold, a two-channel system



$$S_{\pi K,\pi K} = \eta\, e^{2i\delta_{\pi K}}$$
$$S_{\eta K,\eta K} = \eta\, e^{2i\delta_{\eta K}}$$
$$S_{\pi K,\eta K} = i\sqrt{1-\eta^2}\, e^{i(\delta_{\eta K}+\delta_{\pi K})}$$

phase-shifts & inelasticity parameterization

$\eta = 1$ channels uncoupled

experimentally:

S-wave: broad resonances ? *κ*(700), $K^*_0(1430)$
P-wave: narrow resonance *K**(892)
D-wave: narrow resonance $K^*_2(1430)$

all essentially decoupled from *ηK*

coupled-channels in a finite-volume

there is again a discrete spectrum determined by the scattering amplitudes

$$\det\left[\mathbf{t}^{-1}(E) + i\boldsymbol{\rho}(E) - \mathbf{M}(E,L)\right] = 0$$

known 'kinematic' functions

He, JHEP 0507 011
Hansen, PRD86 016007
Briceno, PRD88 094507
Guo, PRD88 014051

spectrum given by the values of *E* which solve this equation

in the single-channel elastic case, this becomes the Lüscher condition we had before

computing the $\pi K, \eta K$ spectrum in LQCD

operator basis :

formally integrate out the quark fields …

$q\bar{q}$ -like

$$\bar{u}\mathbf{\Gamma}s = \bar{u}\,\Gamma D \ldots D\, s$$

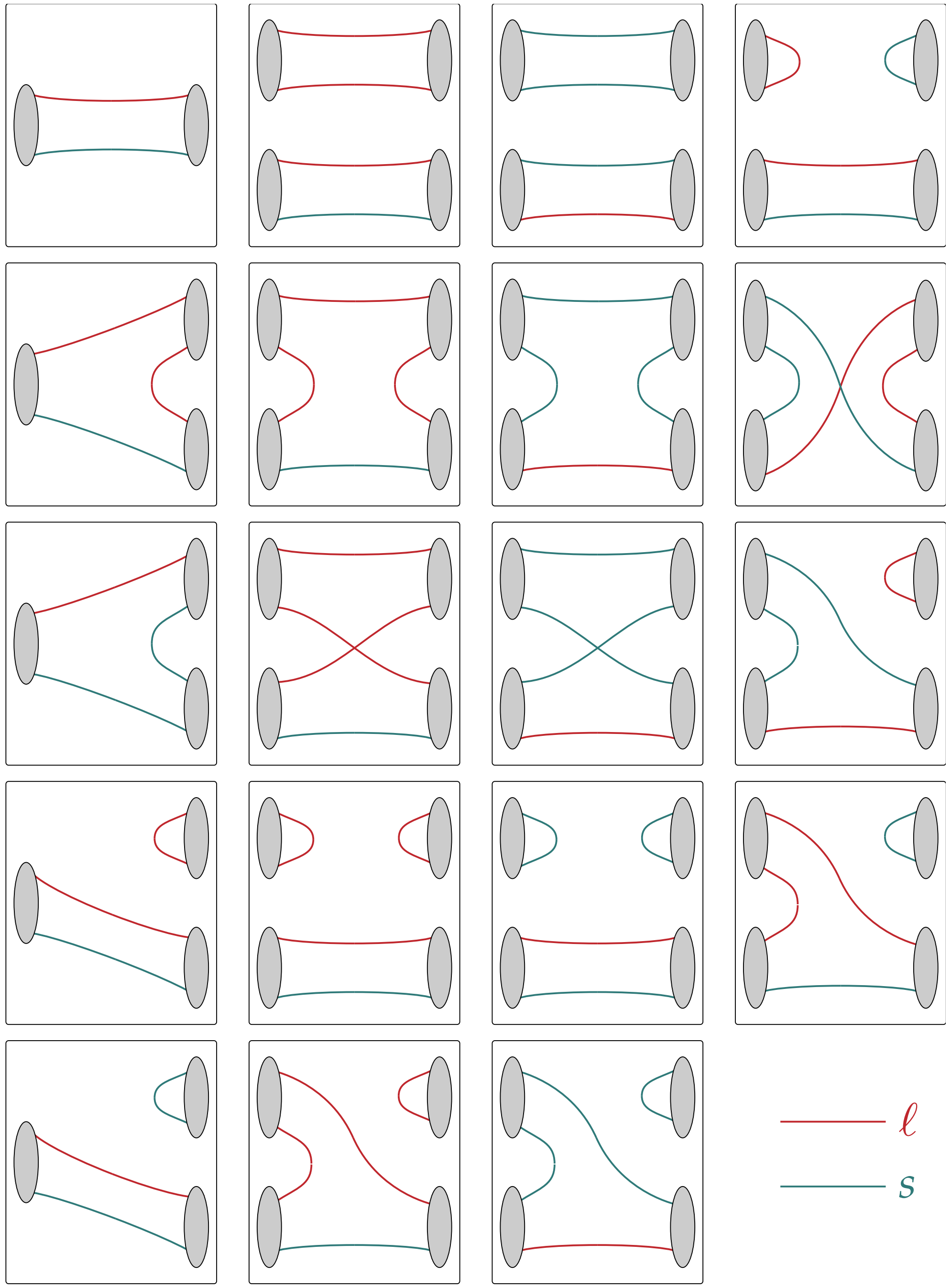
πK -like

$$\sum_{\hat{p}_1,\hat{p}_2} C(\Lambda,\vec{P};\vec{p}_1,\vec{p}_2)\,\pi(\vec{p}_1)K(\vec{p}_2)$$

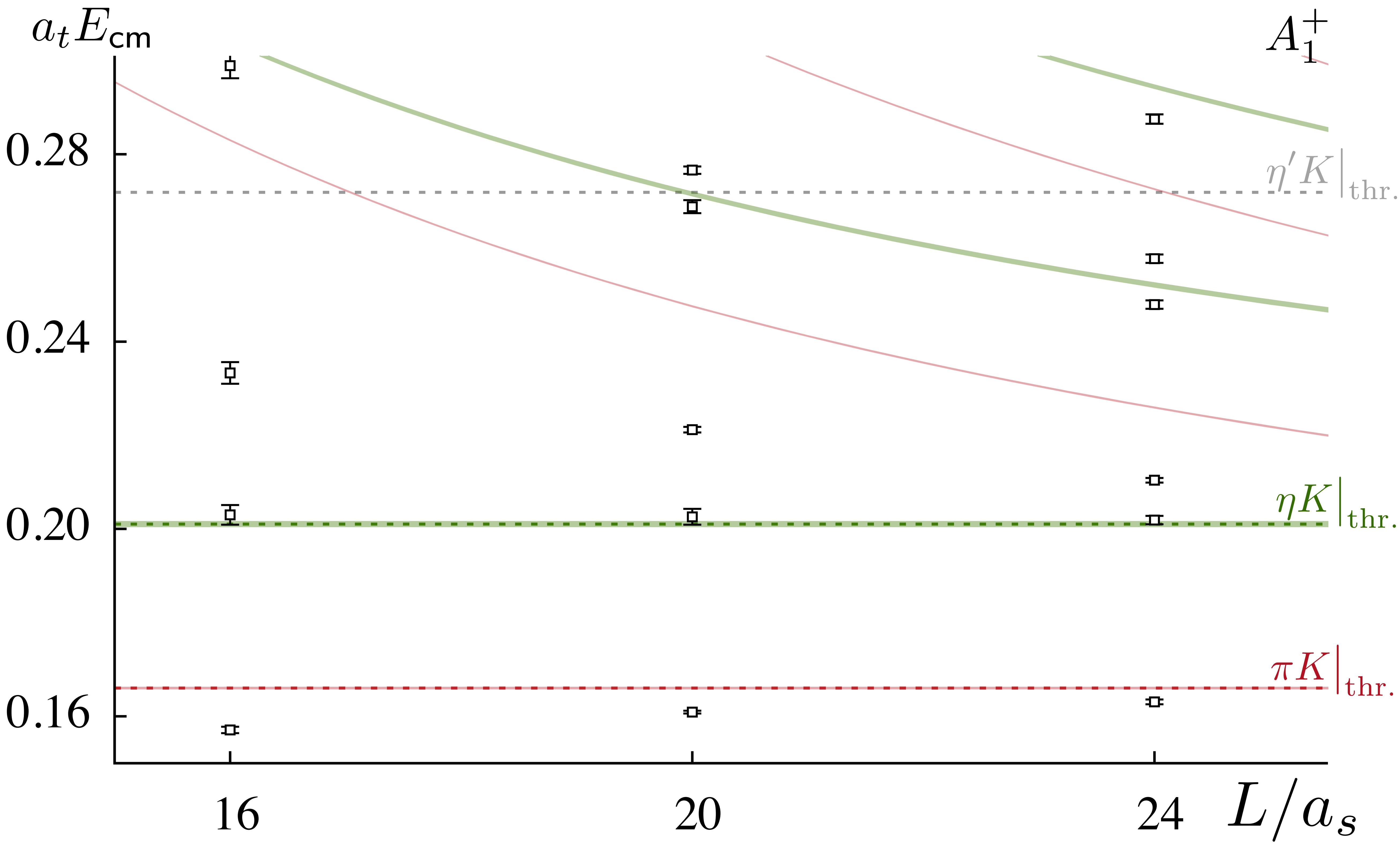
ηK -like

$$\sum_{\hat{p}_1,\hat{p}_2} C(\Lambda,\vec{P};\vec{p}_1,\vec{p}_2)\,\eta(\vec{p}_1)K(\vec{p}_2)$$

WICK CONTRACTIONS

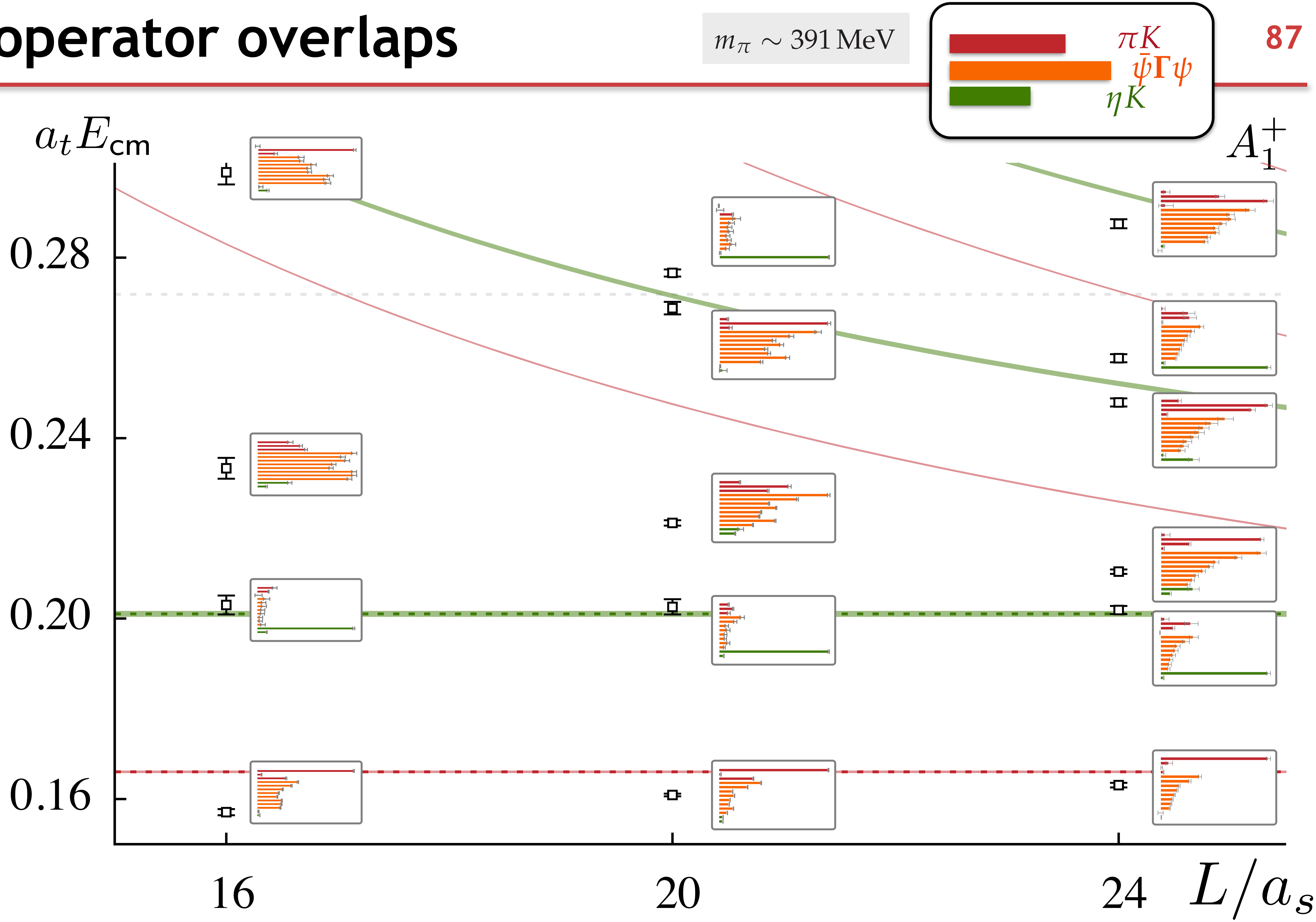


the calculated $\pi K, \eta K$ spectrum



operator overlaps

$m_\pi \sim 391\,\mathrm{MeV}$



coupled-channels in a finite-volume

there is again a discrete spectrum determined by the scattering amplitudes

$$\det\left[\mathbf{t}^{-1}(E) + i\boldsymbol{\rho}(E) - \mathbf{M}(E,L)\right] = 0$$

known 'kinematic' functions

spectrum given by the values of *E* which solve this equation

if **we have the energy levels** – how do we find ***t***(*E*) ?

for each energy level, ½*N*(*N*+1) unknowns = 3 in the two-channel case

so can't just solve level-by-level like we did in the elastic case

how about parameterizing the energy dependence ?

parameterizing *t(E)*

must be a unitarity-preserving parameterization

$$\det\left[\mathbf{t}^{-1}(E) + i\boldsymbol{\rho}(E) - \mathbf{M}(E,L)\right] = 0$$

$$\det\left[\mathrm{Re}(\mathbf{t}^{-1}) + i\,\mathrm{Im}(\mathbf{t}^{-1}) + i\boldsymbol{\rho} - \mathbf{M}\right] = 0$$

must vanish to have solutions

real above threshold

S-matrix constraints are entering the game …

e.g. *K*-matrix form

$$\mathbf{t}^{-1}(E) = \mathbf{K}^{-1}(E) + \mathbf{I}(E)$$

real function

$$\mathrm{Im}\, I_{ij}(E) = -\delta_{ij}\,\rho_i(E)$$

e.g. Chew-Mandelstam form shown by Ian

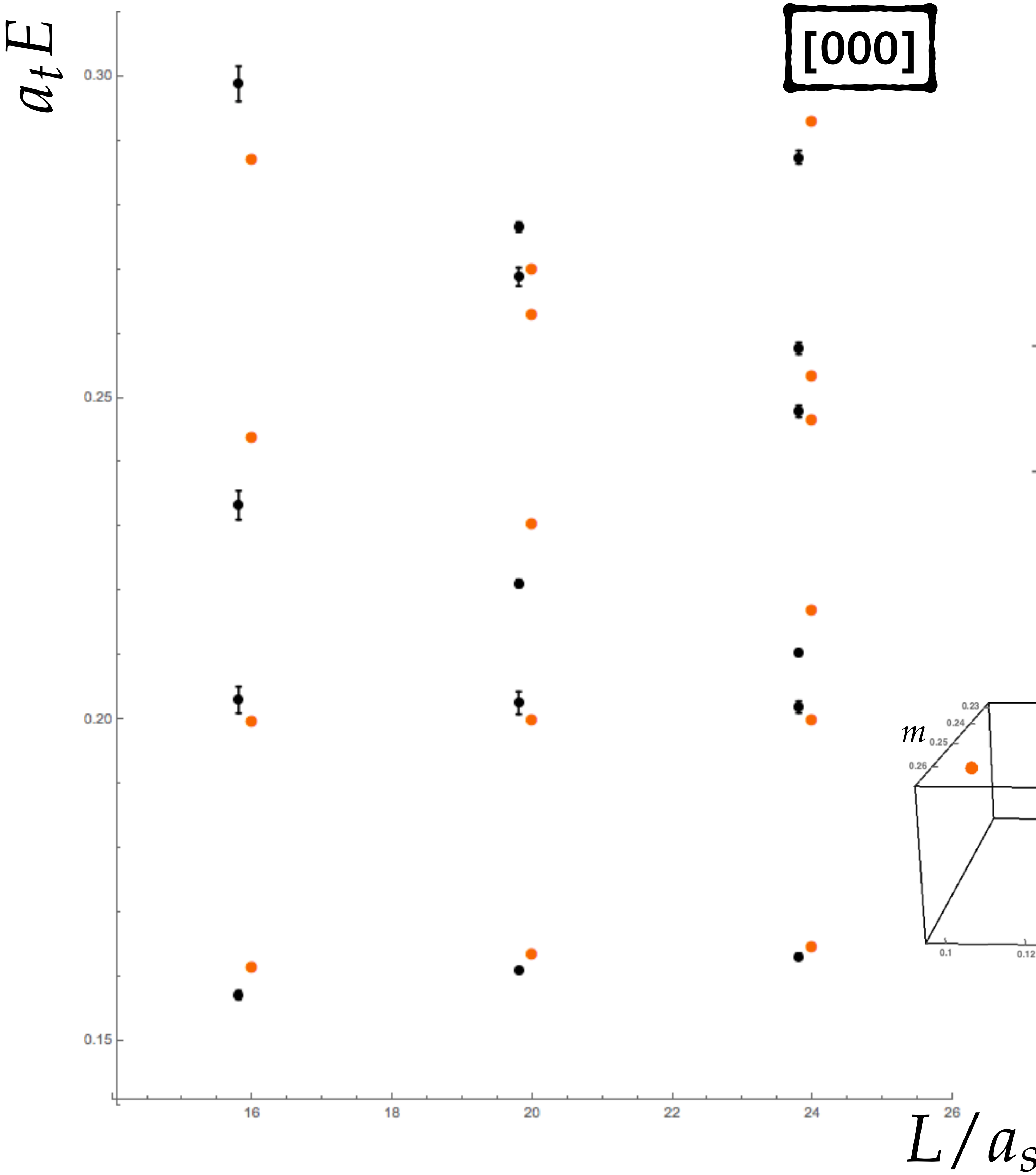
e.g. 6 parameter "pole plus constant" form

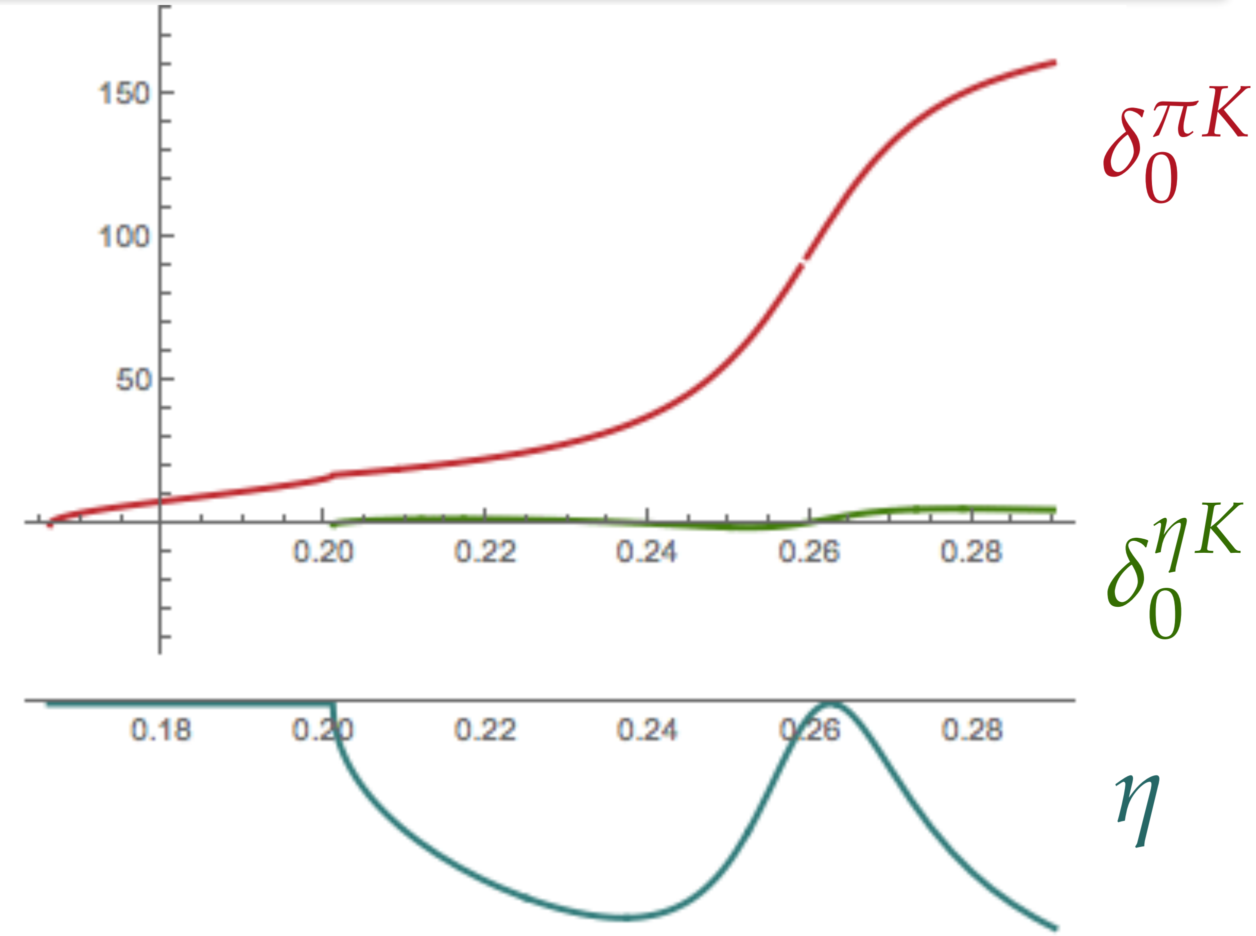
$$K_{ij}(E) = \frac{g_i\,g_j}{m^2 - E^2} + \gamma_{ij}$$

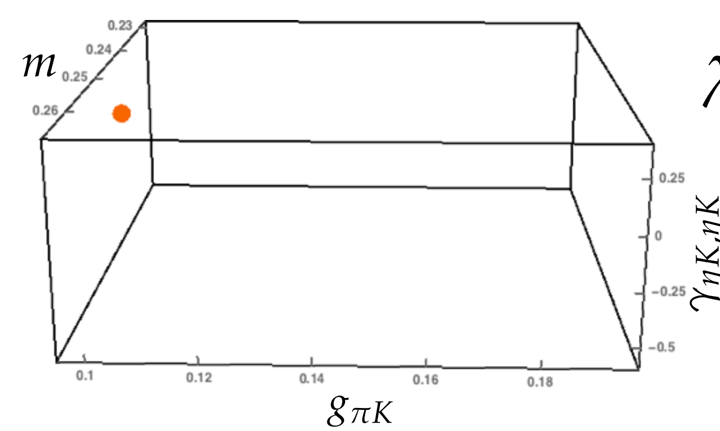
with variables

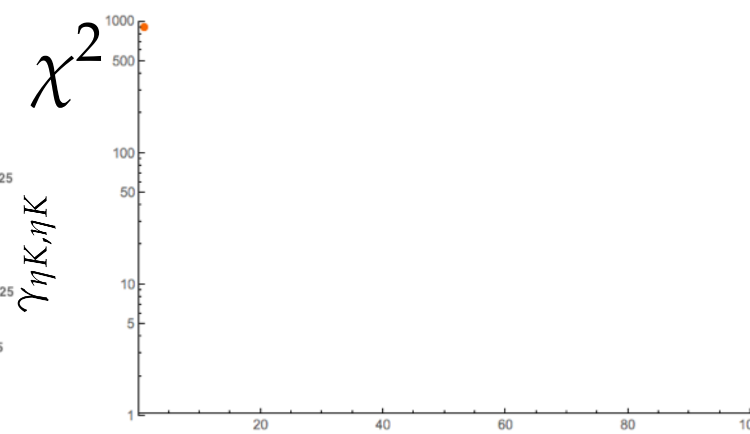
$m,\ g_1,\ g_2,\ \gamma_{11},\ \gamma_{12},\ \gamma_{22}$

a cartoon of the minimization

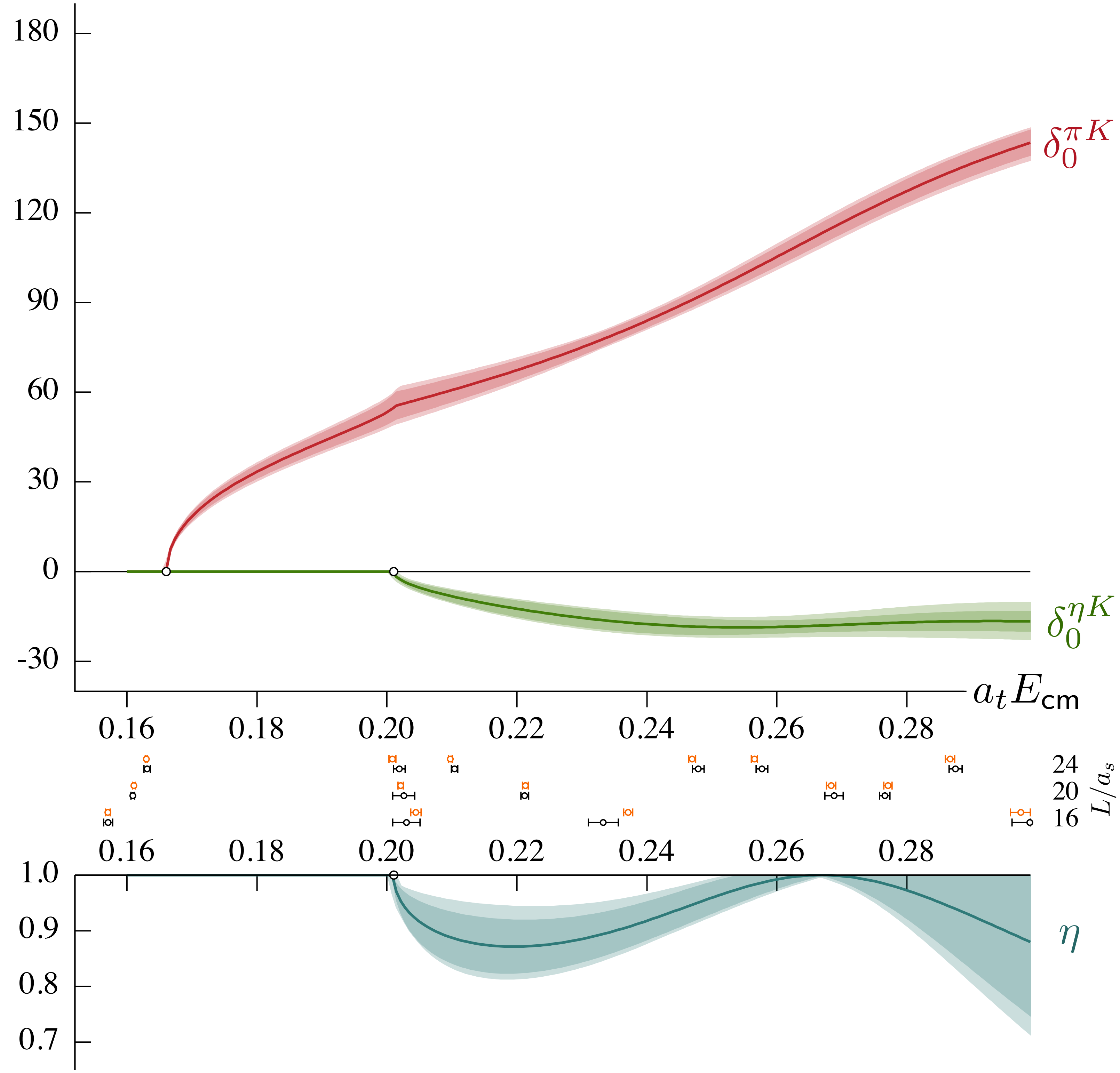








S-wave amplitudes



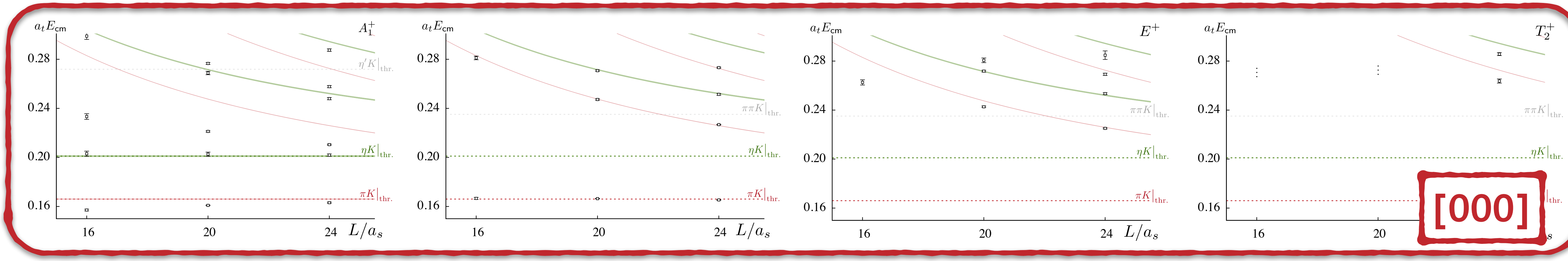
resonant (?) πK scattering

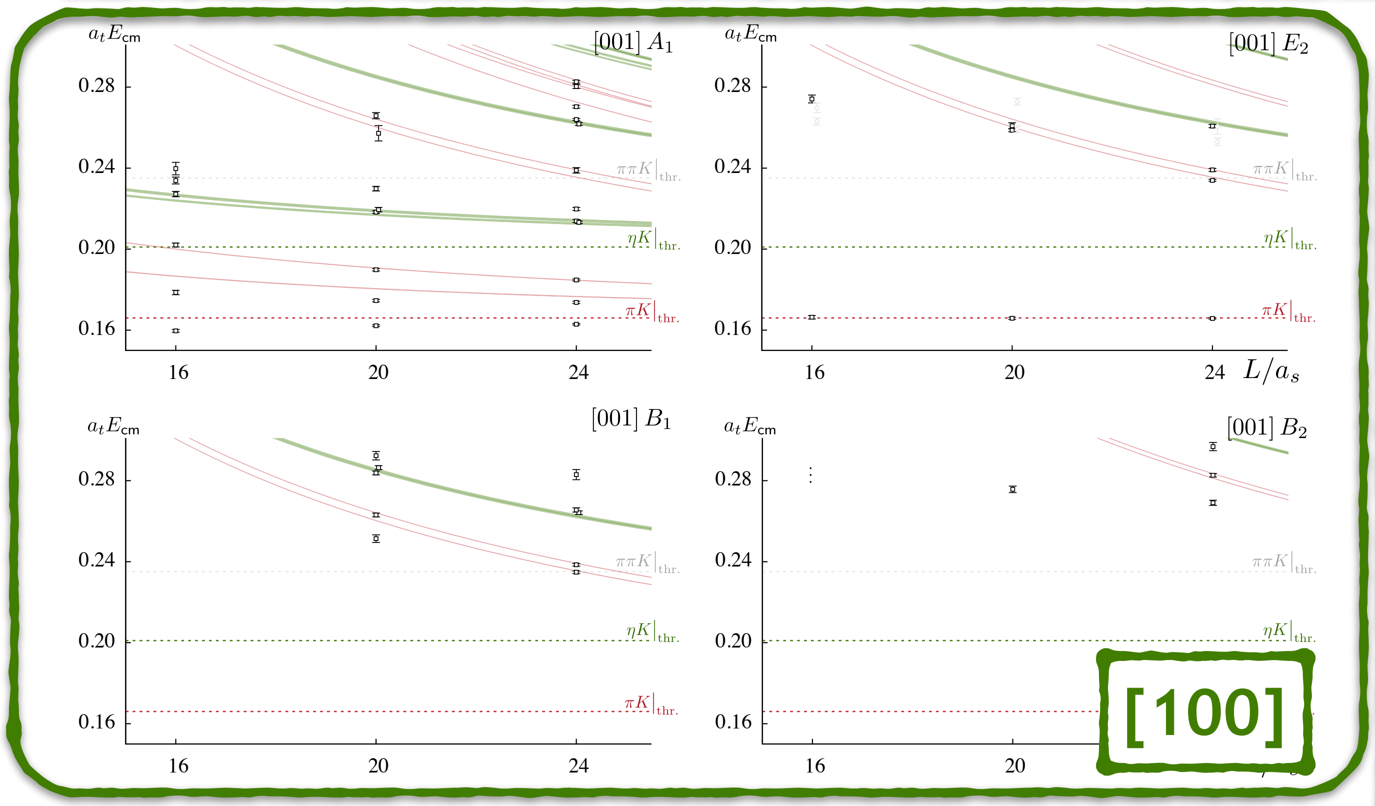
weak ηK scattering

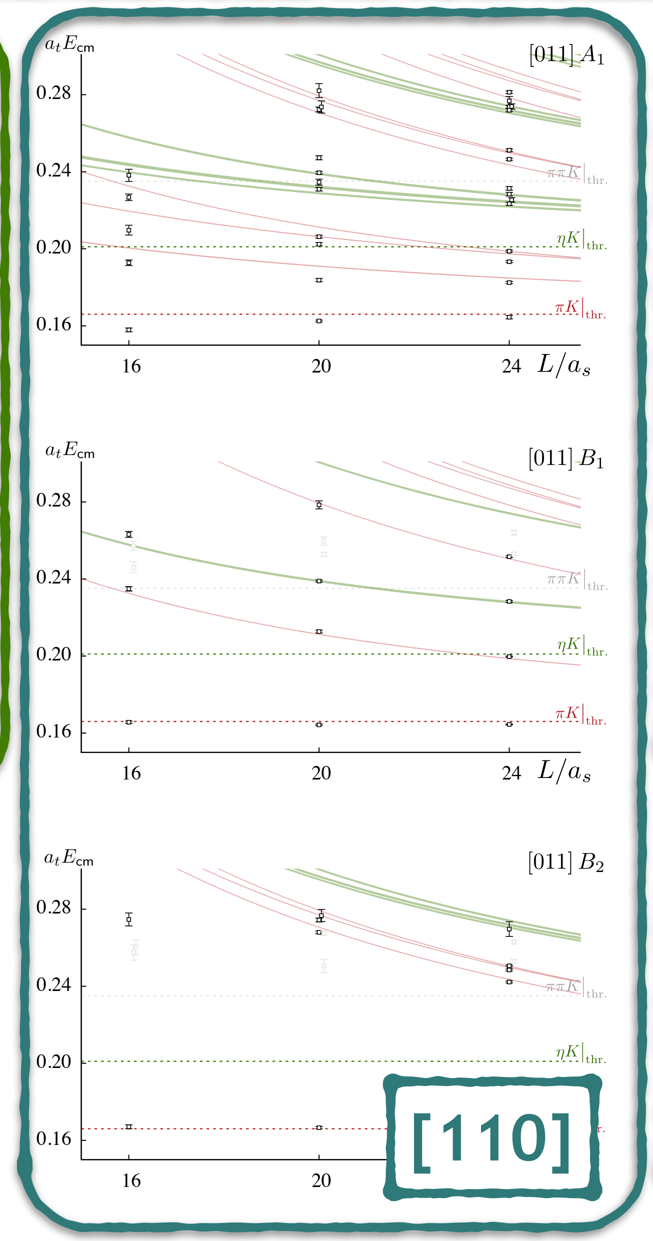
weakly coupled

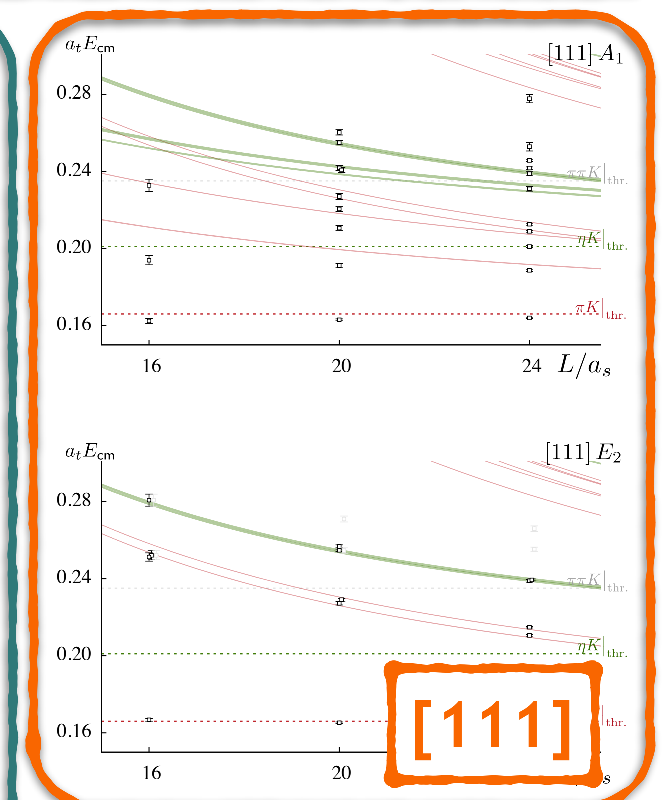
spectra in moving frames

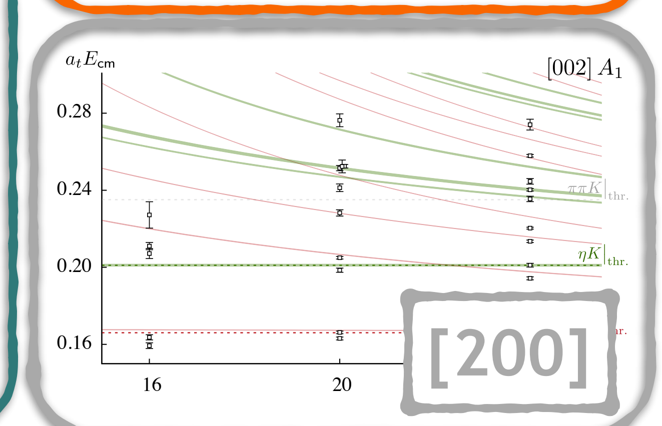
$m_\pi \sim 391\,\text{MeV}$











> 100 levels in the usable energy region

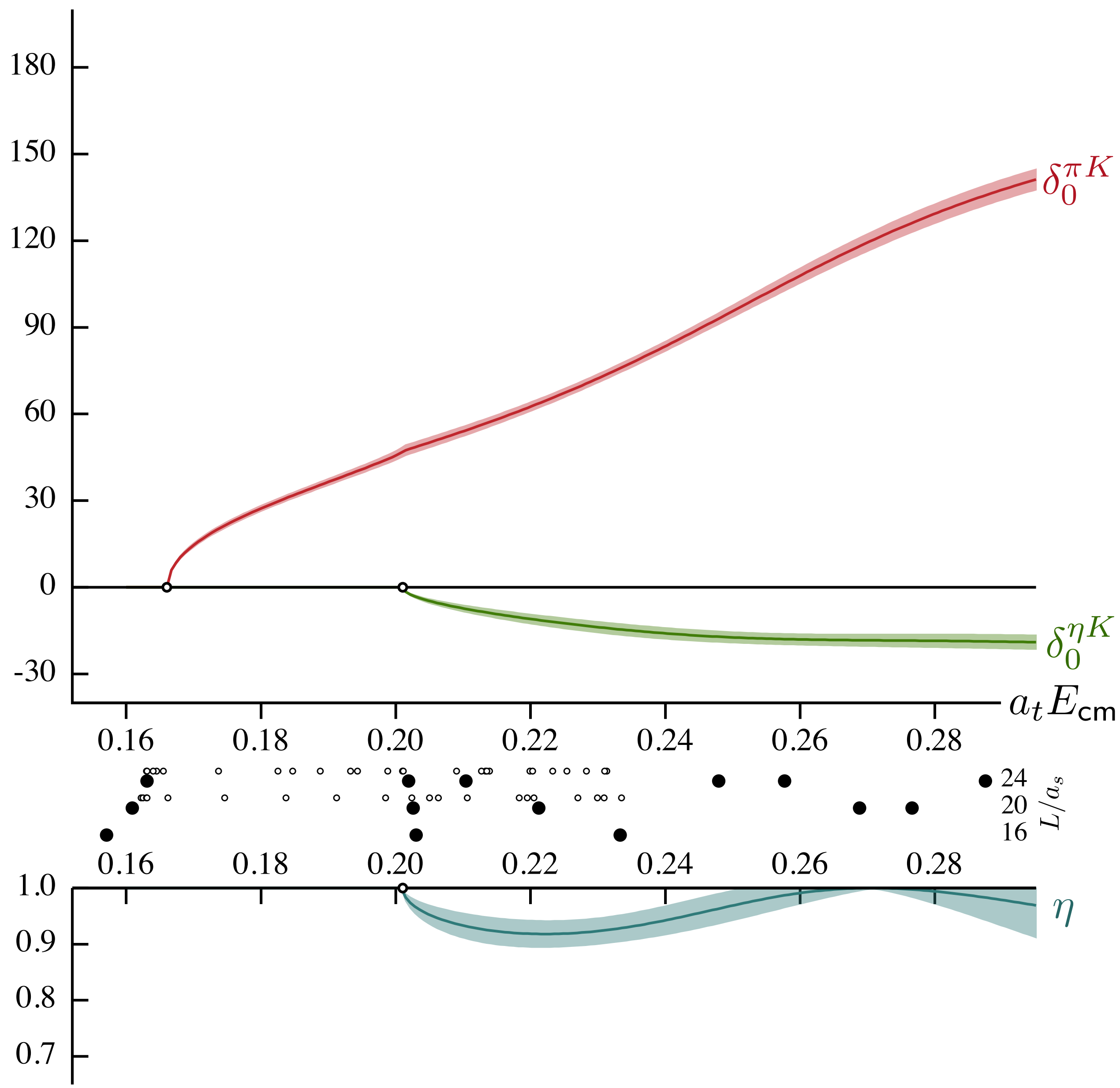
$\pi K/\eta K$ coupled-channel scattering

$m_\pi \sim 391\,\text{MeV}$

describe all the finite-volume spectra

$$\chi^2/N_{\text{dof}} = \frac{49.1}{61-6} = 0.89$$

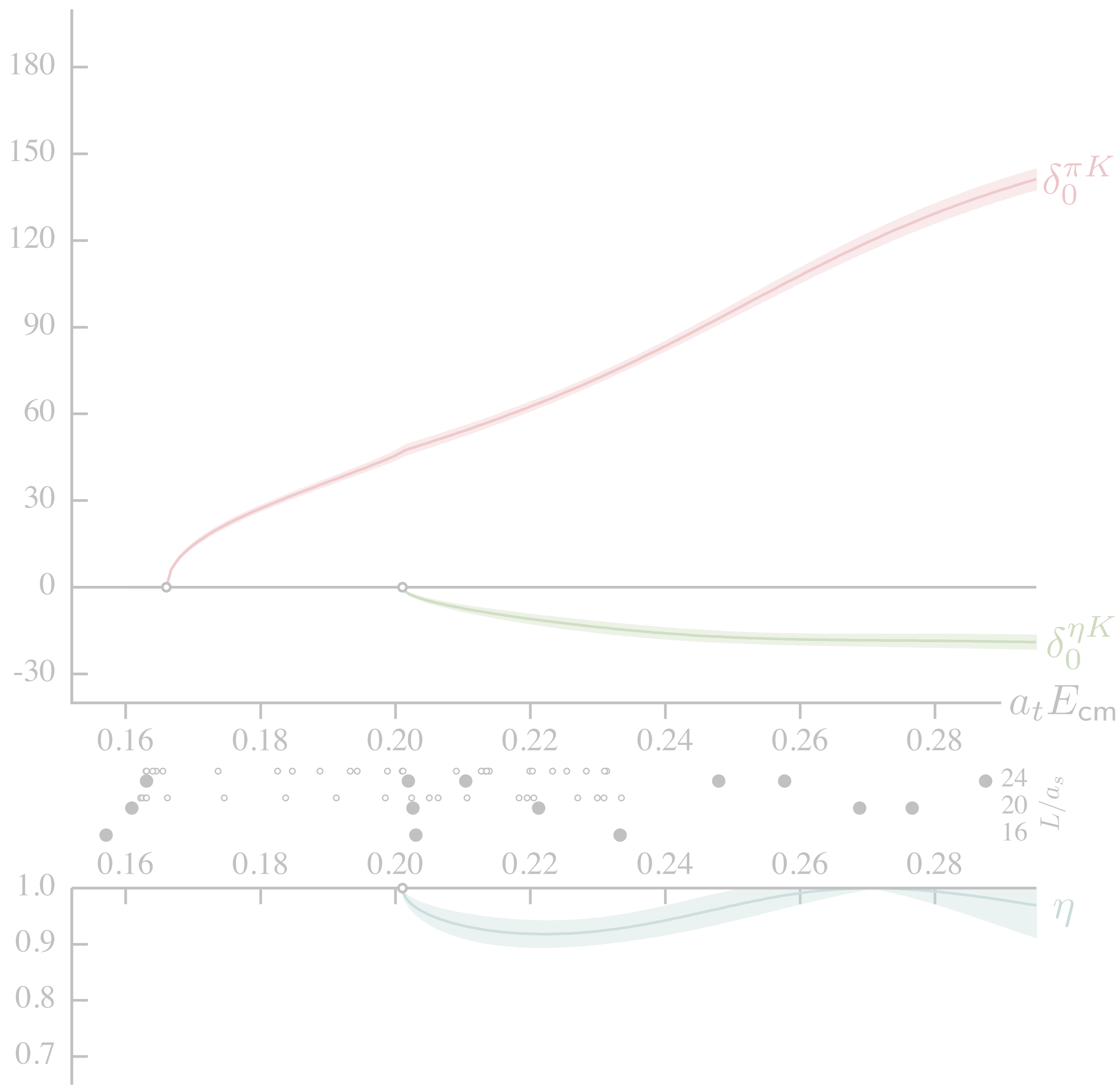
S-WAVE $\pi K/\eta K$ SCATTERING



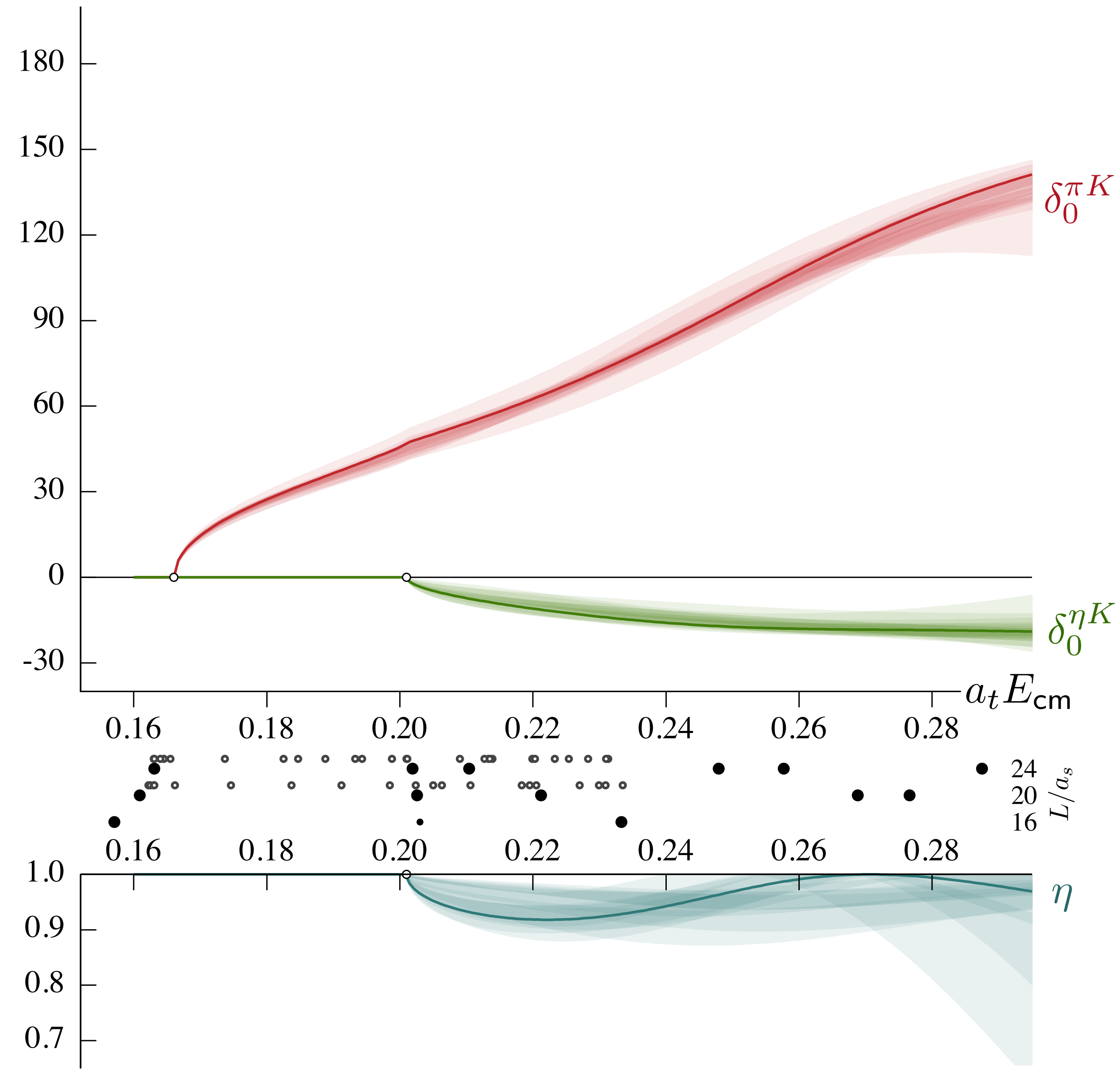
$\pi K/\eta K$ coupled-channel scattering

are the result parameterization dependent ?

– try a range of parameterizations ...



S-WAVE $\pi K/\eta K$ SCATTERING



– gross features are robust

$\pi K/\eta K$ coupled-channel scattering

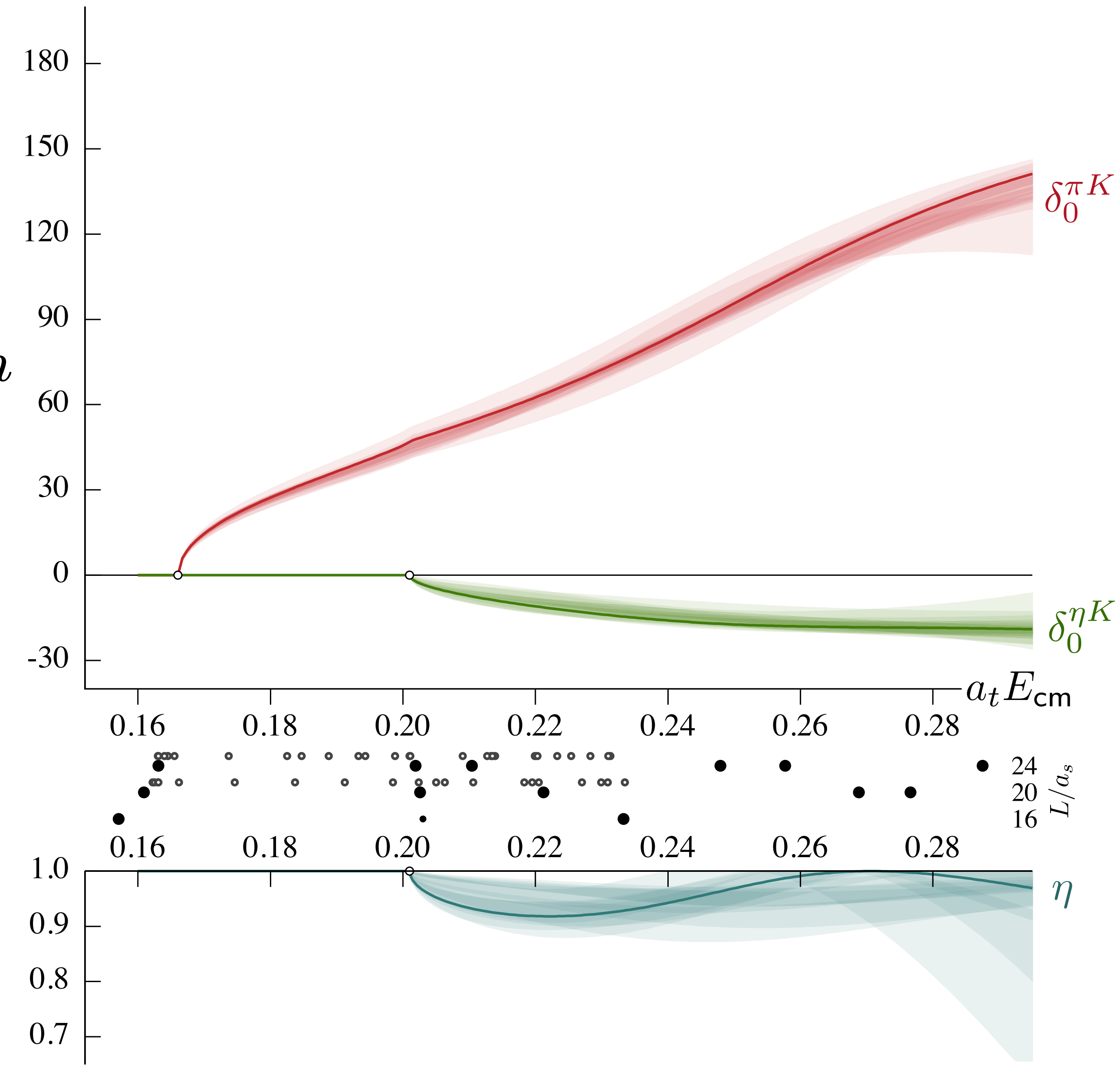
are the result parameterization dependent ?

– try a range of parameterizations ...

$$K^{-1}_{ij}(s) = \sum_{n=0}^{N_{ij}} c^{(n)}_{ij} s^n$$

$$K_{ij}(s) = \sum_p \frac{g^{(p)}_i g^{(p)}_j}{m_p^2 - s} + \sum_n \gamma^{(n)}_{ij} s^n$$

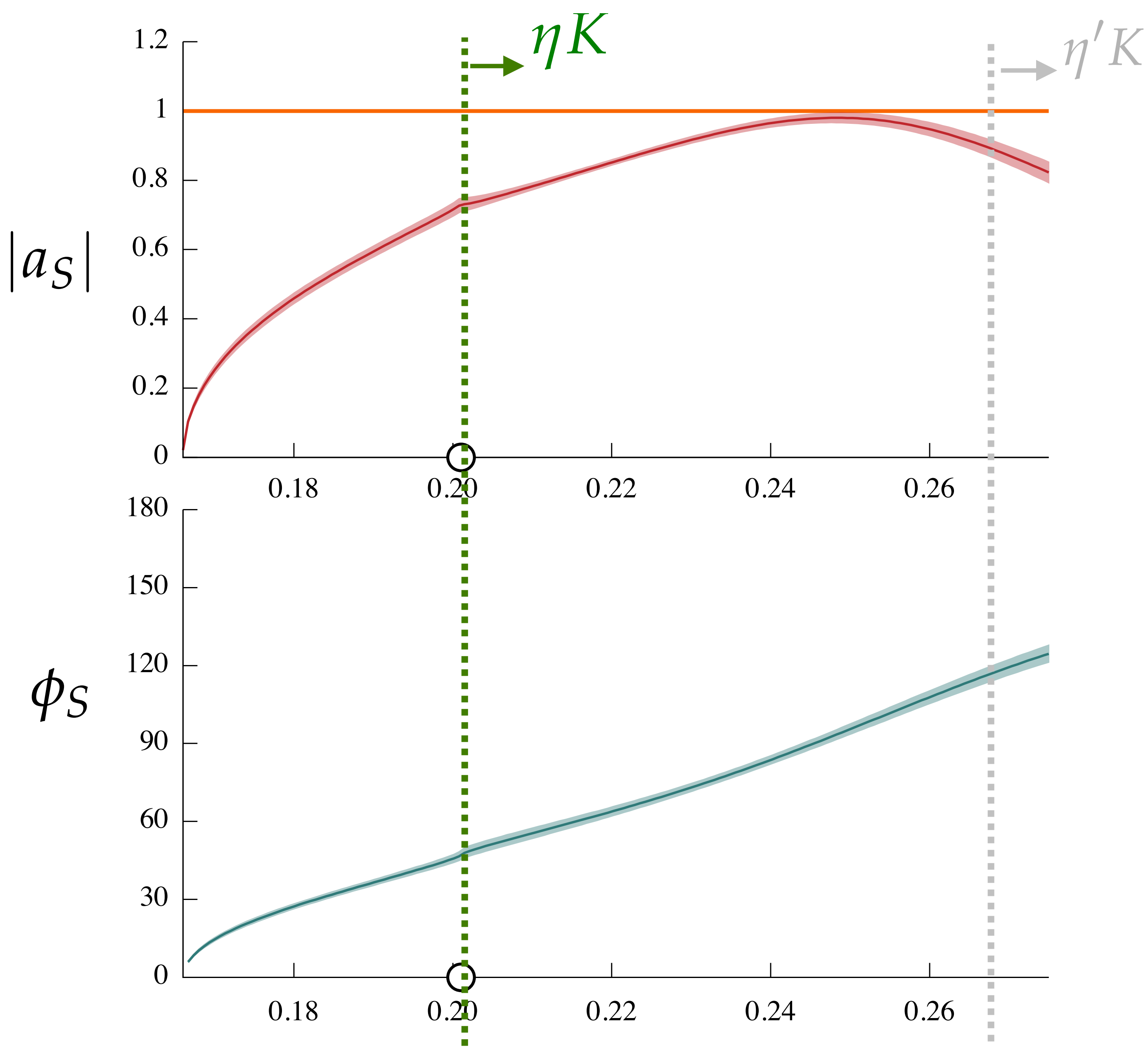
S-WAVE $\pi K/\eta K$ SCATTERING



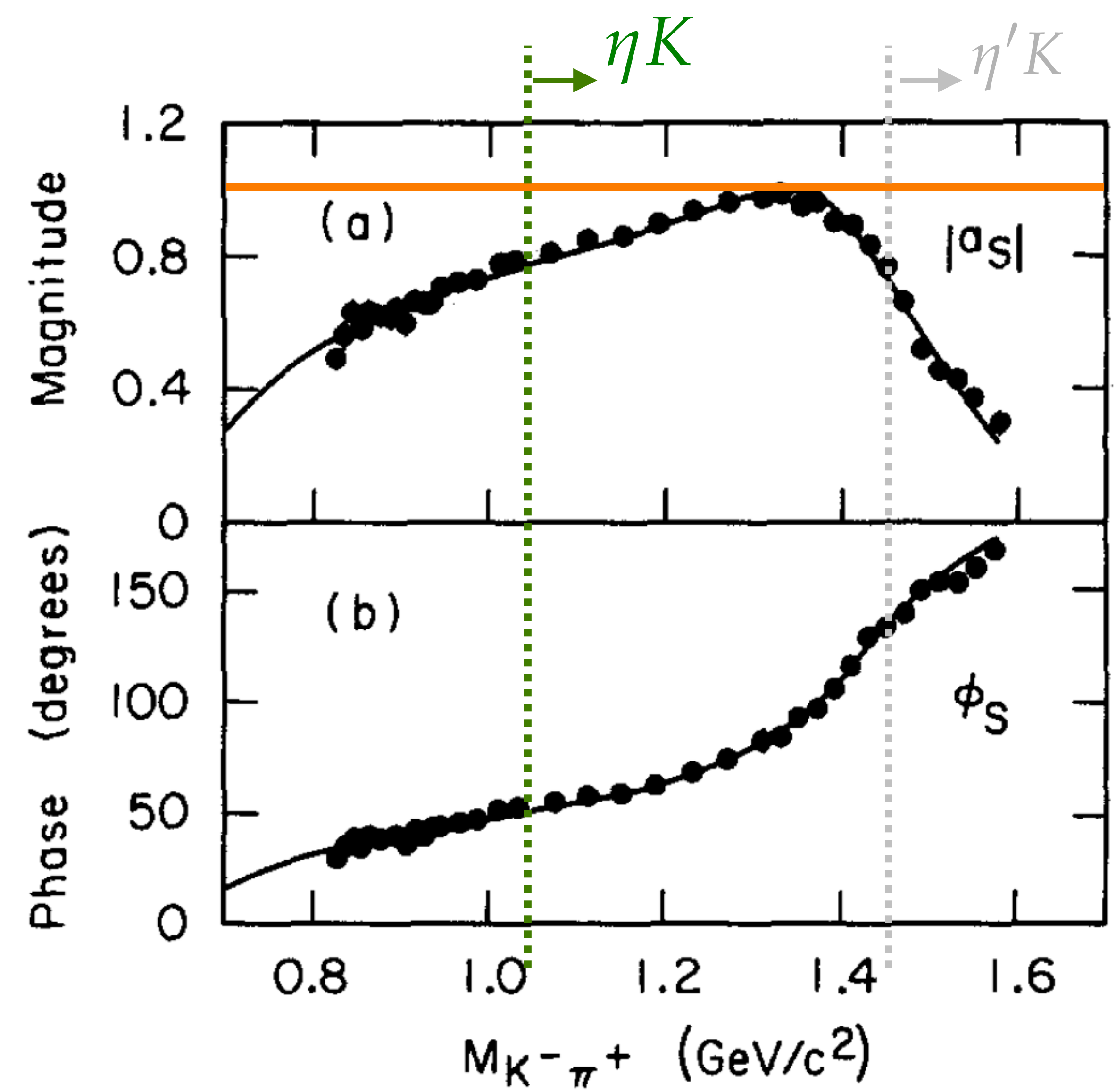
– gross features are robust

versus experimental scattering

S-WAVE $\pi K \to \pi K$ AMPLITUDE



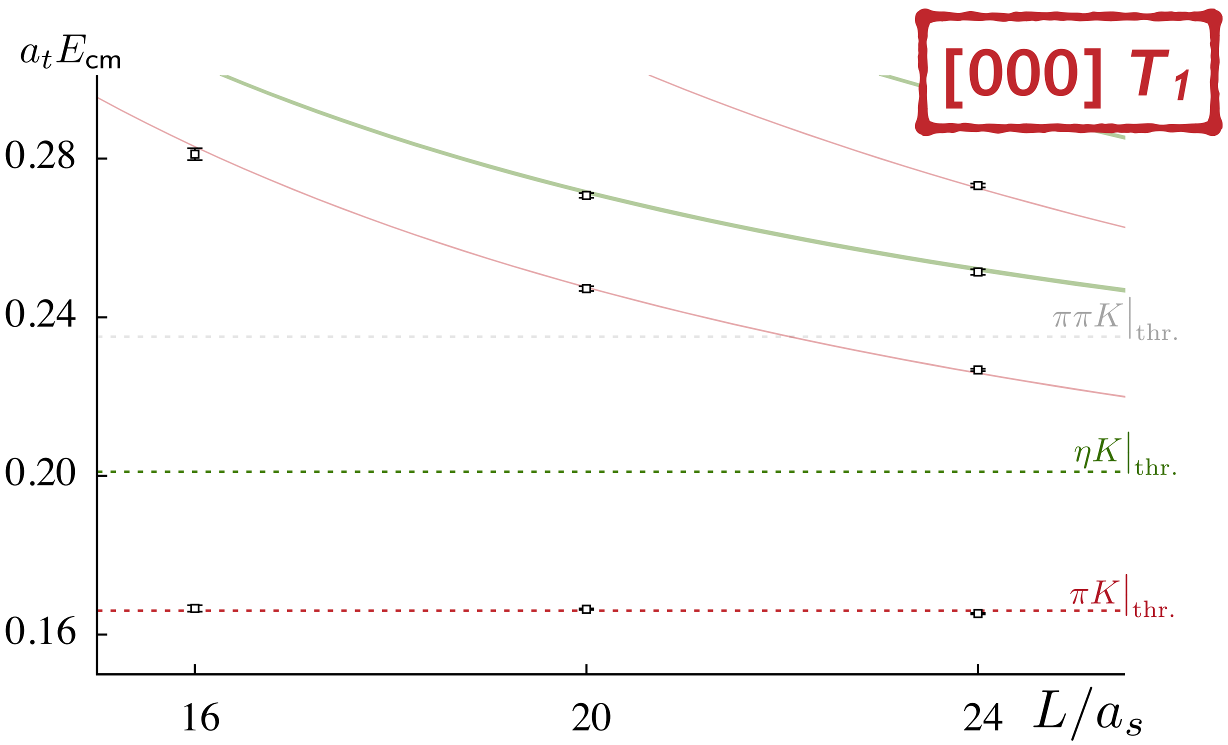
$m_\pi \sim 391\,\mathrm{MeV}$

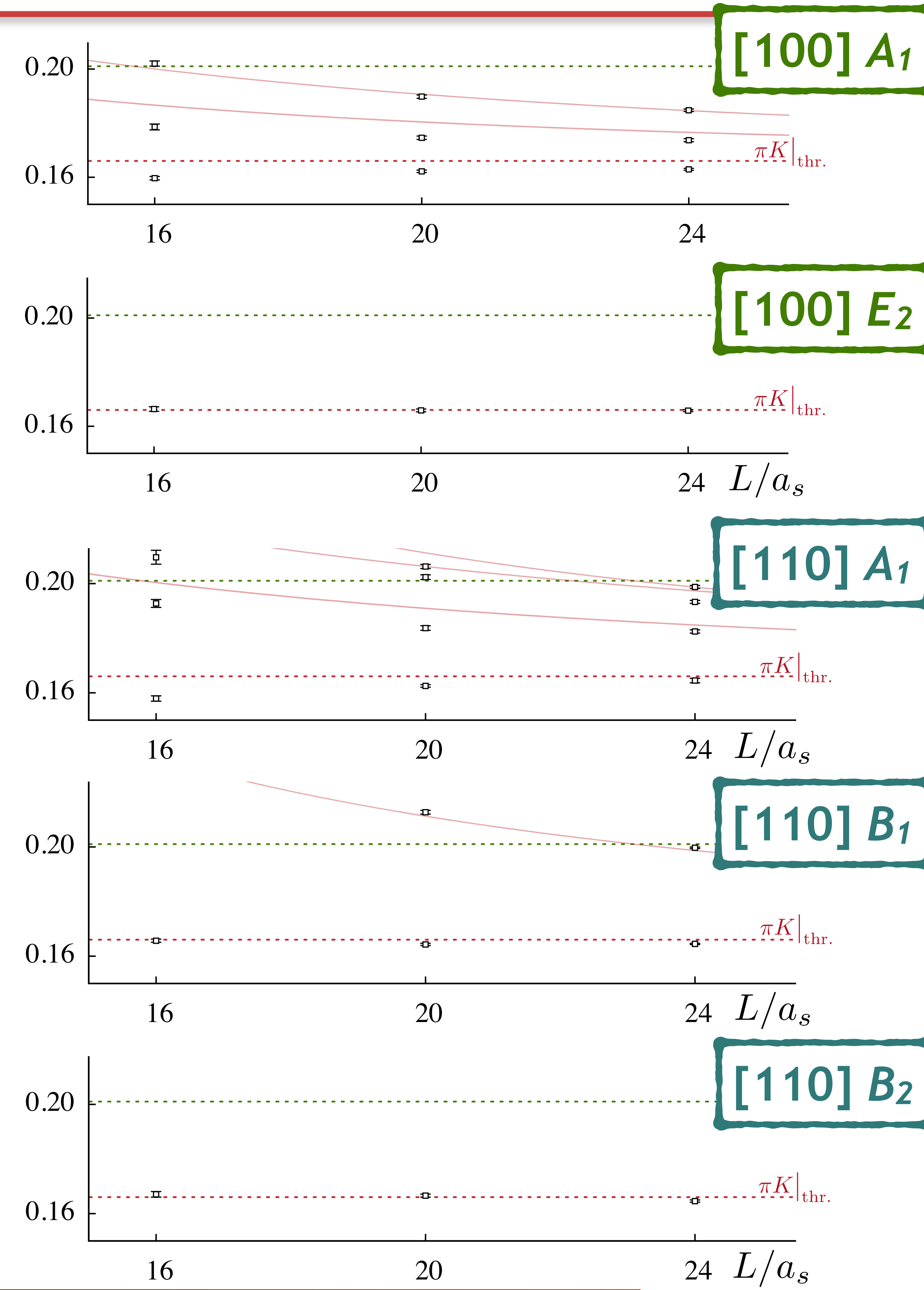


LASS, NPB296 493

P-wave scattering

every irrep containing a subduction of the P-wave has a level very near the πK threshold



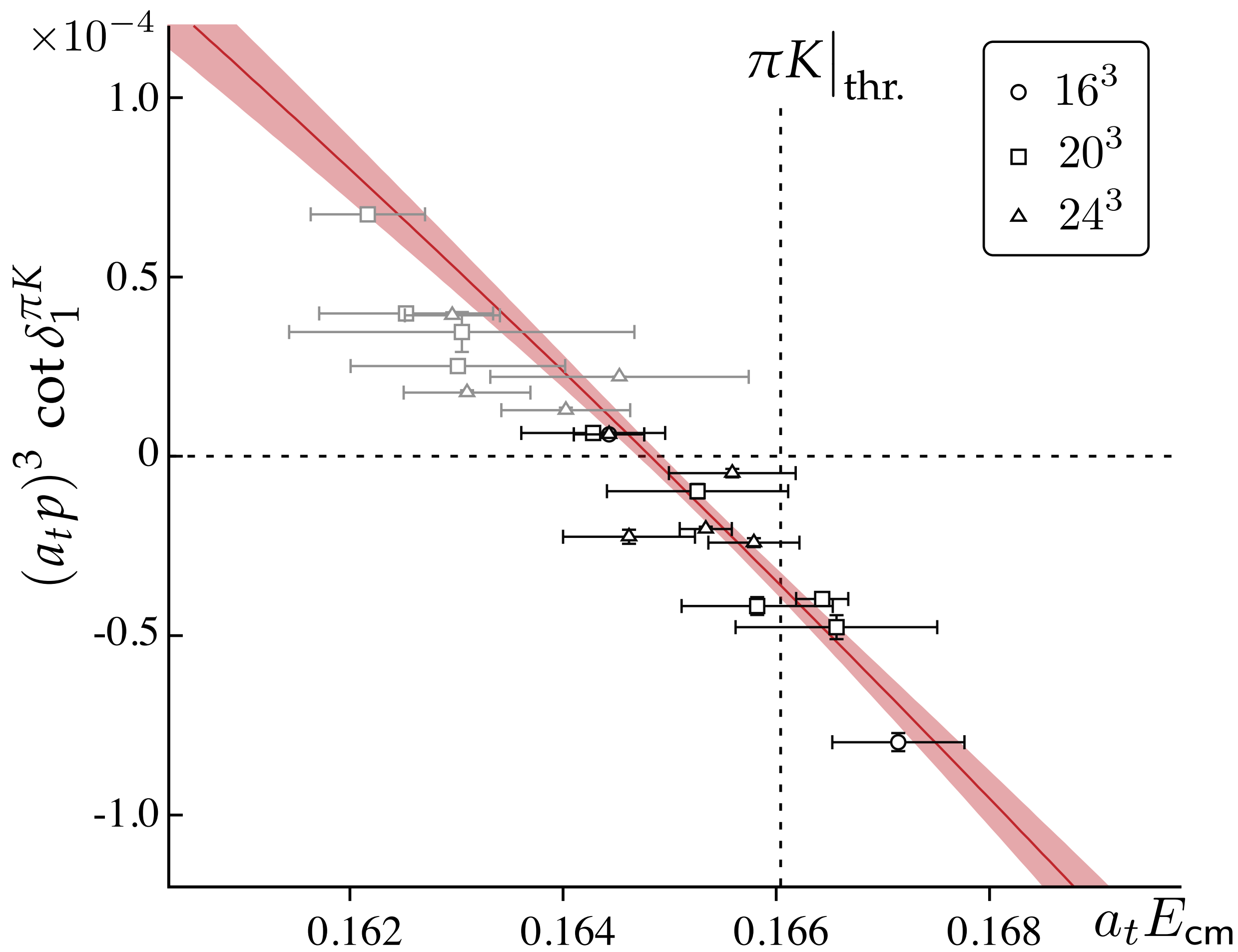


even when there isn't a non-interacting level nearby

suggests a **bound state** near threshold

$\pi K/\eta K$ scattering amplitudes

P-WAVE πK SCATTERING

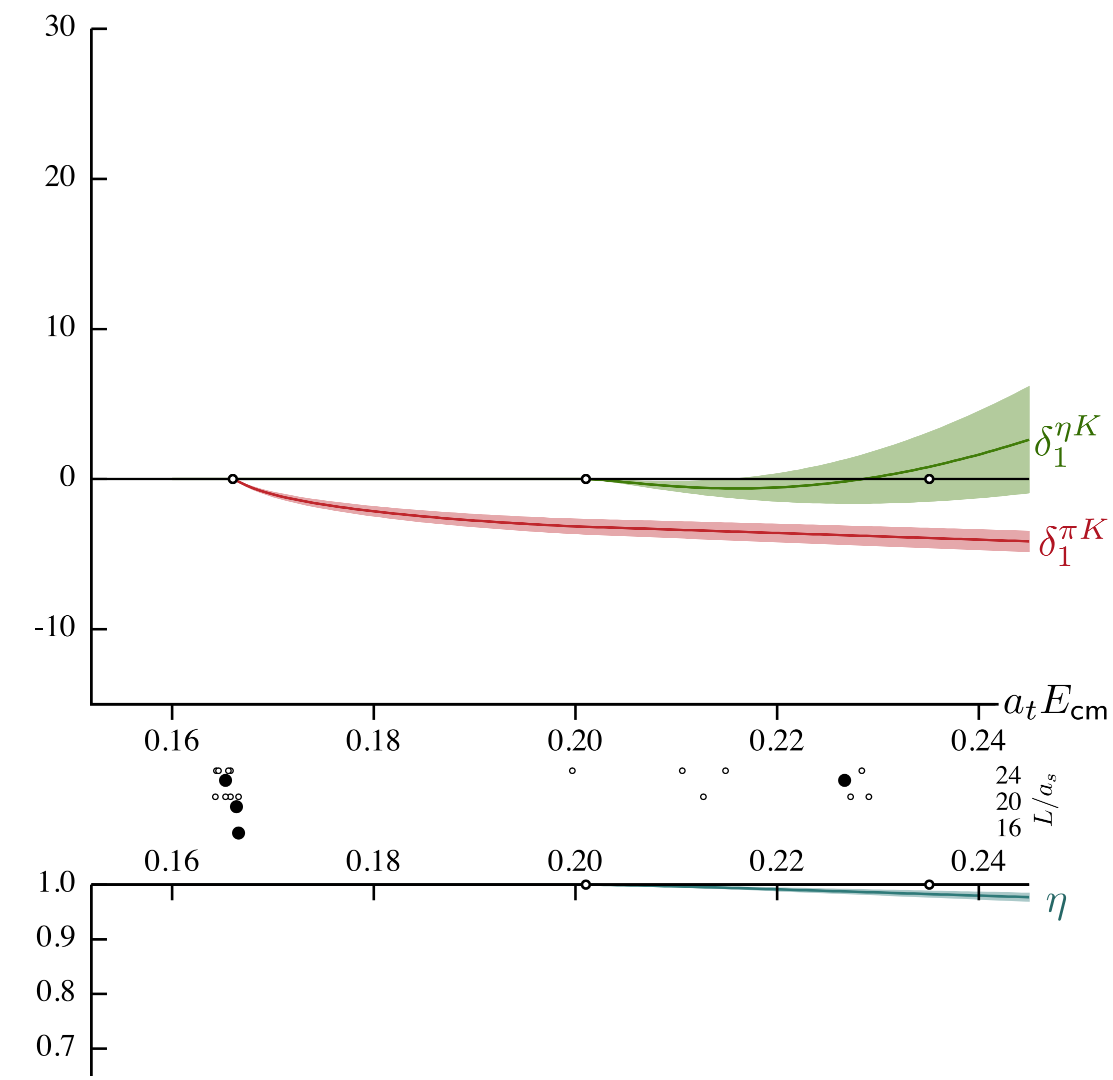


use a Breit-Wigner with a subthreshold mass

$$a_t m(K^\star) = 0.16482(15)$$
$$g = 5.93(30)$$

vector bound-state

P-WAVE $\pi K/\eta K$ SCATTERING



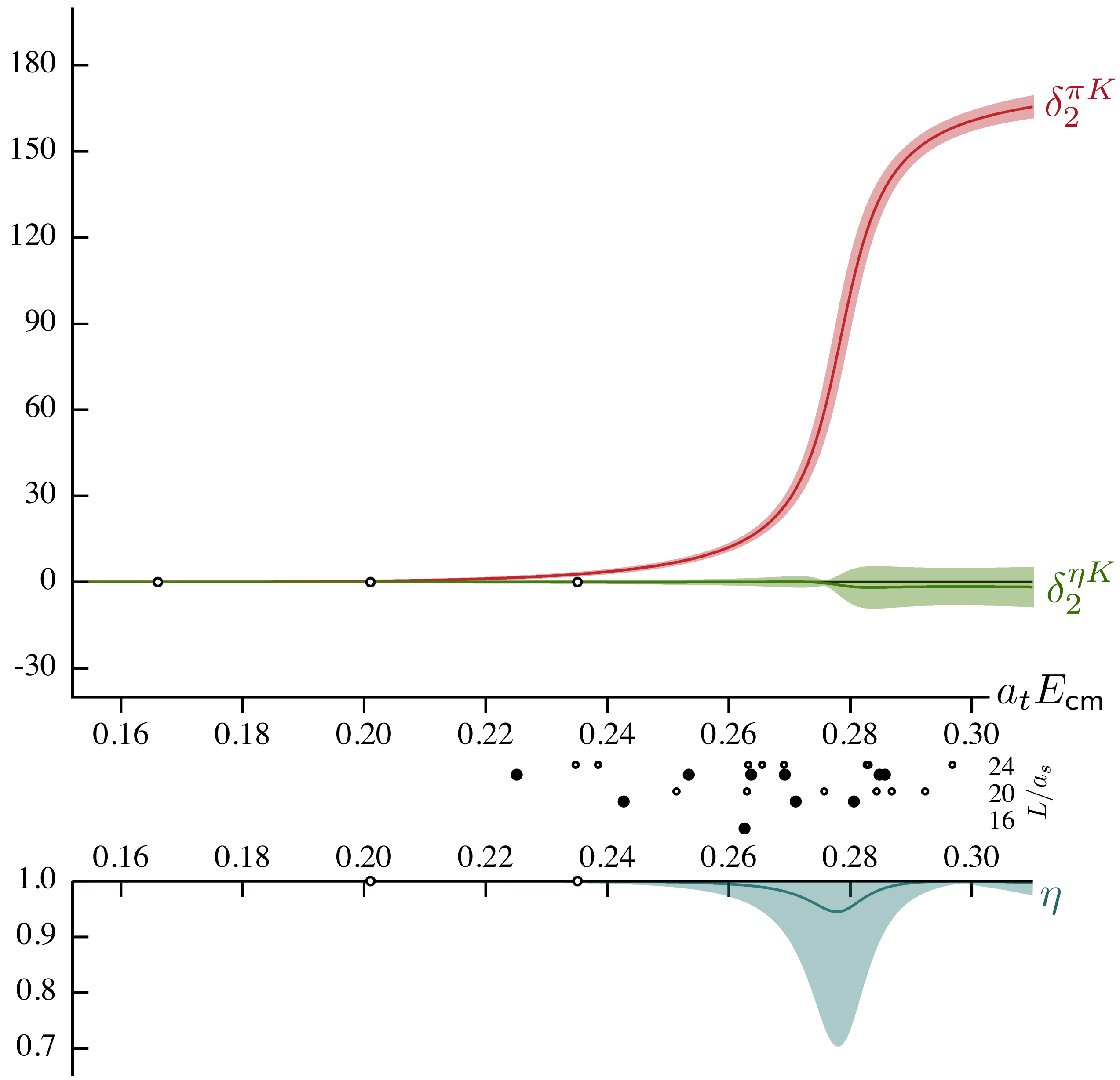
quark mass accident that it lies so close to threshold ...

$g_{\text{phys.}} = 5.5(2)$ *PDG*

$\pi K/\eta K$ coupled-channel scattering

clear narrow resonance in *D*-wave scattering

D-WAVE $\pi K/\eta K$ SCATTERING

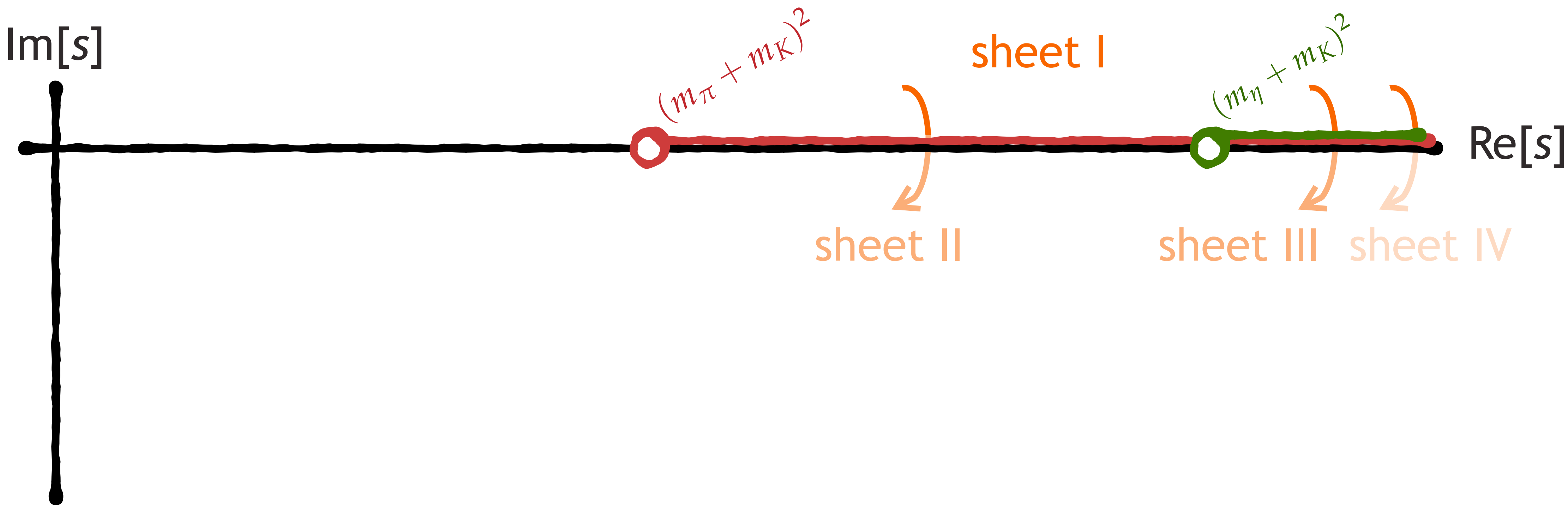


$m_\pi \sim 391\,\text{MeV}$

(you might worry about $\pi\pi K$ in this case)

pole singularities

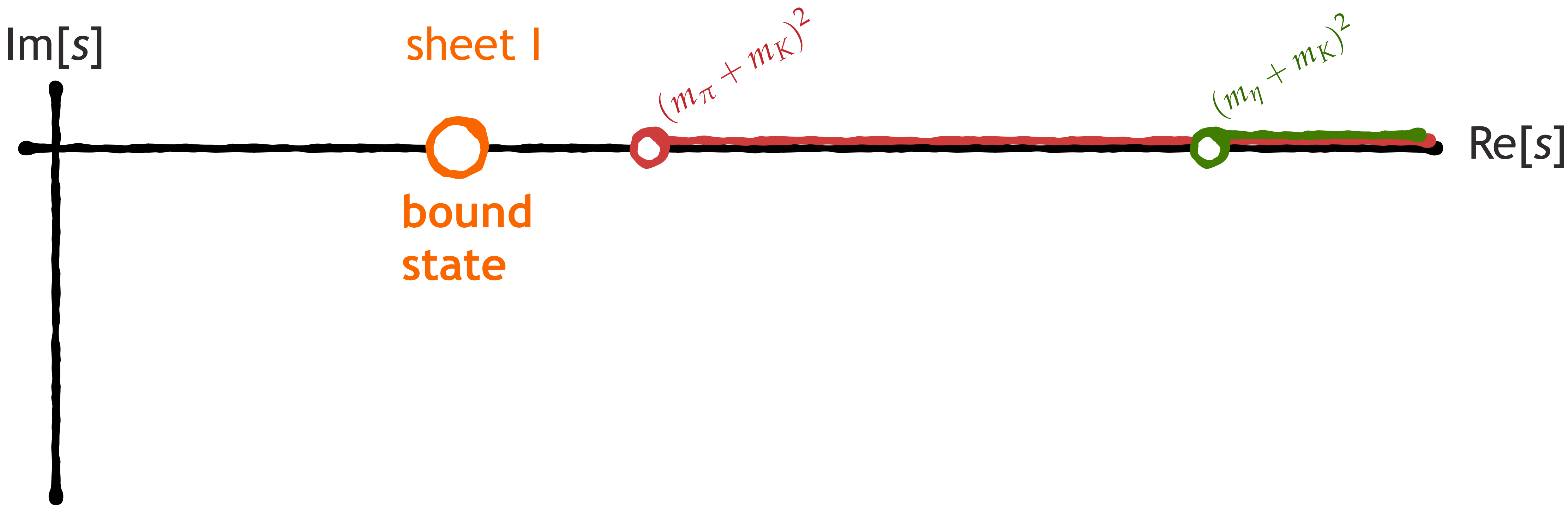
have analytic parameterizations of the scattering amplitudes
– can examine them for the presence of poles at complex *s*



sheet	Im $p_{\pi K}$	Im $p_{\eta K}$
I	$+$	$+$
II	$-$	$+$
III	$-$	$-$
IV	$+$	$-$

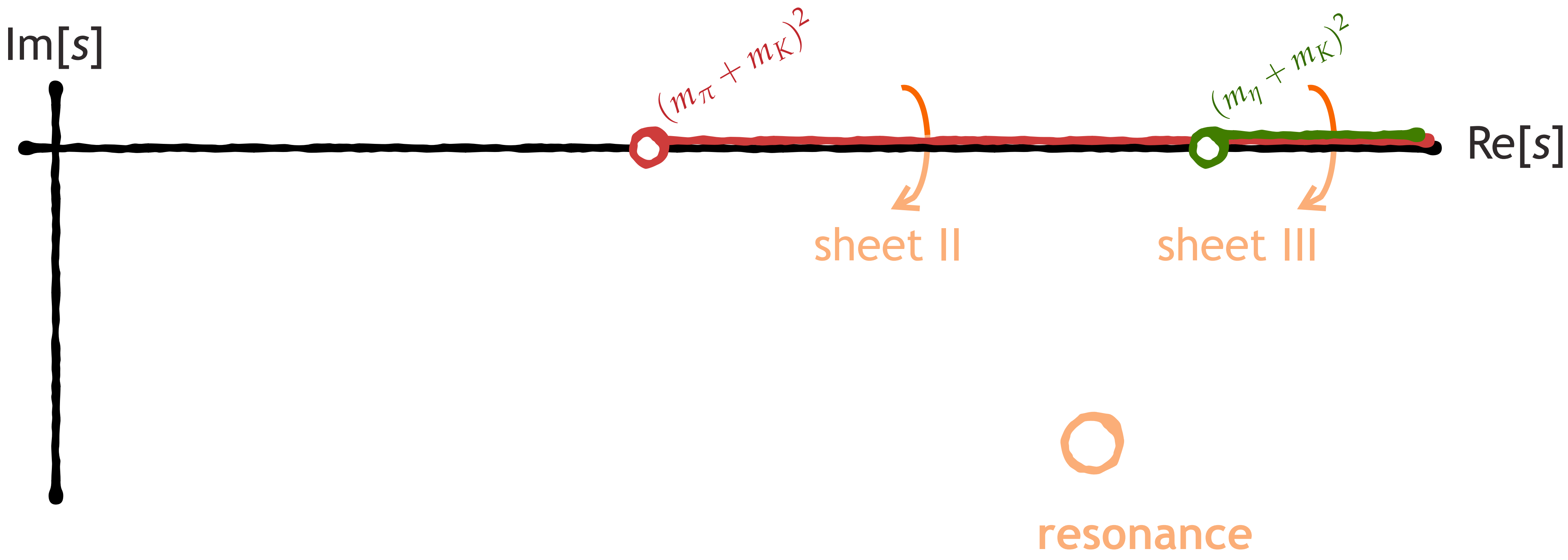
pole singularities

have analytic parameterizations of the scattering amplitudes
— can examine them for the presence of poles at complex s



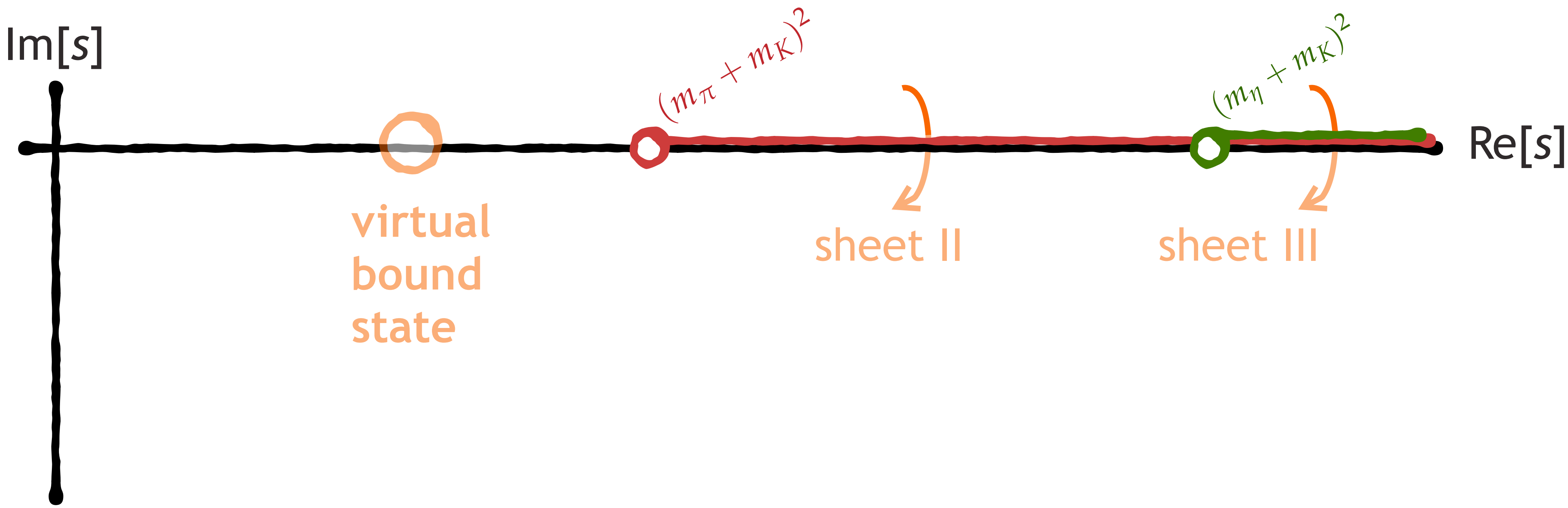
pole singularities

have analytic parameterizations of the scattering amplitudes
— can examine them for the presence of poles at complex *s*



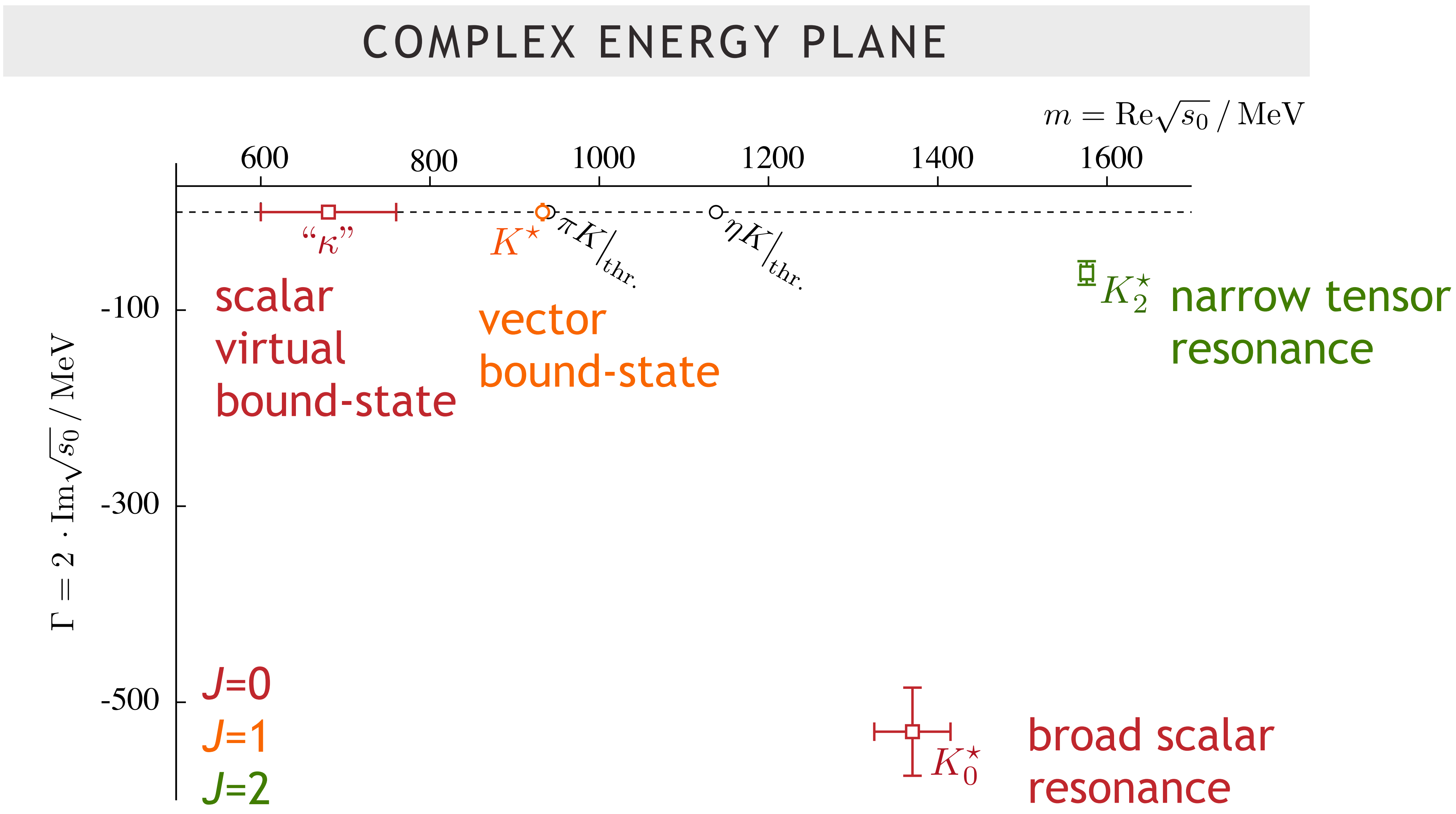
pole singularities

have analytic parameterizations of the scattering amplitudes
— can examine them for the presence of poles at complex s



singularity content

we find ***t*-matrix poles** in each partial-wave

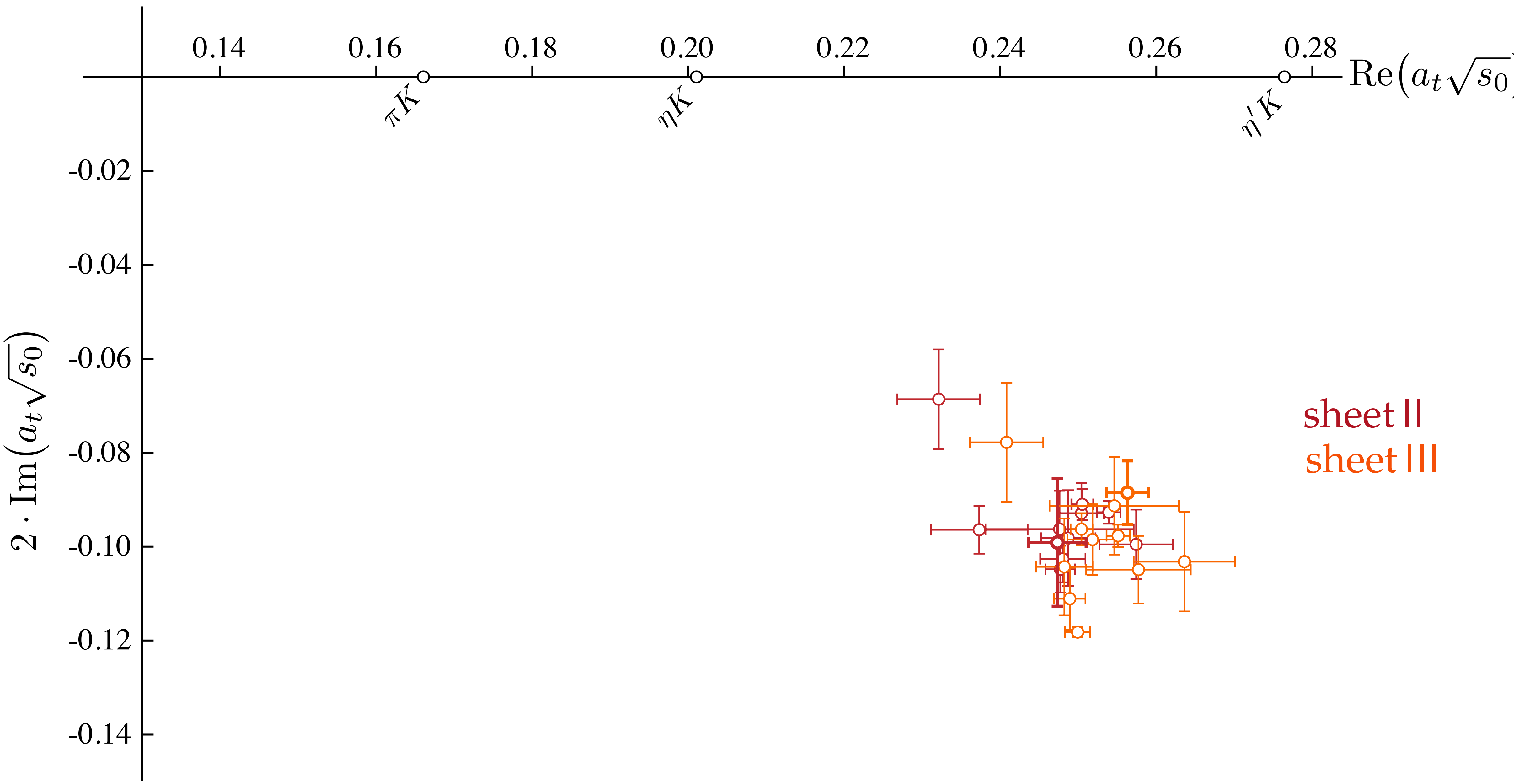


$m_\pi \sim 391$ MeV

PRL 113 182001
PRD 91 054008

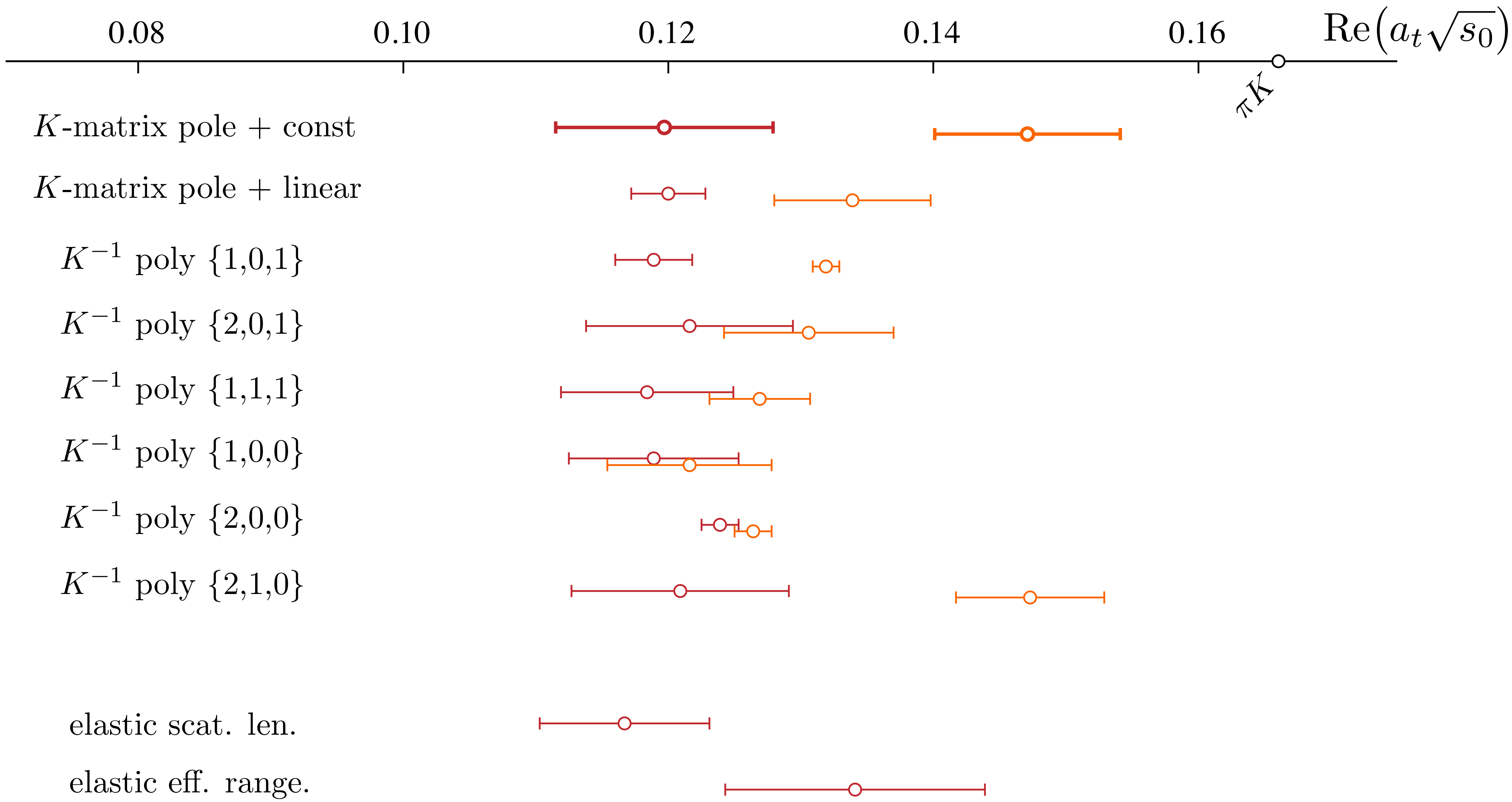
$\pi K/\eta K$ S-wave resonance

- t-matrix pole position under variation of parameterization



$\pi K/\eta K$ S-wave virtual bound state

- t-matrix pole position under variation of parameterization



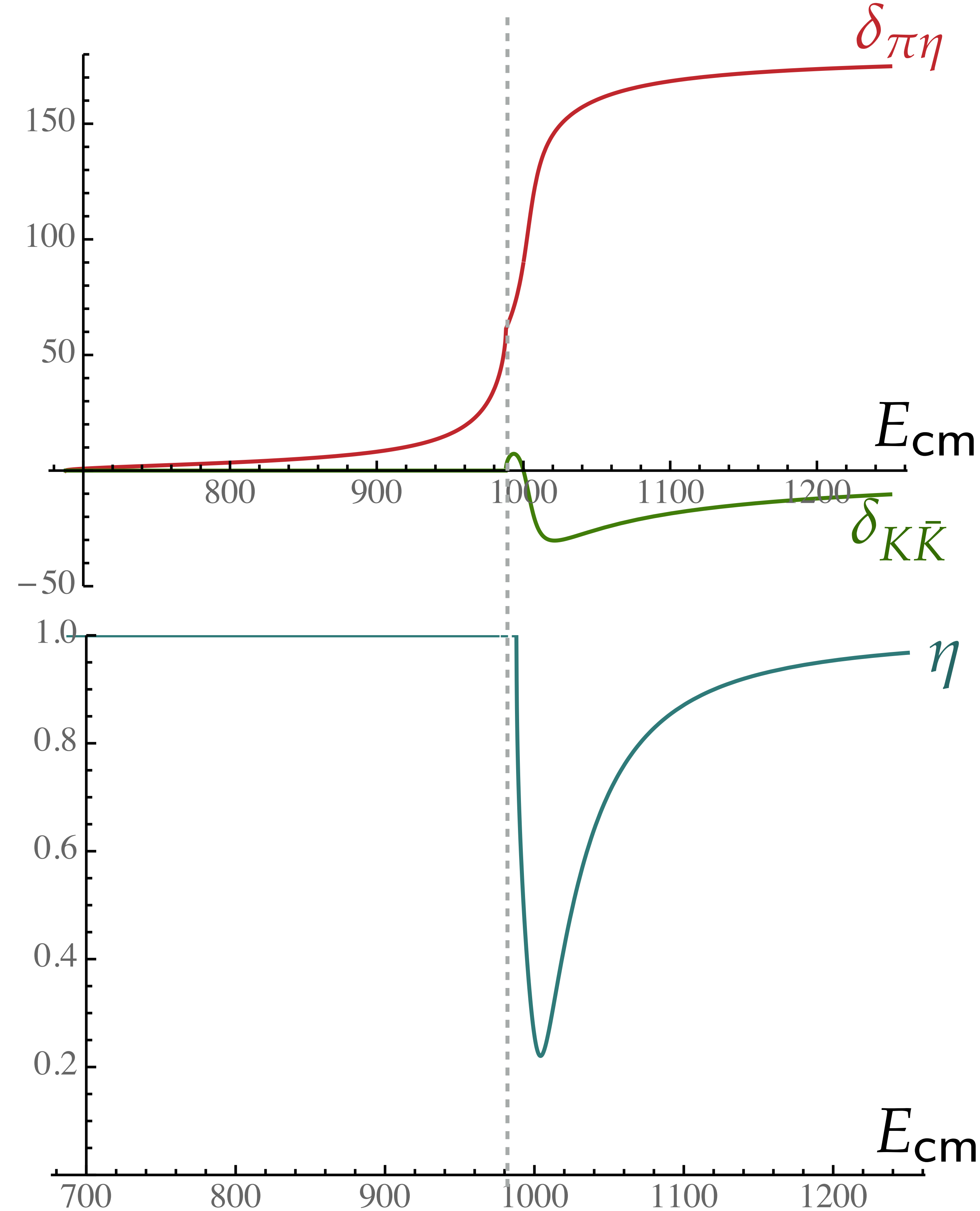
what else can you do ?

with the current technology:
other two-body coupled-channel problems ...

e.g. $\pi\eta,\ K\overline{K}$ $\quad a_0$ resonance
$\pi\eta'$

strongly coupled to both channels
$K\overline{K}$ molecule ?

OBELIX $a_0(980)$ K-MATRIX FIT



what else can you do ?

with the current technology: other two-body coupled-channel problems ...

e.g. $\pi\eta,\ K\bar{K}$ $\pi\eta'$ — a_0 resonance — strongly coupled to both channels; $K\bar{K}$ molecule ?

first study by the end of this year (at m_π~391 MeV)

e.g. $\pi\pi,\ K\bar{K},\ \eta\eta$ — f_0 resonances — $\sigma,\ f_0(980)\ldots$

distant σ pole
$f_0(980)$ as $K\bar{K}$ molecule ?
glueball contributions ?

e.g. $\pi\omega\ldots$ — [ω stable at larger quark masses]

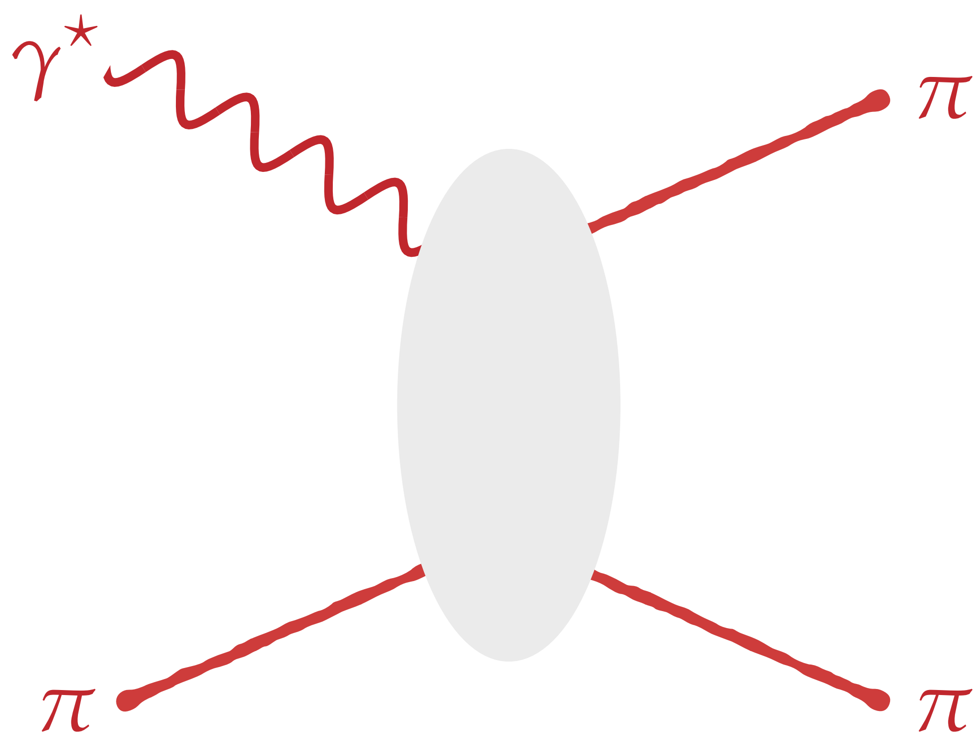
axial resonance physics - coupled *S*,*D*-waves

... use quark mass dependence as a tool ...

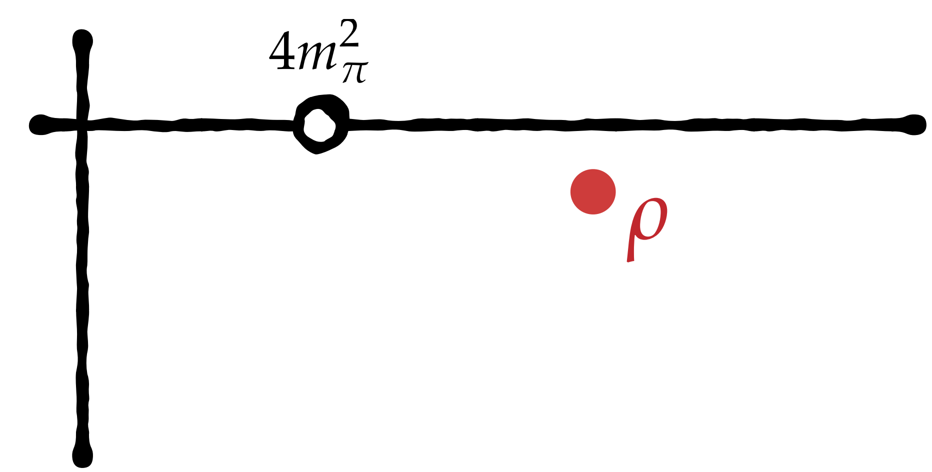
what else can you do ?

coupling to external currents

e.g.



formalism exists to determine the amplitude $A_P^{\gamma^\star\pi\to\pi\pi}(s, Q^2)$



residue at the pole is the (unstable) **ρ→πγ transition form-factor**

$$A_P^{\gamma^\star\pi\to\pi\pi}(s \sim s_\rho, Q^2) \sim \frac{g_{\pi\gamma}^{(\rho)}(Q^2)\, g_{\pi\pi}^{(\rho)}}{\;}$$

coming soon to the arXiv …
Raul Briceno et. al.

what are the challenges ?

as you've heard, many resonances appear in three-hadron final states e.g.

$$a_1 \to \pi\pi\pi$$
$$\eta(1295) \to \eta\pi\pi$$
$$N^\star \to N\pi\pi$$

an open problem is:

how are three-body amplitudes related to the spectrum in a box ?

no complete formalism to date ... so naturally no explicit calculations

try simple channels first ?

$\pi\pi\pi$ isospin=3 ~ non-resonant

$\pi\pi\pi$ isospin=2

~ non-resonant 3-body

~ resonant 2-body 'isobars'

to conclude ...

lattice QCD is a controlled approximation to QCD

implement numerically on big computers

calculate correlation functions → spectra, matrix-elements

field theories in finite-volume have a discrete spectrum

but that spectrum is related to scattering amplitudes

calculate enough spectra and you can infer the scattering amplitudes

these methods now being applied

$\pi\pi$ elastic scattering

first determination of coupled-channel case: $\pi K, \eta K$

coupling to external currents: first calculation will appear soon $\gamma^{*}\pi \rightarrow \pi\pi$

... in all cases utilize constraints from S-matrix theory ...

thank you

Jozef Dudek

Old Dominion University

Jefferson Lab

hadron spectrum collaboration

JEFFERSON LAB

Jozef Dudek
Robert Edwards
Balint Joo
David Richards

Dave Wilson
Raul Briceno

TRINITY COLLEGE, DUBLIN

Mike Peardon
Sinead Ryan

TATA, MUMBAI

Nilmani Mathur

CAMBRIDGE UNIVERSITY

Christopher Thomas

U. OF MARYLAND

Steve Wallace

MESON SPECTRUM

PRL103 262001 (2009)	$I=1$
PRD82 034508 (2010)	$I=1,\ K^\star$
PRD83 111502 (2011)	$I=0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I=0$
JHEP05 021 (2013)	$D,\ D_s$

BARYON SPECTRUM

PRD84 074508 (2011)	$(N,\Delta)^\star$
PRD85 054016 (2012)	$(N,\Delta)_{\rm hyb}$
PRD87 054506 (2013)	$(N\ldots\Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
arXiv:1502.01845	$\Xi^\star_{cc}$

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi\ I=2$
PRD86 034031 (2012)	$\pi\pi\ I=2$
PRD87 034505 (2013)	$\pi\pi\ I=1,\ \rho$
PRL113 182001 (2014)	$\pi K, \eta K$
PRD91 054008 (2015)	$\pi K, \eta K$

"TECHNOLOGY"

PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p}>0$

MATRIX ELEMENTS

arXiv:1501.07457	$M' \to \gamma M$
PRD90 014511 (2014)	$f_{\pi^\star}$