Practice Exercise: Model II 1-factor ANOVA

Question:

The lengths (um) of tick (*Haemaphysalis leporispalustris*) larvae were recorded in samples from four different rabbits. Do tick larvae from different hosts vary significantly in mean size? Do tick larvae lengths vary more among hosts or among individuals from each host? Analyze these data by hand and using SPSS and draw conclusions from those results.

Data:

Host	1	2	3	4
	380	350	354	376
	376	356	360	344
	360	358	362	342
	368	376	352	372
	372	338	366	374
	366	342	372	360
	374	366	362	
	382	350	344	
		344	342	
		364	358	
			351	
			348	
			348	

Hand Calculations:

Host	1	2	3	4
	380	350	354	376
	376	356	360	344
	360	358	362	342
	368	376	352	372
	372	338	366	374
	366	342	372	360
	374	366	362	
	382	350	344	
		344	342	
		364	358	
			351	
			348	
			348	
n_i	8	10	13	6
$\sum X_{ij}$	2978	3544	4619	2168
$\sum X_{ij}^2$	1108940	1257272	1642121	784536

Step 2: $\sum n_i = 37$

$$\sum\sum X_{ij} = 13309$$

$$\sum\sum X_{ij}^2 = 4792869$$

Step 3: $\sum \frac{\left(\sum X_{ij}\right)^2}{n_i} = \frac{2978^2}{8} + \frac{3544^2}{10} + \frac{4619^2}{13} + \frac{2168^2}{6} = 4789090.997$

Step 4: $\text{CT} = \frac{\left(\sum\sum X_{ij}\right)^2}{\sum n_i} = \frac{(13309)^2}{37} = 4787283.2703$

Step 5: $SS_T = \sum\sum X_{ij}^2 - \text{CT} = 4792869 - 4787283.2703 = 5585.7297$

Step 6: $SS_A = \text{Step 3} - \text{CT} = 4789090.997 - 4787283.2703 = 1807.7272$

Step 7: $SS_E = SS_T - SS_A = 5585.7297 - 1807.7272 = 3778.0026$

Step 8: $MS_A = \frac{SS_A}{a-1} = \frac{1807.7272}{4-1} = 602.5757$ $\quad MS_E = \frac{SS_E}{\left(\sum n_i\right) - a} = \frac{3778.0026}{37-4} = 114.4849$

Step 9: $F_{calc} = \frac{MS_A}{MS_E} = \frac{602.5757}{114.4849} = 5.263$

Step 10: $F_{0.05(1),3,33}$ = not in table, so be conservative and use the next lower value that is associated with a smaller sample size, in this case n = 30 rather than n = 33, so: $F_{0.05(1),3,30} = 2.92$

Use the table to get an approximate P value: the calculated F-value of 5.263 is between 0.005 and 0.0025. Use the more conservative value (the larger value in this case), so $P < 0.005$.

Results using SPSS:

Tests of Between-Subjects Effects
Dependent Variable: WIDTH

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	4445442.977	1	4445442.977	7704.906	.000
	Error	1767.487	3.063	576.963		
HOST	Hypothesis	1807.727	3	602.576	5.263	.004
	Error	3778.003	33	114.485		

a Computed using alpha = .05
b .948 MS(HOST) + 5.248E-02 MS(Error)
c MS(Error)

Variance Components

$$n_0 = \frac{\sum n_i - \frac{\sum n_i^2}{\sum n_i}}{a-1} = \frac{37 - \frac{369}{37}}{4-1} = 9.009$$

$$S_A^2 = \left| \frac{MS_A - MS_E}{n_0} \right| = \left| \frac{602.5757 - 114.4849}{9.009} \right| = 54.1781$$

$$S_E^2 = MS_E = 114.4849$$

From SPSS:

Variance Estimates

Component	Estimate
Var(HOST)	54.178
Var(Error)	114.485

Dependent Variable: WIDTH Method: ANOVA (Type I Sum of Squares)

Calculate % Variance Explained:

$$\%\text{ Variance explained by host (among groups)} = \frac{S_A^2}{S_E^2 + S_A^2} = \frac{54.1781}{114.4849 + 54.1781} \times 100\% = 32\%$$

$$\%\text{ Variance explained by error (within groups)} = \frac{S_E^2}{S_E^2 + S_A^2} = \frac{114.4849}{114.4849 + 54.1781} \times 100\% = 68\%$$

How to present homework:
Factor: Individual Rabbit Host, four levels
Dependent Variable: Length of tick larva
Replicates: 6 – 13 tick larvae
Analysis: 1-Factor, Model II ANOVA followed by calculation of variance components.

Assumptions of the test: All assumptions are assumed to be met.

ANOVA Table

Source	df	SS	MS	F	P	% Variance
Host	3	1807.727	602.576	5.263	.004	32%
Error	33	3778.003	114.485			68%
Total	36	5585.730				100%

Conclusions:
Mean lengths of tick larvae differ significantly among hosts at the 0.05 level ($F=5.263$, $P < 0.005$). However, differences among hosts account for less of the total variation in size than do differences attributable to other causes (32% vs. 68%, respectively).