KINEMATICS

1D motion

$$\begin{cases} v = v_0 + at \\ x = x_0 + v_0 t + \frac{1}{2}at^2 \end{cases}$$

$v^2 = v_0^2 + 2a(x - x_0)$

(for free fall $a = -g$)

2D projectile motion

$$\begin{cases} v_x = v_{x0} \\ x = x_0 + v_{x0}t \\ v_y = v_{y0} - g_y t \\ y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \end{cases}$$

DYNAMICS

Second Newton's Law $\vec{F}_{net} = m\vec{a}$

for 1D/2D motion: $F_{net,x} = ma_x$, $F_{net,y} = ma_y$

for uniform circular motion: $F_{net,r} = mv^2/r$

Forces:

Gravitational force $\vec{F}_g = m\vec{g}$

Normal force $N = mg\cos(\theta) + F_{external}$

Tension T

Frictional force $f_s = \mu_s N$ and $f_k = \mu_k N$

Spring force $\vec{F}_e = -k\vec{x}$

Draw a free-body diagram and include ALL forces acting on the body matter

ENERGY

Kinetic energy $K = \frac{1}{2}mv^2$

Change in kinetic energy $\Delta K = K_f - K_i = W = F \cdot d\cos\vartheta$

Gravitational potential energy $U(y) = mgy$

Elastic potential energy $U(x) = \frac{1}{2}kx^2$

Conservation of Mechanical Energy

$K_i + U_i = K_f + U_f$

Conservation of Energy (friction involved)

$K_i + U_i = K_f + U_f + f_k(x_f - x_i)$

Power: $P = \frac{dW}{dt} = \frac{\Delta W}{\Delta t} = Fv$

ROTATIONAL KINEMATICS, DYNAMICS & ENERGY

rotational and translational motion: $\theta = \frac{s}{r}$ $\omega = \frac{v}{r}$ $\alpha = \frac{a}{r}$

period $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$, frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$

$$\begin{cases} \omega = \omega_0 + \alpha t \\ \theta = \theta_0 + \omega_0 t + \alpha t^2/2 \end{cases}$$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

torque $\tau = rF\sin(\phi)$

Second Newton's law $\tau_{net} = I\alpha$ (I is rotational inertia)

Kinetic energy of rotation $K = \frac{1}{2}I\omega^2$

Kinetic energy of rolling $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

Work and power: $W = \tau(\theta_2 - \theta_1)$ $P = \tau\omega$

MOMENT AND IMPULSE

Linear momentum $\vec{p} = m\vec{v}$,

Newton's 2nd Law $\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$

Impulse: $\vec{J} = \vec{p}_f - \vec{p}_i = \vec{F}_{ave}\Delta t$

Conservation of linear momentum: $\vec{F}_{net} = 0$ then $\vec{p}_i = \vec{p}_f$

Elastic collision: $\vec{p}_i = \vec{p}_f$ and $E_i = E_f$

Inelastic collision: $\vec{p}_i = \vec{p}_f$ and $E_i \neq E_f$

Angular momentum: particle $\vec{L} = \vec{r} \times \vec{p}$, rigid body $\vec{L} = I\vec{\omega}$

Newton's 2nd Law $\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$

Conservation of angular momentum: for $\tau_{net} = 0$ $\vec{L} = const$

GRAVITATION

Newton's Law of gravitation $F = G\frac{m_1 m_2}{r^2}$

The free-fall acceleration $g = \frac{GM}{R^2}$

for $M = \rho V = \rho \frac{4\pi}{3} R^3$ then $g = \frac{4\pi}{3} G \rho R$

Gravitational potential energy $U = -\frac{GMm}{R}$

Escape speed: $v_i = \sqrt{\frac{2GM}{R}}$

Orbital motion: $v_c = \sqrt{\frac{GM}{R+h}}$ and $T = \frac{2\pi}{\sqrt{GM}}(R+h)^{3/2}$

SYSTEM OF PARTICLES

Center of mass $\vec{r}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$

Second Newton's Law for a system of particles

$\vec{F}_{net} = M \vec{a}_{CM}$

for $\vec{F}_{net} = 0$ $\quad \vec{v}_{CM} = 0, \quad \vec{r}_{CM} = const$

FLUIDS

Density $\rho = \frac{m}{V}$

Pressure $p = \frac{F}{A}$

Pressure at depth h: $p = p_0 + \rho g h$

Pascal's principle: $F_0 = F_i \frac{A_0}{A_i}$

Archimedes' principle of buoyancy $F_b = m_f g = \rho_f V g$

EQULIBTRIUM

Two Conditions for Equilibrium: $\vec{F}_{net} = 0$ and $\vec{\tau}_{net} = 0$

in physics 231 for most problems $\begin{cases} F_{net,x} = 0 \\ F_{net,y} = 0 \\ \tau_{net,z} = 0 \end{cases}$

Choose ONE object in a time for consideration

Draw a free-body diagram

(show ALL forces acting ON that object)

Choose (wisely) a coordinate system and resolve forces in their components. "Generate" equilibrium equations using the conditions for equilibrium

PERIODIC MOTION

$x(t) = x_m \cos(\omega t + \phi)$

$v(t) = -\omega x_m \sin(\omega t + \phi)$

$a(t) = -\omega^2 x(t)$

$\omega = \frac{2\pi}{T} = 2\pi f$

Restoration Force

$F = ma = -cx$

$\omega = \sqrt{\frac{c}{m}} \quad T = 2\pi\sqrt{\frac{m}{c}}$

Energy

$E = \frac{1}{2} m v_x^2 + \frac{1}{2} c x^2$

Spring

$F = -kx \quad T = 2\pi\sqrt{\frac{m}{k}}$

Pendilum

$F = -\frac{mg}{L} x \quad T = 2\pi\sqrt{\frac{L}{g}}$