Projects

Structure of Project Reports:

1 Introduction. Briefly summarize the nature of the physical system.
2 Theory. Describe equations selected for the project. Discuss relevance and limitations of the equations.
3 Method. Describe briefly the algorithm and how it is implemented in the program.
4 Verification of a program. Confirm that your program is not incorrect by considering special cases and by giving at least one comparison to a hand calculation or known result.
5 Results. Show the results in graphical or tabular form. Additional runs can be included in an appendix. Discuss results.
6 Analysis. Summarize your results and explain them in simple physical terms whenever possible.
7 Critique. Summarize the important concepts for which you gained a better understanding and discuss the numerical or computer techniques you learned. Make specific comments on the assignment and your suggestions for improvements or alternatives.
8 Appendix. Give a typical listing of your program. The program should include your name and date, and be self-explanatory (comments, structure) as possible.

The report should be as concise as possible.

Project 1: A simple projectile motion (no air resistance) (Due on Friday, February 12, 2010 by 13:30)

A British navy ship is about 1 nautical mile from a fort defending Tortuga. (In 17th century Tortuga was one of the largest pirate strongholds). The fortress is located 50.0 meters above the sea level, with the walls as high as 60.0 meters. There is an armory house located 100.0 meters beyond the fort walls.

The captain of the navy ship knows that a direct hit of the armory house (stuffed with barrels of rum) may force pirates to surrender. The ship can fire cannons at the muzzle speed of v=200 m/s. Let's suppose that the captain can disregard air resistance in the problem (We will consider the effect of the air resistance later).

1 At what angle from the horizontal must the cannons be fired to hit the armory? (Use one of methods for solving non-linear equations)
2 Is it important to take into account that the ship is actually moving toward the fort with the speed about 2 knots? (1 knot = 1 nautical mile/hour = 1.852 km/h). The armory's dimensions are 8.0m*8.0m*3.2m.
3 What would you do if you were the captain of the navy ship?
4 Bonus: Evaluate the importance of the effect of air resistance (Back of the Envelope Physics).

Equations:

In the simplest case (with no air resistance) the 2D motion of a projectile is describe by a system of equations

$$x_f = x_i + (v_0 \cos\theta + v_{ship})t$$

$$y_f = y_i + (v_0 \sin\theta)t - \frac{gt^2}{2}$$

Eliminating the time from the equations gives

$$y_i - y_f + v_0 \sin\theta \frac{x_f - x_i}{v_0 \cos\theta + v_{ship}} - \frac{g}{2}\left(\frac{x_f - x_i}{v_0 \cos\theta + v_{ship}}\right)^2 = 0$$

Solving this non-linear equation for the angle θ would give the right shooting angle to hit a target.

In the case $y_i = y_f$ and $v_{ship} = 0$ a very simple analytic solution can be found in most textbook

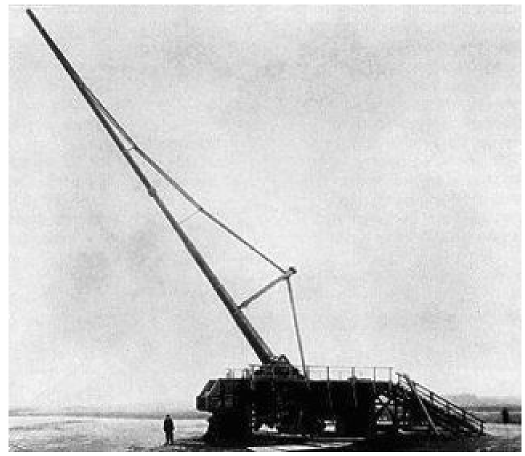
$$\theta = \frac{1}{2}\arcsin\left(\frac{g(x_f - x_i)}{v_0^2}\right)$$

This simple case can be used to test numerical solutions when $y_i = y_f$ and $v_{ship} = 0$.

Project 2: Projectile motion with air resistance (Due on Wednesday, March 5, 2010 by 21:00)

Write a program that simulates the projectile motion in the (x,y) plane with allowing for air resistance, varying air density and wind. The amplitude of air resistance force on an object moving with speed v can be approximated by $F_{drag}=-0.5C\rho_0Av^2$, where ρ_0 stands for air density (ρ_0=1.25 kg/m^3 at sea level), and A is the cross section. The drag coefficient C depends on an object shape and for many objects it can be approximated by a value within 0.05 - 0.5. Use Runge-Kutta method as a primary method for solving a system of differential equations.

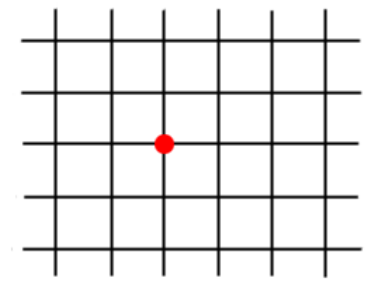
Application: Study the trajectory of shells of one of the largest cannons "Pariskanone" used during the First World War.



1. Determine the angle (between 0 and 90 degrees) that gives the maximum range for "Pariskanone". For this angle calculate time of flight and max altitude of the shells.
2. For the angle calculated in the first part, study the effect of air density, and variable air density on the trajectory (thus you run three calculations: no air resistance, air resistance with constant air density, variable air density).
3. Discuss the accuracy of "Pariskanone", i.e. how much the following effects would affect the accuracy: variations in the air density (day/night temperature, rain), wind, and initial speed.
4. Extra credit. Utilize an adaptive step-size control. Use either the doubling technique with 4^{th} order Runge-Kutta, or Fehlberg's 5^{th} order Runge-Kutta with error estimation.

Reference information: The shell mass - 94 kg., initial speed - 1600m/s, caliber - 210 mm, and the C coefficient is about 0.12. Approximate the density of the atmosphere as $\rho = \rho_0$*exp(-y/y_0), where y is the current altitude, y_0 = 1.0*10^4m, and ρ_0 is air density at sea level (y=0).

Project 3: Random walks in two dimensions (Due on Monday, April 5, 2010 by 13:30)



Part 1: Diffusion (simple random walk). Write a program that simulates a random 2D walk with the same step size. Four directions are possible (N, E, S, W). Your program will involve two integers, K is the number of random walks to be taken and N is the maximum number of steps in a single walk. Run your program with at least K >= 1000. Find the average distance R to be from the origin point after N steps. Plot the mean distance travelled R versus the number of taken steps. Assume that R has the asymptotic dependence as $R \sim N^{\alpha}$, and estimate the exponent α.

Extra credit (1 point): consider a simple random walk in 1D (two directions of motion) and 3D (three dimensions, six directions of motion). What will be the exponent α for 1D and 3D. Compare your results with the 2D case.

Part 2: Random walk on a 2D crystal. Consider a two dimension lattice of size L*L. Randomly place a "random walker" on the lattice and start walking (only four directions are possible: left, right, up, down). As soon as the “random walker” reaches a site outside the L*L area the random walk stops. Find the average number of steps S to get out of the crystal. Is there a connection between S and L?

Part 3: Random walk on a 2D lattice with traps. Consider the same two dimension lattice of size L*L. Now the lattice contains a trap. (An analogy would be a city with (L-1)*(L-1) blocks and a police patrol).

Randomly place a "random walker" on the lattice. When the walker arrives to the trap site, it can no longer move. So, the random walk stops when the walker either trapped or out of the L*L area. Find probabilities to get the walker trapped and to go free. Find also the mean number of steps (“survival time”) before a trap site is reached, or the walker is out of the area, as a function of L.

Explore following scenarios:

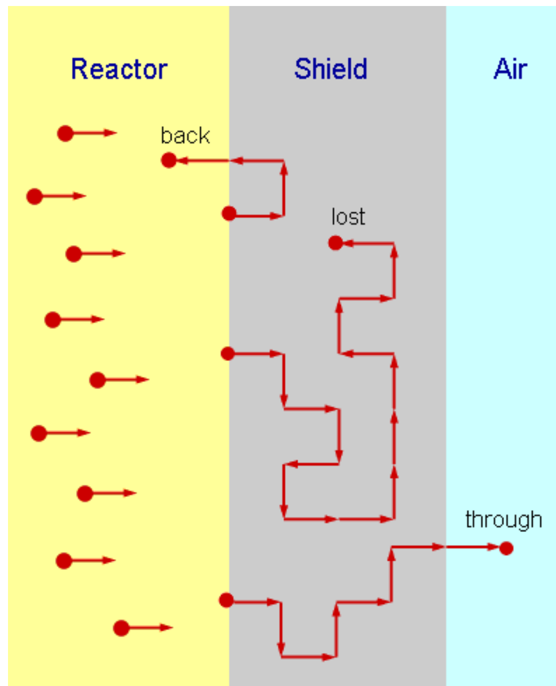
1. A stationary trap located at the center of the area, i.e. with the coordinates (L/2,L/2)
2. A randomly placed stationary trap
3. A randomly moving trap – the trap “walks” randomly with the same speed (one block in a time). Since the trap can not leave the L*L area, when needed, use either the restrictive random walk for the patrol, or the periodic boundary conditions.
4. Extra credit (2 points). There are two randomly moving traps. How will it affect the probability to capture the “random walker”.
5. Extra credit (2 points). A persistent single trap – the trap moves along a closed path (a box around the center with a side S<L – like a moving police patrol). Does the outcome depend on the S/L ratio?

Project 4: Shielding a nuclear reactor (Due on Wednesday, April 14, 2010 by 13:30)

During the World War II scientists in Los Alamos (Manhattan project) had to find how far neutrons would travel in different materials. Results were important for the calculation of critical masses as well as shielding. The physicists knew most of the basic data and their dependences on the neutron energy, namely, the average distances between collisions of a neutron with an atomic nucleus, the probabilities of neutron elastic or inelastic scattering, probability of capture by an atomic nucleus, the energy loss of the neutrons after each collision. However, it was not clear how to use all this information to find a solution. Ulam and von Neumann solved the problem by a novel numerical approach i.e. simulating a path of a neutron using random numbers.

This project below is a very simplified version of the original problem solved by Ulam and von Neumann.

A beam of neutrons bombards a reactor's wall. Considering motion of neutrons as a random walk on (x,y) plane find probabilities for neutrons (as a function of the shield size) a) to be back in the reactor, b) to be captured in the shield, c) to get through the shield.



Conditions:

1 only four directions of motion are possible (left, right, up or down)
2 on the next step the neutron can not step back, but only forward, left or right, and
3 the probability to go forward is two times more than changing a direction
4 on each step the neutron looses one unit of energy
5 initial neutron energy is enough for 100 steps
6 initial neutron velocity is perpendicular to the shield
7 a capture probability on every step is 0.01

Assume that the probability P to get through the shield has the asymptotic exponential dependence as $P \sim e^{-ax}$ with x is the size of the shield. Estimate the exponent a.

Optional (bonus points): Consider the initial neutron energy as a normal distribution with a mean value of 100 steps, and a standard deviation of 20 steps Note: the shield's size is measured in "steps", where one step corresponds to an average distance that neutrons move between collisions.