VOTING COMPLEXITY AND ELECTORAL OUTCOMES: AN AGENT-BASED MODEL OF CONDORCET SOCIAL CHOICE PROBLEMS

By

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Abstract

This paper presents an agent-based model of elections in which there are three possible choices. Previous agent-based models of voting typically have examined the simple two-choice election, while previous formal treatments of three-choice problems have necessitated strict axioms that are both theoretically restrictive and empirically unfounded. The model presented here corrects for both of these limitations. It illustrates how elections may produce socially undesirable outcomes such as frequent reversals of collective decisions and election winners that a majority of voters do not prefer. The model endogenizes two key processes in voter preference formation: the influence of local social networks and the effect of collective choice on an individual's strength of preferences for subsequent elections. I test these dynamics using a genetic algorithm that explores the model's parameter space; this "active nonlinear test" (Miller 1998) illustrates the conditions under which preference cycles and Pareto suboptimal outcomes emerge. I use as an empirical referent the well-documented practice of tactical voting in British parliamentary elections.

After Prime Minister Tony Blair recently called for a general parliamentary election to be held on May 5, 2005, much of the popular press predicted a third successive Labour government under Blair's leadership, albeit one with a reduced majority in the House of Commons. Yet the same press also suggests there is some uncertainty both to the margin of Labour's victory and perhaps even to its inevitability. An important source of this uncertainty is the common practice among British voters to cast a vote not in favor of their preferred political party, but instead to vote against a particular party. In the 1997 and 2001 general elections, for example, many voters from the Liberal party may have cast their ballot for Labour candidates instead, to prevent Tory gains in the Commons ("Where has all the hatred gone?" 2001). Whereas such "tactical voting" may have boosted Labour's majority in 2001 (one estimate suggests tactical voting cost the Tories 43 seats; see "Why the system favours Labour" 2005) observers speculate that in the May 2005 general election voters will abandon such protest voting and instead vote for candidates from their party of choice or cast anti-Labour tactical votes (Baldwin and Rose 2005). This is due both to widespread dissatisfaction with the Labour government's performance over the last four years—particularly its support of American policy toward Iraq—but also because Liberal candidates appear to have reasonable chances in a number of marginal constituencies. *The Economist* suggests that Liberal voters' abandonment of tactical voting, or "tactical unwind," may lead to surprising gains for the Conservative party ("Not drowning. . ." 2004; "Why the system . . ." 2005). While these gains may not be enough to return the Tories to government, it nevertheless highlights the difficulties of predicting the outcome of elections. Traditional indicators of vote choice, like a voter's party identification or his or her responses to a pollster's question, may be unreliable.

It is no insight to observe that voting behavior is complex, particularly in advanced democratic states with established party systems, sophisticated campaign strategies, round-the-clock news media, the internet and bloggers, and spin doctors. Tactical voting is theoretically interesting, however, because the choice that British voters face typifies a wide variety of social choice situations in democratic states. That is, just as British voters must choose among Labour, Liberal or Tory candidates, frequently democratic citizens must decide from among more than two policy options. Such choices among three or more possibilities present unique problems in the aggregation of interests into a social choice, a fact recognized by the Marquis de Condorcet two and a half centuries ago. In brief, Condorcet recognized that in such situations a society may appear unable to make up its collective mind, cycling among the three alternatives, even if individual voters maintain fixed preferences. Condorcet cycles present problems for institutional design in democracies, as well as puzzlement for students and observers of democratic politics.

Condorcet-style decision problems characterize many if not most public policy issues in democratic states. Though parliamentary elections may offer us glimpses of the dynamics of voter preference formation under three-choice conditions, these social choice problems permeate public dialogue, opinion polling, and even referenda. It is therefore somewhat surprising that although researchers have amassed a commendable record of empirical research on two-choice social decisions, we have a relatively sparse empirical record of Condorcet three-choice problems (though formal theorists have devoted considerable attention to them). Much of this empirical record resides in the study of tactical voting in British general elections. The paucity of empirical data reflects two facts. First, because polling tends to be anonymous—whether through opinion surveys or at the ballot box—researchers cannot observe the dynamics of preference formation directly and often must rely upon unreliable operational measurements (see Niemi, Whitten and Franklin 1992, 1993 vice Evans and Heath 1993, Heath and Evans 1994). Second, and more importantly, the process of opinion formation exhibits the properties of a complex adaptive system. Voters in democratic societies are perfectly autonomous in their decisions about public issues or political candidates; they change their minds in response to both local and national factors, and their interactions produce feedback loops (popular support or opposition may generate a bandwagon effect) and nonlinearities that are "path dependent," or strongly conditioned by the previous history of opinion formation in a society. Complex adaptive systems are difficult to study empirically and statistically. This is one reason for the scarcity of valid and reliable empirical data on opinion formation and voting behavior in three-choice elections.

In this paper, I propose an alternative method of exploring the dynamics of preference formation in democratic societies. Building on the insights of complex systems theory, I develop an agent-based model of preference formation among voters faced with three possible choices. Because the paper uses the combined inductive and deductive techniques of agent-based modeling, it avoids many of the problems of validity and reliability that plague empirical researchers. Of course, for this very reason the model does not offer a test of hypotheses, at least as we conventionally think of them in the social sciences. Complex systems theorists have developed algorithmic techniques for exploring the parameter space of agent-based models, however; these techniques are useful inductive tests of hypotheses. I use one such technique, Miller's active nonlinear test (1998), to identify the conditions under which tactical voting will arise. The model offers a second advantage, both over empirical studies and existing agent-based models of voting. Because the model looks explicitly at Condorcet choice problems with more than two possible social choices, it offers a generalization of previous agent-based models of voting. I assert that this generalization offers researchers a tool for understanding tactical voting, preference cycling and a number of other empirical phenomena about which existing agent-based models of preference formation are mute.

Condorcet's Paradox and Social Choice Theory

Condorcet illustrated an important fallacy of the composition of social choices: even though individuals may have ordered and transitive preferences among three social options, the collective choice itself may not be transitive. That is, when faced with more or two alternatives, any of the three (or more) options may defeat any other option in pairwise voting, even if individual preferences are perfectly transitive. Condorcet's paradox is perhaps best elaborate by Arrow (1963) whose impossibility theorem illustrates that no social choice rule exists that satisfies the desirable properties of nondictatorship, the inclusion of all individual preferences, Pareto optimality (no individual can be better off by options other than the social choice), and the independence of irrelevant alternatives (that is, individual votes depend only on preferences over feasible outcomes). Condorcet's paradox illustrates two important conceptual points for politics in democratic societies. First, electoral institutions cannot be both politically neutral and inclusive: because any choice can defeat any other in pairwise voting, the order of pairwise voting alone determines the winner. Second, the paradox illustrates the possibility of preference "cycles" in which a society regularly changes its collective choice among alternatives. Hence social choices are intransitive even when individual preferences satisfy the condition of transitivity.

Consider the following example of a Condorcet-style three-choice social problem, adapted from Gaubatz (1995: 538-40; see also Radcliffe 1993a). In a simple world of three voters with three options, under any decision rule a societal choice will arise that will be a second-best option for two of the voters. Table 1 shows three voters (*A*, *B* and *C*) who must choose among three nominal policy options (here labeled Green, Yellow and Blue). Each of the three voters has ordered and transitive preferences. "Ordered" simply means that each voter decides among options and ranks them from most preferred to least preferred; no voter is indifferent between any two options. Transitivity is simply the principle that if someone prefers *i* to *ii*, and *ii* to *iii*, then that person also prefers *i* to *iii*. Assume these conditions hold for the notional voters in table 1. Voter A prefers choice "blue" to "green" and "green" to "yellow." Likewise voters B and C rank their preferred colors. Finally, assume that when polled, each voter is sincere and will cast a vote for his or her most preferred outcome.

[Table 1 about here]

If we polled the voters to determine their preferred colors, what would society choose? The answer depends upon our election rule. If we assume a majority rule, then no color achieves a majority. Yellow, green and blue each receive a single vote. "Pairwise" voting produces some surprising results. Suppose we choose between blue and green first, with the winner in a "runoff" with yellow. Clearly blue will win the first election because both voters *A* and *B* prefer blue to green. In the run-off election, yellow would defeat blue since both voters *B* and *C* prefer yellow to blue. Yet yellow is an odd societal choice from a normative perspective, since ideally we want our election to select the so-called Condorcet winner: the choice that will defeat any other possible choice in pairwise voting. Yellow is not the Condorcet winner because two of the three voters have an alternative first choice. Voter *C* prefers green to yellow, and voter *A* prefers both blue and green to yellow. Thus, the solution is not Pareto optimal because at least one voter may be better off with an alternative choice. This rationale obtains for any other pairwise election one might hold, whether it is green versus yellow or blue versus yellow. In these pairwise voting scenarios, two paradoxes arise. First, the option reserved for the second round of pairwise voting always wins, as yellow did in our example. Hence in principle, *any* possible choice may emerge as the collective choice. Second, for any social choice arrived at through this election rule, two of the three voters have a stronger preference for an alternative choice. Hence society's choice is intransitive: green is preferred by two voters to yellow, two prefer yellow to blue, which in turn is favored by two voters over green. Condorcet's paradox and various extensions of it illustrate that political institutions matter because, as in the pairwise example, they shape societal choices in a three-choice situation irrespective of individual preferences. The Condorcet social choice problem illustrates, furthermore, the possibility of electoral "cycles." Since the social choice is intransitive, the choice itself may change regularly depending on how a society aggregates individual preferences into a collective choice.

Social choice problems and preference cycling are not merely abstract problems. Stearns (1999) identifies cases where the Supreme Court has produced opinions that a majority of justices oppose. DeMarzo (1993) finds cycling preferences plague rules for shareholder governance of publicly traded firms. Gaubatz (1995) has illustrated that the inchoate nature of American public opinion toward humanitarian intervention may reflect a preference cycle. These studies all illustrate the possibility of preference cycles even when individual preferences meet the strict assumption of transitivity. Yet a number of researchers have found evidence that voter preferences may in fact be intransitive, suggesting that social choice problems are even messier than the paradigmatic Condorcet example suggests (for example, see Radcliffe 1993b; for opposing evidence see Hansen 1998). Clearly, social choice theory offers both important formal insights into the paradoxes of voting as well as some empirically verified findings about the surprising prevalence of preference cycles. Indeed, DeMarzo (1993: 725) argues "this problem is so pervasive that in the standard one-person/one-vote environment, majority voting equilibria do not exist in general!"

Recent research on Condorcet problems has relaxed some of the classical assumptions to explore their effects on intransitivities. For example Jones et al. (1993) argue that modern computing power allows researchers to relax the assumption of "strong" preferences (i.e. voters have strict preferences among any given pairs, and are indifferent between no choices). Yet Jones et al. preserve the assumption of individual transitivity. To my knowledge, no research has yet explored the implications for social choices of a relaxation of the assumption of transitivity, despite the considerable empirical evidence that individual preferences in fact may not satisfy this assumption. In addition to empirical research on the structure of individual preferences (Radcliffe 1993b), there are good reasons to assume while individual preferences may be transitive, their votes may not be. Social choice theorists typically assume that voters are "sincere"; that is, their preferences necessarily determine their vote. Yet a number of studies have found that individuals often vote contrary to their preferences (Stearns 1999). Studies of tactical voting in British elections find ample evidence of the practice, even if researchers disagree about measures (Galbraith and Rae 1989; Niemi et al. 1992, 1993; Evans and Heath 1993; Heath and Evans 1994; Fieldhouse et al. 1996). In effect, these researchers find voters' preferences are in fact intransitive because voters do not necessarily cast a ballot for their most preferred choice.

The conceptual definition of tactical voting suggests an important difference between preferences and vote choice. Heath and Evans (1994) define tactical voting as "any case where a voter votes for a party other than his or her preferred one in order to reduce the chances of a disliked party winning that constituency" (p. 558). Likewise Franklin, Niemi and Whitten (1994) argue that tactical voters are "those who preferred not to waste their vote on a party that could not win, and so would give it to a less preferred party in order to defeat a third party that was seen as even less desirable" (p. 549). Both of these definitions suggest vote choice is a function not only of an individual's (ordered) preferences, but also of the voter's assessment of the distribution of preferences within society. How do voters assess the distribution of preferences? We know that vote choice may reflect the influence of spouses and neighbors, the media, and previous election results among other sources. Hence an individual's vote choice reflects both internal factors (preferences) as well as social structural factors (the individual's imperfect knowledge about the distribution of preferences within society). Once we relax two assumptions—that of transitivity of preferences, and that of the necessary relationship between preferences and vote choice—social choice problems in Condorcet-style three-choice elections become considerably more interesting. They also become considerably harder to study.

I seek to understand how intransitive preferences at the individual level may give rise to the social choice problems of preference cycling and collective choices that the violate Condorcet criterion. Existing studies are largely mute on this question. Empirical studies of tactical voting suffer both from disagreements about measurement, and from a failure to engage the broader social choice literature (though see Tsebelis 1986). Social choice theory typically utilizes formal methods, complemented with empirical validation of the assumptions of transitive and ordered preferences. Only recently have researchers used Monte Carlo methods to study the consequences of weak preference orders on social choice problems (Jones et. al. 1995). The problem of individual-level intransitivity presents computational challenges similar to those faced when relaxing the assumption of strong preferences. I opt for a different research design. Rather than empirical, formal or Monte Carlo methods, I explore the issue of individual intransitivity using an agent-based model.

Preferences and Voting as a Complex Adaptive System

Electoral institutions arguably exhibit the properties of a complex adaptive system. Complex systems theorists suggest that these systems share important organizational characteristics, which in turn lead such systems to exhibit adaptive behavior. Complex systems consist of a large number of autonomous actors, interacting independently because of the absence of central authority. In this massively parallel structure, autonomous agents exhibit multiple levels of organization, with actors at one level forming the building blocks at other levels (Waldrop 1992). This decentralized structure allows complex systems to evolve, change, grow, adapt and even to anticipate as actors incorporate knowledge about the system into their local decision routines (Holland 1992). Complex adaptive systems thus exhibit systemic behaviors and properties produced by local interactions alone: even simple, conditioned behavior among autonomous agents may produce complex systemic behavior (Holland 1992: 17-30). Complex systems theorists refer to these systemic behaviors as "emergent" properties; typically these emergent behaviors are what most interest researchers.

Even someone who is only a causal observer of elections and voter behavior may draw the obvious parallels between elections and complex adaptive systems. In democratic societies, voters by definition are free from the coercion of centralized authority; they are perfectly autonomous in their decision-making. Voters form multiple levels of organization, from community civic groups to interest groups, coalitions, movements, and political parties. Public opinion both shapes and responds to public policies. Both polls and elections produce emergent behaviors of practical, normative and theoretic interest: they surprise us (Dewey in fact did not defeat Truman) and may seem unjust (as many felt about the 2000 presidential election in the United States). It is thus little surprise that a number of researchers have applied the theories and methods of complex systems to the problems of elections (Wilensky 1998, Kottanau and Pahl-Wostl 2004; Reed 2004).

I build on this tradition of agent-based modeling of democratic elections with two extensions of existing voting models. First, the existing applications of complex systems theory typically have focused on binary social choice problems; they consequently offer no insight on Condorcet three-choice problems. This clearly is a deficiency given the prevalence of Condorcet dynamics not only in elections but also in opinion polls, corporate governance, and other institutions. My extension thus offers a generalization of preference and voting dynamics. Second, just as formal theorists do, agent-based modelers have tended to treat preferences and votes as necessarily identical. I have already noted that empirical research has found ample evidence to question this assumption. Even more problematic, however, is the inability of such agent-based models to explore the reciprocal effects of elections on voter behavior. The outcomes of elections and electoral systems themselves may affect preferences and subsequent votes (Morton and Williams 1999). One may more accurately characterize existing agent-based models as models of preference formation and aggregation rather than as models of elections. I develop an agent-based model of elections that avoids these problems. As I illustrate below, the model provides an explanation for Condorcet preference cycles even under weak conditions of intransitive individual preferences. The model also incorporates several preference formation algorithms to illustrate the conditions under which tactical voting is most prevalent.

The Model

I develop the model using NetLogo 2.1, an java-based integrated modeling environment that uses an object-oriented architecture to simulate the massively parallel organization of complex adaptive systems (Wilensky 1999). I populate a world with 500 agents and randomly assign each agent an ordered set of preferences among three nominal social choices: blue, green and yellow. At each step in the model, agents change their preferences according to different rules that the modeler may vary. Agents must cast a vote for their nominal social choice at fixed periods within a given run of the model, though the modeler may alter the periodicity of elections for different runs. Elections occur no more frequently than every ten steps in the model, and no less frequently than every 100 steps. The election rule is plurality: the winning social choice is that which receives the most votes. Within these broad parameters, I allow both agent preferences and voting strategies to vary.

Voter Preferences

At any given time, agents' preferences satisfy classic assumptions of ordering and weak transitivity. Yet each agent's preferences are mutable and hence may exhibit intransitivity over time; at one step a voter may prefer green to blue, but will prefer blue to green at the next step. This is because I allow voters to change their "minds." In the model, agents poll their neighbors at each step. Strictly speaking, they poll only their von Neumann neighbors (any other agents who are to their left, right, top, bottom and diagonals, for a total of eight possible neighbors). When a given voter finds its opinion is in the minority in the neighborhood, the agent will reorder its preferences with some probability set by the modeler. Agents will reorder by promoting the neighborhood winner one rank but will not necessarily adopt the neighborhood winner as its top choice. For example, an agent that prefers blue > green > yellow, but finds yellow is the neighborhood winner, will reorder its preferences to blue > yellow > green.

Voters in the model consequently reorder their preferences in a stochastic rather than deterministic manner. For any given voter, preferences depend upon the neighborhood winner, the voter's previous preference ordering, and the voter's "sensitivity" to the local winner. This sensitivity is analogous to the strength of a voter's preference. Voters with strong partisan attachments or ideological positions are insensitive to the preferences of others, for example, while moderate voters with weak affiliations may be more considerate of others' preferences and more likely to change their minds. The model endogenizes this sensitivity as a probability of preference change. For example, if the modeler selects voter sensitivity as .05, any given voter will reorder its preferences only about one time in 20 on average.

As each run evolves the model allows voter sensitivity to vary according to the outcomes of previous elections. Voters may be "winners" or "losers" depending upon whether or not their preference was the winner of the last election. I assume that losers and winners change their preferences with different probabilities, and build into the model a number of algorithms that the modeler may activate or de-activate. One implementation, the "good winners" option, instructs winners to accommodate the wishes of the losers. Winning voters do this by reordering their preferences two to five times more frequently than losers do. Conversely, "sore losers" are stubborn and refuse to change their preferences. In this implementation, voters who lose the previous election never change their preferences until they win an election. The model includes a "tolerance" variable for losers, who may have to lose several elections before they become sore losers. Both the good-winners and sore-losers algorithms produce voters with weaker preferences (winners) and stronger preferences (losers). The algorithms differ in one important way: in the good-winners algorithm, losers will reorder their preferences, albeit proportionally less frequently than winners. In the sore-loser algorithm, by contrast, losers never reorder their preferences.

I implement an additional algorithm to explore "boot-strapping" strategies among losers. In a three-choice election, more often than not two losing factions will outnumber the winners. For example, 400 voters may vote for blues, 300 for green and 300 for yellow, but the "anti-blue" voters outnumber the "pro-blues" by three to two. To simulate the process of voters constructing anti-incumbent coalitions, I allow losers to poll only those neighbors who also are losers. This implementation thus allows the anti-blue majority, for example, to reorder their preferences in response to each other while being insensitive to the winning voters. The coalition of losing voters thus may bootstrap their way into a winning collective choice.

Together these implementations connect individual preferences with social preferences, but in an imperfect way. Voters reorder their preferences according to the neighborhood winner, but with some probability less than one. This probability in turn varies according to the social choice at the last election and whether or not the voter's preference accords with the winning choice. In other words, although voters reorder their preferences according to the *local* distribution of preferences, their strength of preferences (and hence the probability of changing preferences) depends upon *global* distribution of preferences at the time of the previous election. It is important to note, furthermore, that both winners and losers have limited knowledge about the distribution of preferences in society as a whole. Their knowledge is limited to the results of the previous election, a snapshot in the ongoing evolution of societal preferences. So while a voter may know his choice did not prevail at election time, in subsequent steps his or her knowledge of overall support for his or her top choice is incomplete. In this respect, voters have current knowledge of local preferences only; the information about overall social preferences decays between elections and is least informative immediately prior to a subsequent election.

Vote Choice

The model endogenizes a routine for tactical voting. If the modeler chooses to activate this algorithm, at each election winners will cast a ballot for their most preferred options. Losers will vote, on the other hand, for their second-preferred outcome. For example, a loser who prefers yellow > green > blue will vote green instead of yellow. Because the model's social choices are nominal, this implementation of tactical voting is generic and cannot distinguish between "instrumental" versus "expressive" tactical voting (Franklin et al. 1994), or "inverse" tactical voting (Tsebelis 1986). These debates over inverse, expressive and instrumental tactical voting are more about individual motives than about the vote choice itself. For this reason, I view the model's algorithmic implementation of tactical voting as consistent with the conceptual definition: voters cast a ballot for a lower ranked choice.

The model allows the researcher to run each of the five implementations of preference formation and voting rules individually or in any combination. Table 2 summarizes the preference formation and voting algorithms for the model.

[Table 2 about here]

Initial Findings

My interest is to explore whether or not the agent-based model will produce the intransitive preferences that typify three-choice social decisions. My assumptions differ from formal treatments of Condorcet problems because I allow for intransitive individual preferences over time. Under what conditions might preference cycles emerge?

I start with a baseline run of the model, in which there are no good winners, no sore losers, no coalitions or tactical voting. In the baseline run, voters reorder their preferences with a probability of .05, or one in every 20 times their neighborhood choice differs from their top choice. In this run the model holds elections every ten steps. Figure 3 produces the results from one such baseline run. From an initial random distribution of preferences among 500 voters, the model converges around the consensus choice of blue after about 70 elections. Although the social choice reversed a couple of times in the early elections, eventually blue develops an advantage that tips the social choice as more voters come to prefer blue. These blue voters in turn influence their neighbors, resulting in the model's convergence around a unanimous choice. Though the length of convergence may differ among runs, this convergence around a unanimous choice typifies the baseline of the model, in which voters change their preferences only in response to local factors (their immediate von Neumann neighbors).

[Figure 3 about here]

Though the baseline run converges on a unanimous equilibrium choice, several of the model's implementations of voting strategies produce the unstable cycles of social choices that characterize Condorcet problems. Figure 4 illustrates a run of twenty elections in a society of good winners, while figure 5 illustrates a run of twenty elections when society has sore losers. In both runs, all three options win a number of elections. In the good-winners run, blue wins ten of the 20 elections, green wins four, and yellow wins six. Likewise blue is the winner eight times in the sore-losers run, green wins six times, and yellow wins five times (blue and green tied in one election). It is worth noting that the coalitions implementation also produces Condorcet cycles, arising from the differential sensitivity of losers to the preferences of other losers versus those of winners. I omit the figure for the sake of brevity; visually it is similar to figures 4 and 5. Clearly, the differences in strengths of preferences between winners and losers produce regular cycles of reversal in the outcome of elections. These cycles appear to arise regularly for the sore-losers, good-winners and coalitions implementations: the important causal factor appears to be the relative differences in the strengths of preferences of winners and losers. These differences in strength of preferences in turn depend upon each voter's position as either a winner or loser in the previous election. Electoral outcomes thus indirectly give rise to Condorcet cycles, at least under the conditions of frequent elections.

[Figures 4 and 5 about here]

It may be, however, that during infrequent elections such cycles are less likely because each voter's knowledge of the current distribution of preferences erodes the longer the time between elections. For example, figure 6 illustrates a run of the good-winners algorithm with elections occurring every 50 steps in the model instead of every ten. Interestingly, the relatively infrequent election locks out one social choice (yellow). Though the collective choice changes regularly, the elimination of yellow as an option reduces the social choice problem to a simple two-choice situation. In effect, the longer period between elections reduces the Condorcet three-choice problem to a simple bipartite election.

[Figure 6 about here]

How does tactical voting affect results of Condorcet elections? The model suggests some intriguing findings. If elections are held frequently, it appears that tactical voting solves Condorcet problems by electing (and re-electing) a majority choice. As figure 7 illustrates, however, tactical voting tends to result in the re-election over time of a least preferred social choice. Blue wins every election in this particular run of the model. Surprisingly, however, after the seventh election (time = 70) society as a whole prefers both yellow and green to blue. This is a surprising finding. Tactical voting leads to an equilibrium social choice, but one that is not the Condorcet winner. It is important to note that a few other runs of the tactical-voting implementation produce an equilibrium social choice that is consistent with the Condorcet preference winner. Thus it is unclear to what degree the results presented in figure 7 represent a common occurrence. Even so, it is striking to note that tactical voting produces an equilibrium outcome that is at least a second-best option for the majority of voters. Just as anti-Tory tactical voting in the British general elections of 1997 and 2001 helped keep Labour in power, tactical voting in the model seems to support the incumbent social choice.

[Figure 7 about here]

Exploring the Parameter Space: Active Nonlinear Test

This simple agent-based model thus produces intransitive social preferences and cycles of indecision that characterize three-choice problems. It also illustrates that voters who vote tactically may prevent such intransitivities and produce equilibrium but suboptimal outcomes. I base these initial findings simply on a few configurations of the parameters in the model. Yet the model includes eight parameters—time between elections, voter sensitivity, good winners, winner sensitivity, sore losers, loser tolerance, coalitions, and tactical voting—with 288,000 possible combinations. Under which combinations of parameters do Condorcet cycles occur most frequently? Likewise, what set of parameters leads to the most tactical voting? To answer these questions, I use a genetic algorithm developed by Miller (1998) to explore the model's parameter space. Miller's active nonlinear test anneals a model's parameters according to a fitness criterion the modeler selects a priori. My implementation of the active nonlinear test starts with a list of 40 randomly generated parameter sets. Using these initial parameter sets the algorithm runs the model 40 separate times. At the end of the first generation, the algorithm undertakes a selection tournament from among the 40 parameter sets, selecting those that performed best according to a specific fitness criterion. The algorithm also incorporates a genetic crossover routine, in which parameter sets selected by the tournament selection procedure swap parameter values with sets not selected. This genetic crossover occurs with a probability that declines over generations (thus allowing the algorithm to converge on a specific parameter set). My implementation of the active nonlinear test conducts the selection tournament and genetic crossover for 40 generations, for a total of 1,600 runs of the model. Miller has shown that the active nonlinear test efficiently explores the parameter space of nonlinear models provided the fitness landscape has a single or a few fitness peaks. If not, the test may converge on suboptimal peaks in the fitness landscape, though the genetic crossover routine minimizes this risk.

I conduct two runs of the active nonlinear test on 1,000 voters using two fitness criteria. First, I test the voting model using a criterion for preference cycles: I operationalize “cycles” as the percentage of elections that reverses the social choice of the previous election. Because I constrain the algorithm to runs of only 20 elections, the maximum possible number of election reversals is 19 (since the first election by construction cannot reverse a previous choice). So the maximum possible fitness for this operational measure is 0.95, or 19 reversals out of 20. Second, I use a measure of tactical voting that averages the number of voters who vote tactically over the duration of each run. Because the sum of the differences between preferences and votes will always equal zero, I use the sum of squared differences between preferences and votes for all three social choices. I then take the square root of the sum of squared differences and divide by the number of steps in the model to find the average number of tactical voters per step. I use this measure of tactical voting as a fitness criterion in the active nonlinear test. Figure 8 illustrates how the active nonlinear test anneals the parameter set: in both cases, after about 10 generations the algorithm has converged upon parameter sets that maximize cycles in societal preferences (figure 8(a)) and tactical voting (8(b)).

[Figure 8 about here]

The results of the active nonlinear test suggest that Condorcet-style preference cycles emerge most frequently when voters are relatively sensitive to their neighbor's preferences. The test converges on a voter sensitivity of 0.27—about a one in four chance that voter will reorder its preferences to reflect the preferences of its neighbors). Though this is fairly frequent, it is nevertheless somewhat counterintuitive: one would expect that a higher level of voter sensitivity would lead to preference cycles since individuals would always be changing their minds. The explanation may be that greater voter sensitivity to local preferences leads the model to converge quickly on a social winner; in the middle ranges of voter sensitivity, by contrast, voters change their mind frequently enough to produce intransitive societal choices but not frequently enough to produce a consensus. Interestingly, the test illustrates that Condorcet cycles emerge most frequently in a world of both good winners and coalitions. The good winners in the model are four times as sensitive as losers. Highly sensitive good winners lead to a rapid erosion of support for the prevailing social choice, while coalitions allow losing voters to bootstrap their preferences into a winning choice. The test finds that neither sore losers nor tactical voters produce preference cycles. This finding about sore losers is somewhat surprising, since my preliminary investigations illustrated that the sore-losers implementation alone produces preference cycles. One possible explanation is that the active nonlinear test has converged on a suboptimal parameter set; more likely, however, is that the fitness landscape has multiple fit nodes. Other runs of the active nonlinear test may just as well converge on the sore-losers implementation as an optimal parameter. Figure 9 presents the final run of the active nonlinear test's exploration of preference cycles.

[Figure 9 about here]

Tactical voting is most prevalent when elections are frequent, voters are relatively insensitive, and political coalitions allow losers to bootstrap. Interestingly, the active nonlinear test finds that tactical voting produces an equilibrium social choice, but one that does not reflect overall societal preferences. Figure 10 presents a typical run of the model using the parameter set that the active nonlinear test found to produce the most tactical voting. As this run's history shows, tactical voting under these conditions produces a few reversals of social choice before settling on an equilibrium choice. Although social preferences converge around yellow, however, the vote converges on green. So voters who cast a ballot for a second-best option produce an irony. Their votes produce a stable equilibrium social choice, but one that is not the Condorcet winner among voter preferences. Tactical voting thus elects an outcome that no one prefers. The reason for this simple: although voters change their preferences in response to the distribution of preferences in their immediate neighborhood, they cast their vote based on imperfect information about global preferences. Tactical voting thus becomes a self-fulfilling prophecy. Elections lead voters to overestimate the strength of overall support for an undesirable outcome. These misperceptions lead to the election and re-election of the suboptimal choice, even though a majority of voters prefer an alternative. Ironically, this information dynamic appears to be analogous to Kuran's finding (1991) of "preference falsification" in totalitarian states, whereby citizens underestimate popular opposition to the authoritarian government. In the case of tactical voting, one might term this "voting falsification."

[Figure 10 about here]

Interestingly, this lock-in to suboptimal outcomes does not appear to hold when elections occur less frequently. I re-ran the model using the same parameters identified by the active nonlinear test, except that the model held elections ever 50 steps instead of every ten. As figure 11 shows, infrequent elections lead to the elimination of one of the social choices. The greater time between elections allows voters' preferences to converge based on local information, and for losers to bootstrap into winning coalitions (though at the expense of the elimination of one of the losing choices). Infrequent elections thus reduce the amount of information voters have about the global distribution of preferences. Instead, the Condorcet three-choice problem reduces to a simple two-choice problem.

[Figure 11 about here]

Conclusions

These findings suggest that formal treatments of Condorcet social choice problems err when they assume that voters are sincere: that is, voters' preference orders necessarily determine their vote. When one relaxes the assumption of sincerity and allows for tactical voting, three-choice elections exhibit surprising properties. Tactical voting appears to produce equilibrium social choices that avoid the intransitive preferences and cycling pluralities that typify elections in three-choice situations. In this respect, tactical voting appears to "solve" Condorcet problems in some normative sense, if cycling and intransitive preferences are democratically undesirable. In another respect, however, tactical voting creates new problems for three-choice elections. In my agent-based model, tactical voting produces winners that are neither the majority nor even the plurality choice among social preferences. This problem arises because of differences in how social preferences inform a voter's individual preferences on the one hand, and his or her vote on the other. When one treats the two choices as distinct, it becomes clear that a voter's imperfect knowledge of social preferences overall may produce election outcomes that are Pareto suboptimal. A plurality, majority or even consensus of voters may prefer some social choice that differs from the equilibrium election winner. Elections may produce equilibrium or Pareto optimal choices, but not both.

These are surprising but admittedly preliminary findings. To have more confidence in them, one could extend the model in several respects. First, given the possibility of multiple fitness nodes, repeated iterations of the active nonlinear test may determine the frequency with which the model converges on the parameter sets I have identified in this paper. It may be, as I noted above, that the sore-losers implementation produces cycling majorities just as frequently as the good-winners implementation. I thus require a reasonable sample of active nonlinear tests to get some sense of the fitness landscape of the model, both in terms of preference cycles and tactical voting. Second, the findings suggest that the frequency of elections play an important role in how voters form their preferences and cast their votes. In the case of preference cycles, infrequent elections lead voters to converge on a unanimous choice. Paradoxically, in the case of tactical voting, infrequent elections reduce tactical voting and tend to lock out one social choice, thus reducing the three-choice problem to a two-choice one. The model needs to explore in greater detail the impact of the frequency of elections on cycles and tactical voting. Third, one interesting extension of the model is to vary the election rules. Currently the model uses a simple plurality rule, though this need not be so. One can easily extend the model to require strict majority election rules, "runoff" elections, and single transferable voting rules.

As I write this, there remain only five days until the British general election on May 5, 2005. Does the agent-based model of tactical voting provide any hints about the possible outcome of the election? Will the Tories gain from anti-Labour voting and "tactical unwind"? Will the

Liberals prosper at the expense of Labour? Though the agent-based model is not specifically a model of British general elections, it nevertheless suggests that prognostications of Labour's losses overstate the actual decline in the number of Parliamentary seats—and hence overstate Liberal and Tory gains. Tactical voting may be a self-fulfilling prophecy: the model illustrates how it tends to strengthen incumbents rather than weaken them. If Labour enjoys a surprisingly strong election on May 5th, it may reflect the fact that voters are not necessarily sincere. This is not to denigrate Labour; it merely suggests that voters' knowledge of overall social preferences is imperfect, and that tactical voting consequently imperfectly matches preferences to electoral outcomes. This may be good news for Tony Blair.

Tables and Figures

Table 1: The intransitivity problem and preference cycles

	Order of preferences				
Voter	*1*		*2*		*3*
A	Blue	>	Green	>	Yellow
B	Yellow	>	Blue	>	Green
C	Green	>	Yellow	>	Blue

Table 2: Preference formation and voting rules

Rule	*Algorithm*
Baseline	Poll all neighbors; re-order preferences with probability *p*. Vote is top preference.
Good winners	Poll all neighbors; re-order preferences with probability *p* if loser, or probability *g*p* if winner (with *g* set by the modeler). Vote is top preference.
Sore losers	Poll all neighbors: re-order preferences with probability zero if loser, or probability *p* if winner. Vote is top preference.
Coalitions	If "winner," poll all neighbors: re-order preferences with probability *p*. If "loser," poll only neighbors who also are losers: re-order preferences with probability *p*. Vote is top preference.
Tactical voting	Re-order preferences according to one or more of the rules above. Vote is second preference.

Figure 3: Typical baseline run to convergence

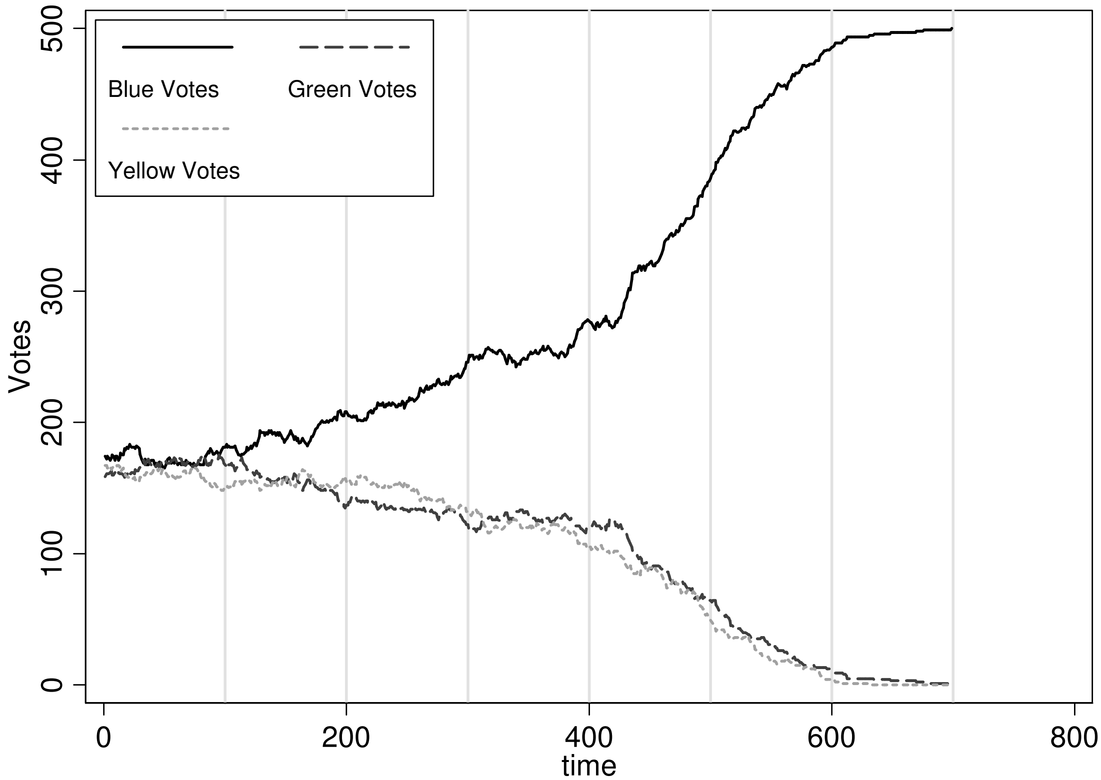


Figure 4: Condorcet cycles arising from good-winners implementation

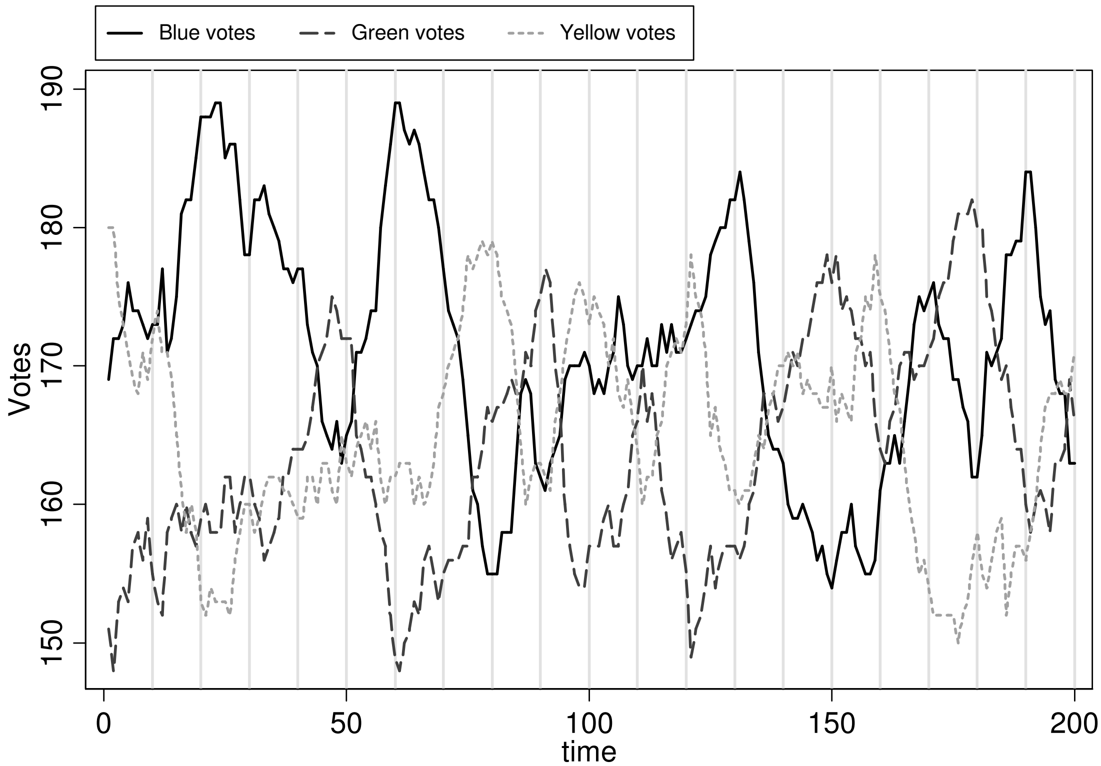


Figure 5: Condorcet cycles emerging from sore-losers implementation

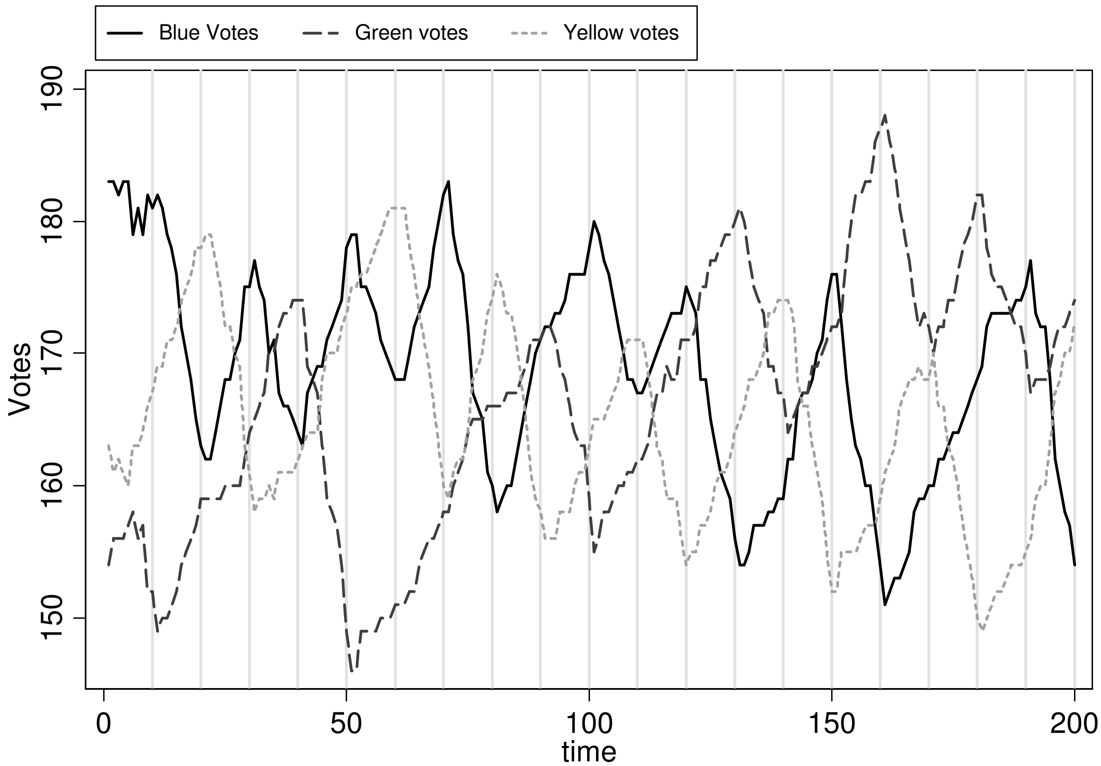


Figure 6: Good-winners implementation, infrequent elections

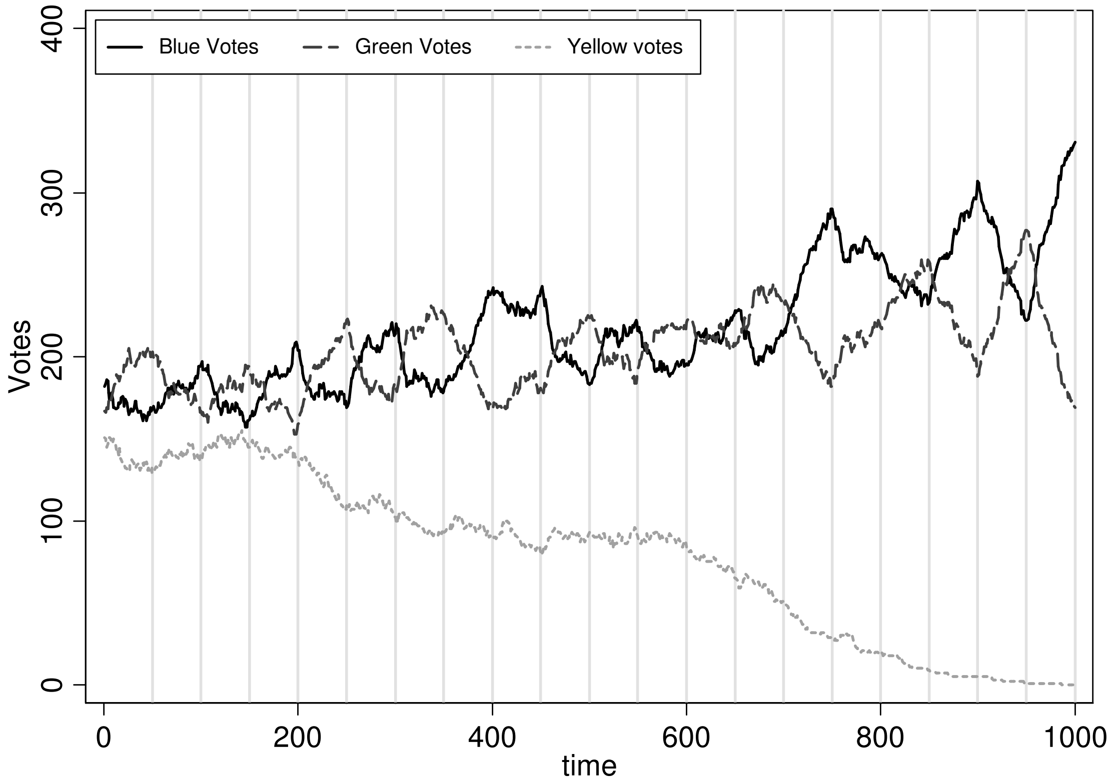


Figure 7: Tactical voting in frequent elections

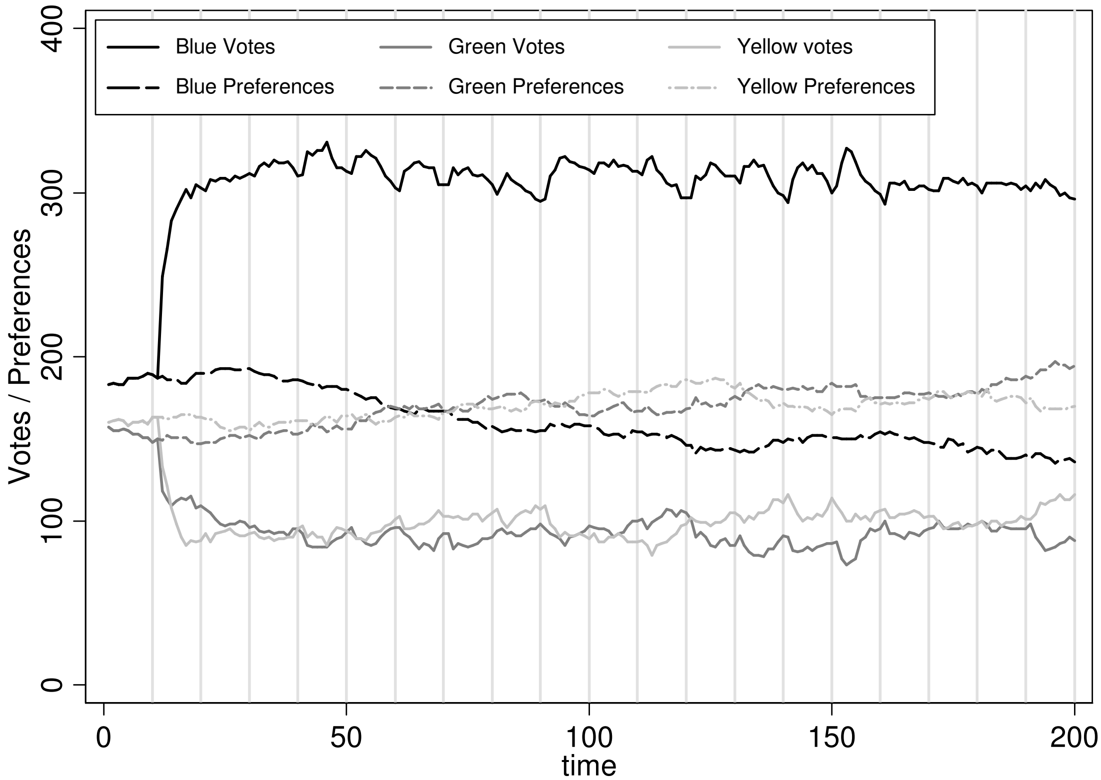


Figure 8: The active nonlinear test's annealing of the model's parameters for (a) maximum preference cycles; and (b) maximum tactical voting

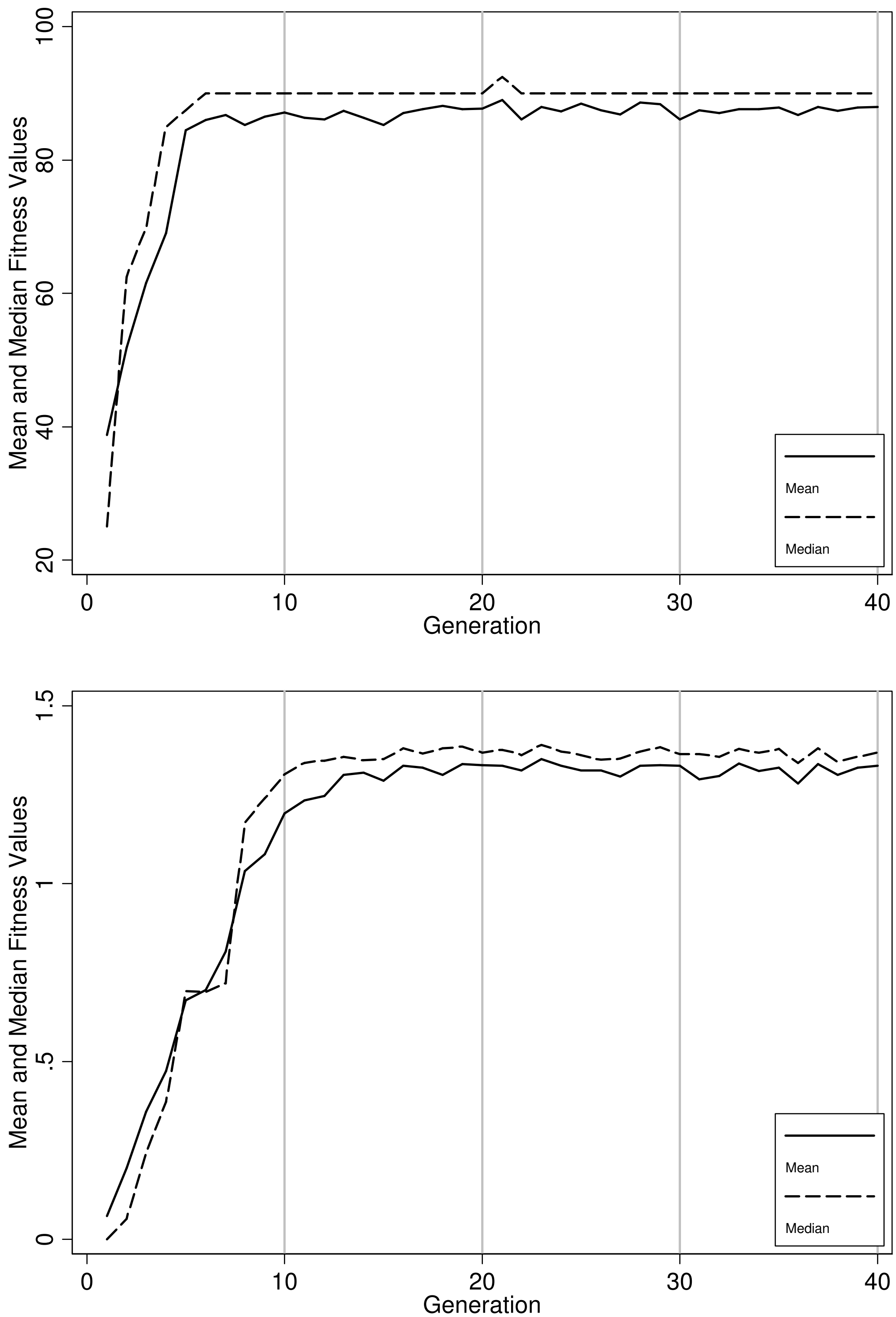


Figure 9: Final run of the active nonlinear test's simulation of preference cycles

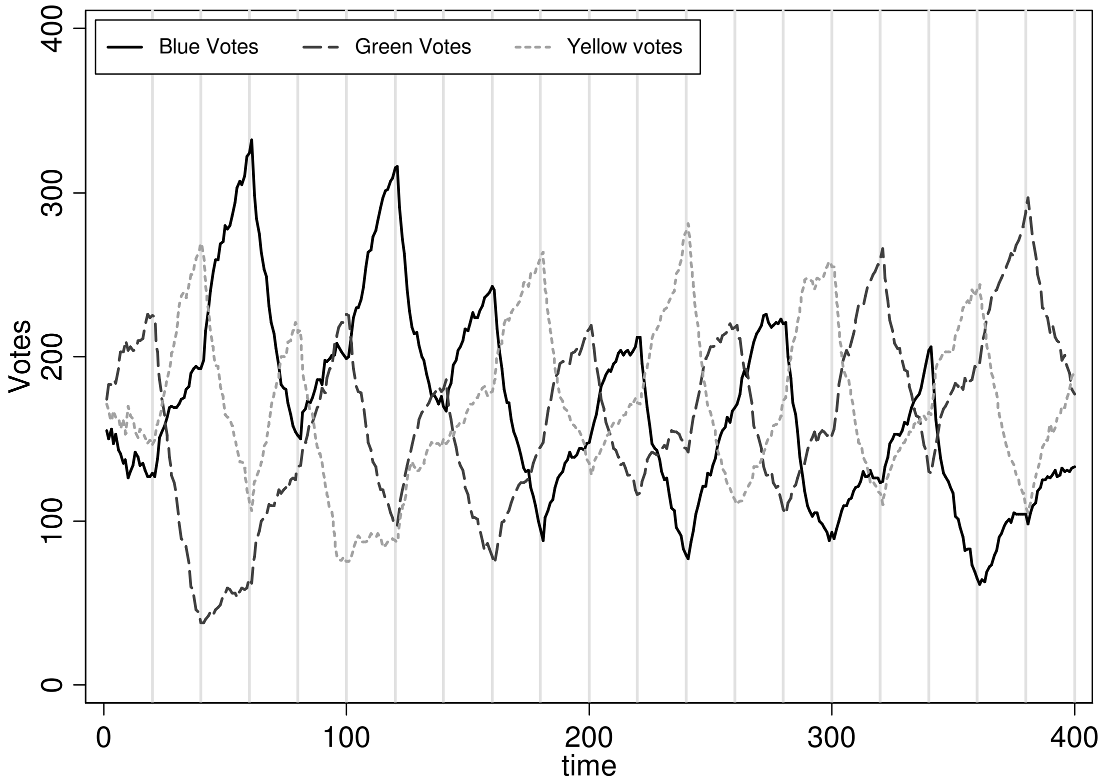


Figure 10: Sample run of the active nonlinear test's fit parameters for tactical voting

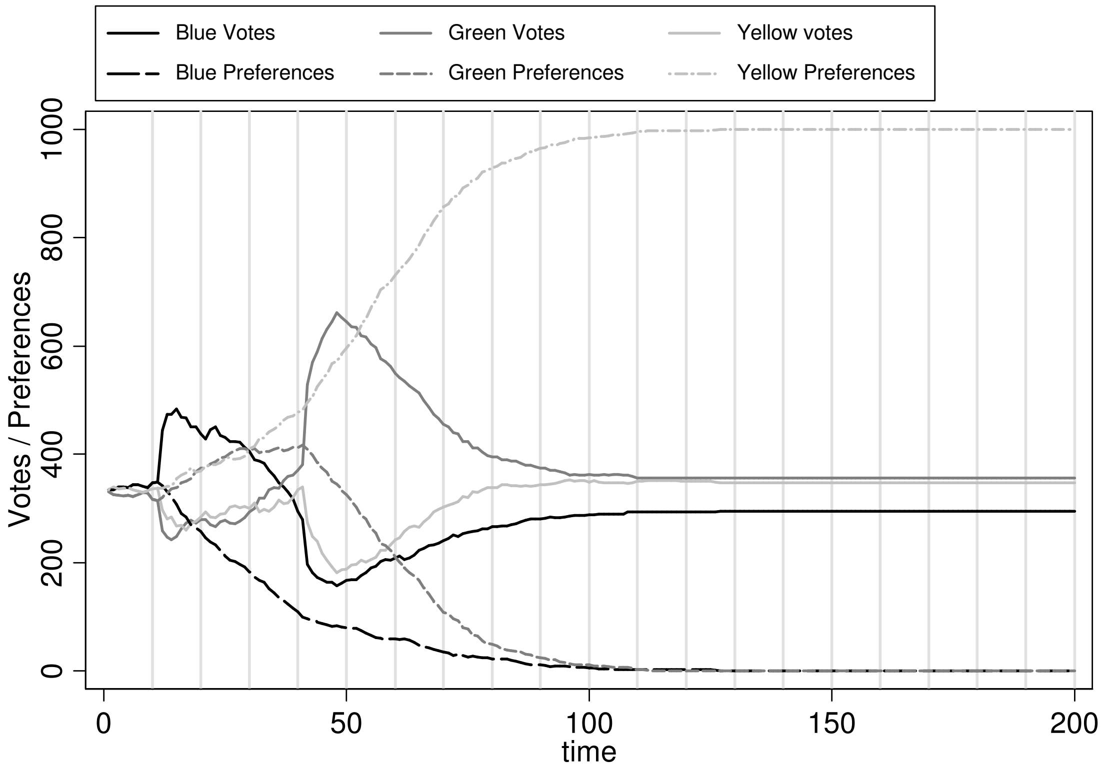
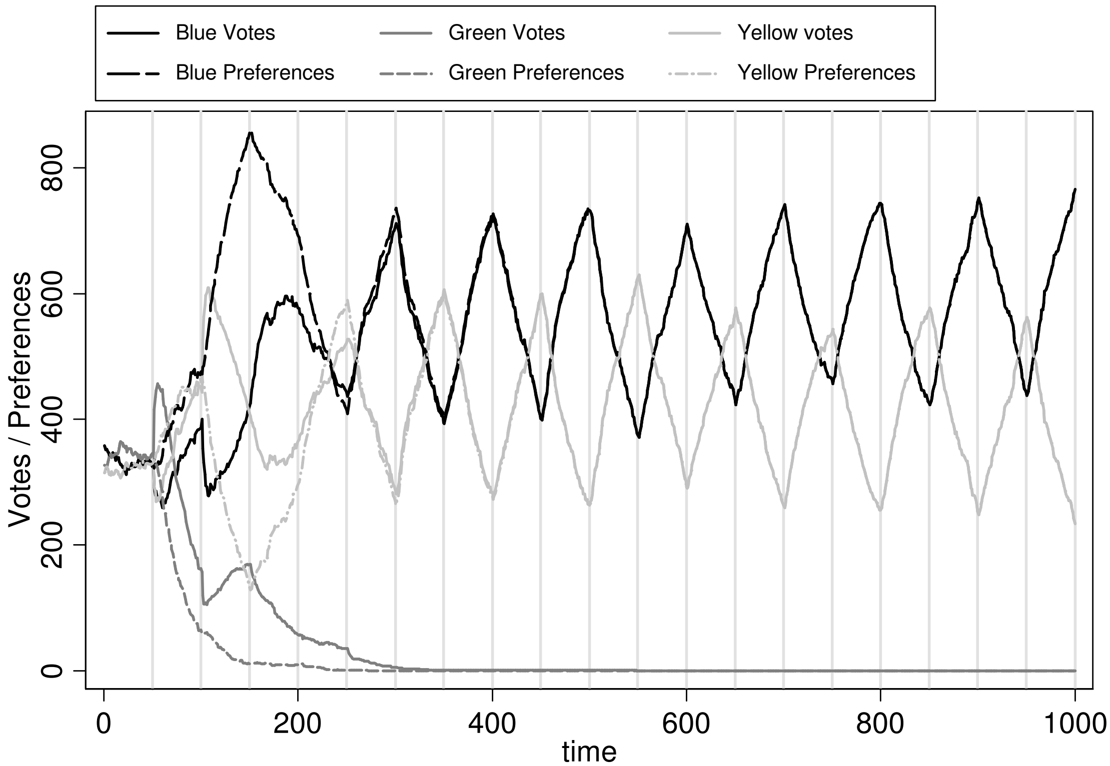


Figure 11: Tactical voting in infrequent elections



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