# One-Factor ANOVA Example Calculations

**Examples of Model I and Model II Study Questions:**

Model I:

A. Are the mean numbers of zooplankton ml$^{-1}$ different in the Elizabeth, James, and York Rivers?
B. Are there differences in the amount of cell damage resulting from the five most common cryopreservation techniques?

Model II:

A. Are the mean numbers of zooplankton ml$^{-1}$ different in rivers in Virginia? Since we cannot sample from every river in Virginia, test this by sampling from three randomly chosen rivers.
B. Is the level of cell damage caused by a particular cryopreservation technique more different among tissue samples or within a single sample?

**Example Calculations:**

Note that a single datum (like the first value in group 1, 36) is denoted $X_{ij}$ where $i$ is the group and $j$ is the observation number within the group. The number of observations within a group is $n$ and the number of groups is $k$.

| Treatment Group | | |
|---|---|---|
| 1 | 2 | 3 |
| 36 | 46 | 40 |
| 39 | 47 | 50 |
| 43 | 47 | 44 |
| 38 | 47 | 48 |
| 37 | 43 | 50 |

**Step 1:**

| | | | |
|---|---|---|---|
| $\sum X_{ij}$ | 193 | 230 | 232 |
| $n_i$ | 5 | 5 | 5 |
| $\sum X_{ij}^2$ | 7479 | 10592 | 10840 |

**Step 2:**

| | |
|---|---|
| $\sum\sum X_{ij}$ | 655 |
| $\sum n_i$ | 15 |
| $\sum\sum X_{ij}^2$ | 28911 |

**Step 3:**

$$\sum \frac{\left(\sum X_{ij}\right)^2}{n_i} = \frac{(193)^2}{5} + \frac{(230)^2}{5} + \frac{(232)^2}{5} = 28795$$

**Step 4:** Correction Term (CT)

$$\text{CT} = \frac{\left(\sum\sum X_{ij}\right)^2}{\sum n_i} = \frac{(655)^2}{15} = 28602$$

**Step 5:** Total Sum of Squares ($SS_T$)

$$SS_T = \left(\sum\sum\left(X_{ij}^2\right)\right) - CT = 28911 - 28602 = 309$$

**Step 6:** Among-Groups Sum of Squares ($SS_A$)

$$SS_A = \text{Step 3} - CT = \sum \frac{\left(\sum X_{ij}\right)^2}{n_i} - CT = 28795 - 28602 = 193$$

**Step 7:** Within-Groups (Error) Sum of Squares ($SS_E$)

$$SS_E = SS_T - SS_A = 309 - 193 = 116$$

**Step 8:** Mean Square Calculations

Among Groups Mean Square: $MS_A = \frac{SS_A}{k - 1} = \frac{193}{2} = 96.5$

Within Groups Mean Square: $MS_E = \frac{SS_E}{\left(\sum n_i\right) - k} = \frac{116}{12} = 9.7$

**Step 9:** Calculate the F-statistic

$$F_{calc} = \frac{MS_A}{MS_E} = \frac{96.5}{9.7} = 9.95$$

**Step 10:** Compare $F_{calc}$ to the critical value from the F-table (B.4):

numerator degrees of freedom: $v_1 = k - 1 = 2$

denominator degrees of freedom: $v_2 = \sum n_{ij} - k = 15 - 3 = 12$

Using 0.05 significance level,

$F_{\alpha(1), v1, v2} = F_{0.05, 2, 12} = 3.89$

You can also get an approximate P value from the table by reading across the row to find the α interval in which the calculated F falls. In this case, it is between α = 0.005 and 0.001

**Step 11:** State your conclusions

Since 9.95 > 3.89, we reject $H_0$ at the 95% confidence level (or the 0.05 significance level) and conclude that:

Model I: There is a significant difference among the treatment (group) means.

OR

Model II: There is significant variation among the groups.

**Step 12:** Present your results in an ANOVA table

| Source | df | SS | MS | F | P |
|---|---|---|---|---|---|
| Among Groups | 2 | 193 | 96.5 | 9.95 | < 0.005 |
| Error | 12 | 116 | 9.7 | | |
| Total | 14 | 309 | | | |

**Step 13:** Follow-up calculations

For a significant F test:

Model I: follow-up with a multiple comparison procedure to identify which groups were different from the others.

Model II: follow-up with calculation of variance components to identify how much of the variation is due to the treatment.

For a non-significant F test, you may want to follow-up with a post-hoc power calculation based on the minimum biologically-significant difference.