Old Dominion University
Mathematics-Statistics Department
Fall 2011 Semester
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Stat 630 Homework 1

Due date: September 10^{th}

Explain your answers.

1. Suppose X and Y are random variables with joint probability density function of the form $f(x,y) = 3x$, for $0 \leq y \leq x \leq 1$, and zero elsewhere.

   (a) Sketch the support set of X and Y.

   (b) Compute $P(0 \leq X \leq 0.5 \text{ and } Y \geq 0.25)$

   (c) Compute the marginal densities of X and Y.

2. Suppose X and Y are random variables with joint probability density function of the form $f(x,y) = x + y$, for $0 \leq x \leq 1$, and $0 \leq y \leq 1$ and zero elsewhere.

   (a) Find the marginal distribution of X and Y.

   (b) Compute $E(X), E(Y), Var(X)$ and $Var(Y)$.

   (c) Compute $Cov(X,Y)$ and ρ.

   (d) Compute $E\left[(2X-Y)^2\right]$.

3. Suppose X and Y are random variables with joint probability density function of the form $f(x,y) = Kxe^{-(x+y)/2}$, for $x \geq 0$, and $y \geq 0$ and zero elsewhere.

   (a) What is the value of K?

   (b) Compute $Cov(X,Y.)$

   (c) Find $E[h(X,Y)]$ where $h(X,Y) = xy^2$.

4. Suppose X and Y random variables that are discrete and can only take three values: $(-1,0),(0,1),$ and $(1,0)$ with probability $1/3$ each.

   (a) Find the marginal distribution of X and Y.

   (b) Compute $E(X), E(Y), Var(X)$ and $Var(Y)$.

   (c) Compute ρ.

5. Suppose X and Y are discrete random variables with $P(X = x \text{ and } Y = y) = \frac{x+2y}{18}$ for $x = 1, 2$ and $y = 1, 2$.

   (a) Find the marginal distribution of X and Y.

   (b) Find $E(X), E(Y), Var(X)$ and $Var(Y)$.

   (c) Compute ρ.

6. Consider the model $X_t = 0.01 + 0.2X_{t-2} + Z_t$ where Z_t is a Gaussian white noise with mean 0 and variance 0.02. Find the mean and variance of X_t. Compute the lags 1 and 2 autocorrelations of X_t.

Let me know if you have any questions.