I pledge to support the Honor system of Old Dominion University. I will refrain from any form of academic dishonesty or deception, such as cheating or plagiarism. I am aware that as a member of the academic community, it is my responsibility to turn in all suspected violators of the Honor Code. I will report to a hearing if summoned.

Student Signature: ____________________________________________

Student Name (BLOCK CAPITALS): ______________________________

UIN Number: __________________________________________________

Please turn in this examination paper, with the pledge above signed with your answer books.

1. This examination contains 26 problems from the following six areas:
   
   A. MATH (only two from this group)  
      A1 A2 A3 A4
   
   B. CIRCUITS & ELECTRONICS  
      B1 B2 B3
   
   C. DSP/CONTROLS/COMMUNICATION  
      C1 C2 C3 C4 C5 C6
   
   D. EMAG/QUANTUM ELEC./LASERS  
      D1 D2 D3 D4
   
   E. SOLID STATE/PHYS. ELEC./  
      PLASMA ELECTRONICS  
      E1 E2 E3
   
   F. COMPUTER SYSTEMS  
      F1 F2 F3 F4 F5 F6

2. You must answer five questions (no more than two from MATH group).

3. Answer in the blue books provided. Use a separate book for each question. Put the title and number of question on front of each book (ex. MATH A-1)

4. Return all the 26 problems.

5. You will be graded on five questions only.

6. The examination is “closed-book”; only writing material and a scientific calculator are allowed. No formula sheet is allowed. Formulas are included where needed. No material shall be shared without prior permission of the proctor(s).

7. You have four hours to complete this examination.
A function $f(z)$ is said to be analytic in a domain $D$ if $f(z)$ is defined and differentiable at all points of $D$.

If $f(z) = |z|^2$, use Cauchy-Riemann equations to show whether $f(z)$ is analytic.
Consider the function $f(x) = x^2 - y^2$.

a) Make a sketch of $f$. You do not need to be numerically accurate, but you need to capture the qualitative shape of $f$.

b) Compute the vector field $A = \nabla f$.

c) Make a sketch of $A$. Again, you do not need to be numerically accurate, but capture $A$ qualitatively.

d) Compute $\text{curl}(A)$.

e) It is possible to compute $\text{curl}(\nabla f)$ even without knowing $f$? What is the result, and how can it be interpreted?
PROBLEM A3 – MATH

**Linear Algebra**

Let $A = [a_{ij}]_{1 \leq i, j \leq n}$ be a square matrix of dimension $n \times n$.

1. Write the equation that determines the eigenvalues and eigenvectors of matrix $A$ and the expression of the characteristic polynomial of matrix $A$.

2. State Cayley-Hamilton theorem and argue that $A^n$ is a linear combination of matrices $I, A, ..., A^{n-1}$.

3. Show that eigenvectors corresponding to distinct eigenvalues of matrix $A$ are linearly independent.

4. Define similar matrices and show that, if matrix $A$ has $n$ distinct eigenvalues, then it is similar to a diagonal matrix. What are the elements of this diagonal matrix and how is the similarity transformation matrix obtained?
Suppose \( s \geq 1 \) is a fixed integer. Let \( x \) be a random variable with probability density function

\[
f(s, x) = \frac{x^{s-1}}{(s-1)!} e^{-x}, \quad x \geq 0
\]

and zero otherwise.

(a) What properties must \( f(s, x) \) satisfy to be a valid density function?

(b) Define \( \Gamma(s, x) = \mathbb{P}\{x > x\} \). Determine a first-order, linear differential equation that \( e^x \Gamma(s, x) \) must satisfy.

(c) Find a series solution to the differential equation in part (b).

(d) Use the result from part (c) to produce an explicit formula for \( \mathbb{P}\{x > x\} \).

(e) Compute \( \mathbb{P}\{x \leq 1\} \) when \( s = 5 \).
The op amp shown in the circuit is ideal. There is no energy stored in the circuit at the time it is energized. If \( v_g = 16,000 \ u(t) \ V \), find

a. Find \( V_o(s) \).

b. Find \( v_o(t) \).

c. How long does it take to saturate the operational amplifier?

d. How small the rate of increase in \( v_g \) must be to prevent saturation?
PROBLEM B2 – CIRCUITS AND ELECTRONICS

**Sinusoidal Steady State Power**

The load impedance $Z_L$ for the circuit shown in the figure is adjusted until maximum average power is delivered to $Z_L$.

a) Find the maximum average power delivered to $Z_L$.

b) What percentage of the total power developed in the circuit is delivered to $Z_L$?
PROBLEM B3 – CIRCUITS AND ELECTRONICS

Electronics

For the circuit in below figure,
(a) find the values of R and results in \( V_D = 0.8 \text{V} \). The MOSFET has \( V_{th} = 0.5 \text{V} \), \( \mu_n \alpha_{ox} = 0.4 \text{ mA/V}^2 \), \( \frac{W}{L} = 0.72 \mu \text{m}/0.18 \mu \text{m} \), and \( \lambda = 0 \).

Assume that the above circuit is connected to a transistor Q2 which is identical to Q1.
(b) Find the value of R2 that results in Q2 operating at the edge of the saturation region when calculated value of R and \( V_D = 0.8 \text{V} \) are applied.
Given the 2D image \( f(x,y) \) below, answer the following questions.

\[
f(x,y) = \cos(2\pi x + 6\pi y)
\]

a) Determine the Fourier transform \( F(u,v) \) of \( f(x,y) \) and sketch the spectrum below. Use circles to indicate the locations of impulses. (4 pts)

b) Suppose this signal is sampled uniformly with sampling intervals \( \Delta x=\Delta y=\Delta=1/4 \). Sketch the spectrum of the sampled signal below. (3 pts)
c) If the sampled signal is filtered using an ideal low-pass filter $h(x,y)$ with frequency response:

$$H(u,v) = \begin{cases} 
1 & \text{if } \sqrt{u^2 + v^2} < \frac{f_s}{2} \\
0 & \text{otherwise}
\end{cases}$$

where $f_s=1/\Delta$. Sketch the spectrum of the filtered signal below and provide the spatial domain expression of the filtered signal $f_{\text{filtered}}(x,y)$. (3 pts)
Digital Signal Processing

1. (10 points) For an LTID system, the impulse response of the system is known as $h[n]$. If we denote the zero-state response of the system for $u[n]$ as $g[n]$, prove

$$g[n] = \sum_{k=-\infty}^{n} h[k]$$
2. (10 points) Based on the pole-zero location of the systems, roughly sketch the amplitude response of these filters (from 0 to π).

(a) (4 points)

(b) (4 points)

(c) (2 points) What kind of filter is it in (a) and (b) (low-pass, high-pass or band-pass)?
Suppose that in order to meet the following control specifications:

Percent Overshoot \( \leq 12\% \)
Settling Time \( \leq 0.5 \text{ sec.} \)

that a lead controller is to be designed with pole at \( s=-60 \) and a zero at \( s=-10 \), that is, the lead compensator is of the form \( G_c(s) = K_c \frac{s+10}{s+60} \). The root locus for \( K_c > 0 \) is given in Figure 2.

a) Derive the range of values of \( K_c \) so that all the design specifications are met. Explain.

b) Select the value of \( K_c \) from the range determined in part (a) that gives the smallest steady state error to unit steps in \( r(t) \) and wind disturbance, \( d(t) \). What is this error?

c) For this value of \( K_c \), approximately what are the gain and phase margins? Are the margins better than when \( K_c=1 \) (see Figure 3)? Explain why or why not. REMEMBER TO INCLUDE ALL YOUR WORK IN YOUR SOLUTIONS BOOKLET.

**REVIEW**

For a prototype second order open-loop transfer function \( G(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \) the following unit step response relations are useful:

- Percent overshoot = \( 100 \exp(-\zeta\pi / \sqrt{1 - \zeta^2}) \)
- 2\% settling time \( \approx 4 / (\zeta\omega_n) \)
Figure 2. Root locus for closed-loop poles with a lead compensator.

Figure 3. Bode plots of loop gain when \( K_c = 1 \). The magnitude plot crosses the \( w=0.1 \) rad/sec line with a magnitude of 30.3 dB.
Communication systems

Consider a sinusoidal carrier signal \( c(t) = A_c \cos(2\pi f_c t) \) modulating the information-bearing signal \( m(t) = a \cos(2\pi f_m t) \), where \( f_c >> f_m \).

1. Write the expression of the modulated signal \( u_{AM}(t) \) when conventional amplitude modulation is used, define the modulation index, and state what condition is required to prevent overmodulation.

2. Write the expressions of the pre-envelope signal \( u_{AM}^+(t) \) and of the complex envelope \( \tilde{u}_{AM}(t) \) for signal \( u_{AM}(t) \) in Part 1, and determine its corresponding in-phase and quadrature components \( u_I^{AM}(t) \) and \( u_Q^{AM}(t) \).

3. Write the expression of the modulated signal \( u_{PM}(t) \) when phase modulation is used, define the modulation index \( \beta_p \), and state the condition required for narrowband phase modulation.

4. Write the expressions of the pre-envelope signal \( u_{PM}^+(t) \) and of the complex envelope \( \tilde{u}_{PM}(t) \) for signal \( u_{PM}(t) \) in Part 3, and determine its corresponding in-phase and quadrature components \( u_I^{PM}(t) \) and \( u_Q^{PM}(t) \).

5. Assuming that the condition required for narrowband phase modulation in Part 3 is satisfied, write an approximate expression for \( u_{PM}(t) \) that is similar to that of \( u_{AM}(t) \) in Part 1. Comment on the similarities/differences between the two modulated signals (the narrowband phase modulated signal and the conventional amplitude modulated signal).

USEFUL MATHEMATICAL FORMULAS

Trigonometric identities:

\[
\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x \\
\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \\
\cos(x) \cos(y) = \frac{\cos(x - y) + \cos(x + y)}{2} \\
\sin(x) \sin(y) = \frac{\cos(x - y) - \cos(x + y)}{2} \\
\sin(x) \cos(y) = \frac{\sin(x - y) + \sin(x + y)}{2}
\]

Euler’s formula

\[
e^{jx} = \cos(x) + j \sin(x)
\]

Hilbert transform pairs

\[
\mathcal{H}\{\cos 2\pi f_c t\} = \sin 2\pi f_c t \\
\mathcal{H}\{\sin 2\pi f_c t\} = -\cos 2\pi f_c t
\]
Communication Networks

1. (5 pts) The channel bandwidth is 6 MHz. There are four levels of symbols.

(a) If the channel is noiseless, what is the maximum achievable rate?

(b) If the channel has noise and the SNR is 10 dB, what is the maximum achievable rate now?

2. (5 pts) Consider a network where node C is connected to nodes B, D, E. Distance vector routing is used. The following vectors just come to node C: (5, 0, 8, 12, 6, 2) from B, (16, 12, 6, 0, 9, 10) from D, and (7, 6, 3, 9, 0, 4) from E. The cost of the links from C to B, D, and E, are 6, 3, 5, respectively. Please compute the new routing table for node C. Give both the next-hop and the cost.
PROBLEM D1 – PHYSICAL ELECTRONICS I

Electromagnetics

A microwave oven typically operates at 2.45 GHz. A bottom round steak has a relative permittivity of 40 (or $\varepsilon' = 40\varepsilon_0$, $\varepsilon_0 = 8.8542 \times 10^{-12}$ F/m, $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m) and a loss tangent $\tan \delta = 0.3$ at the operating frequency.

(a) Calculate the complex wavenumber $k$.

(b) Calculate the penetration depth $d_p$ in steak.

(c) Calculate the intrinsic impedance $\eta$.

(d) Let the microwave be represented by a time-harmonic uniform plane wave: $\vec{E} = \hat{x}E_0 \exp(-jkz)$ and $\vec{H} = \hat{y}(E_0/|\eta|) \exp(-jkz - j\phi)$, where $k$ is the complex wavenumber, and $\eta = |\eta| \exp(-j\phi)$ is the complex intrinsic impedance. Given the electric-field intensity just beneath the surface of the steak, $|\vec{E}|_{z=0} = 1$ MV/m, find the time-average power density just beneath the surface and that inside the steak for a depth of $d_p$. (Hint: Calculate the time-average Poynting vectors at $z = 0$, and $z = d_p$, respectively.)
Electromagnetics

Two conducting spheres of radii \( a \) and \( b \) are connected by a long and thin conducting wire as shown in below figure. A charge of \( q \) is placed on this structure, i.e. \( q_a + q_b = q \). Find the charge on each sphere and calculate the \( E \) field on the surface of each sphere.
Lasers

A diode laser has the following characteristics:

Optical cavity of length 250 μm
Peak radiation at 1550 nm
Refractive index of semiconductor diode material 4
Optical gain bandwidth 2 nm.

(a) What is the laser mode spacing in nm? (3 points)

(b) How many modes can lase and what are their wavelengths in nm? (4 points)

(d) What is the reflection coefficient at each face of the semiconductor surface/air interface? (3 points)
PROBLEM D4 – PHYSICAL ELECTRONICS I

Optical Fiber Communications

Find the external quantum efficiency for a Ga_{1-x}Al_xAs laser diode with (x = 0.03) which has an optical-power-versus-drive-current relationship as shown in the following figure (use the 20° C curve.)

![Figure 4-31](image)

**FIGURE 4-31**
Temperature-dependent behavior of the optical output power as a function of the bias current for a particular laser diode.
A Boron Diffusion is used to form the base of a npn transistor in a 0.18 Ohm cm n-type Si wafer. A solid solubility limited Boron Pre-deposition is performed at 900 C for 15 min followed by a 5 hour drive-in at 1100 C.

a) Find the boron surface concentration graphically from Fig. 4.6 and calculate the B impurity dose after the pre-deposition step. Use $D_1 = 1.45 \times 10^{-15}$ cm$^2$/sec for Boron diffusivity at 900 C.

b) Calculate the B surface concentration and the junction depth following the 5 hour drive-in step. Apply Fig. 4.8 to obtain the n-type background dopant concentration. Use $D_2 = 2.96 \times 10^{-13}$ cm$^2$/sec for Boron diffusivity at 1100 C.

Figure 4.8: Room temperature resistivity in n- and p-type Silicon as a function of impurity concentration.
Fig. 4.4: Graphical comparison of Gaussian and complementary error function (erfc) profiles.

\[ x = \frac{x}{2\sqrt{Di}} \]

Normalized distance from surface, \( \bar{x} \)
**PROBLEM E2 – PHYSICAL ELECTRONICS II**

**Physical Electronics**

**Question 1**
For a silicon abrupt junction with \(N_A = 10^{18} \text{ cm}^{-3}\), \(\varepsilon_{max} = 4 \times 10^5 \text{ V/cm}\) at reverse bias \(V_R = 30\text{V} \) (\(T = 300\text{K}\)), calculate the n-type doping concentration.

**Question 2**
Calculate the theoretical saturation current, \(I_S\) of an ideal silicon p-n junction having following specifications:
\(N_D=10^{18} \text{ cm}^{-3}\), \(N_A=5\times10^{16} \text{ cm}^{-3}\), \(\tau_p=\tau_n=5 \times 10^{-7}\text{s}\), \(D_p = 10 \text{ cm}^2/\text{s}\), \(D_n = 21 \text{ cm}^2/\text{s}\) and a device area of 2\times10^{-4} \text{ cm}^2.
Also calculate the forward current at 0.5 V.

\[
J_p = q\mu_p \left( \frac{1}{q} \frac{dE}{dx} \right) - kT \mu_p \frac{d\rho_p}{dx} \quad \frac{d^2\psi}{dx^2} = -\frac{d\phi}{dx} = -\frac{\rho_s}{\varepsilon_s} = -\frac{q}{\varepsilon_s} \left( N_D - N_A + p - n \right).
\]

\[
V_{bi} = \psi_n - \psi_p = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \quad N_A x_p = N_D x_n. \quad W = x_p + x_n. \quad \varepsilon_m = \frac{q N_D x_n}{\varepsilon_s} = \frac{q N_A x_p}{\varepsilon_s}.
\]

\[
V_{bi} = \frac{1}{2} \varepsilon_m W. \quad W = \frac{2 \varepsilon_s \left( N_A + N_D \right)}{q N_A N_D} V_{bi}. \quad \varepsilon(x) = -\varepsilon_m + \frac{q N_B x}{\varepsilon_s}, \quad \varepsilon_m = \frac{q N_B W}{\varepsilon_s}.
\]

\[
C_j = \frac{\varepsilon_s}{W} \sqrt{\frac{q \varepsilon_s N_B}{2 (V_{bi} - V)}} \quad V_{bi} = \frac{kT}{q} \ln \frac{p_{po} n_{no}}{n_i^2} = \frac{kT}{q} \ln \frac{n_{no}}{n_{po}}, \quad n_{no} = n_{po} e^{q (V_{bi} - V)/kT}.
\]

\[
p_{po} = p_{no} e^{q V_{bi}/kT}, \quad n_n = n_{po} e^{q (V_{bi} - V)/kT}, \quad n_p = n_{po} e^{q V/kT}.
\]

\[
J = J_p(x_n) + J_n(-x_p) = J_0 (e^{q V/kT} - 1), \quad J_0 = \frac{q D_p p_{no}}{L_p} + \frac{q D_n n_{po}}{L_n},
\]

Silicon (300 K): \(N_C = 2.86 \times 10^{19} \text{ cm}^{-3}\); \(N_V = 2.66 \times 10^{19} \text{ cm}^{-3}\); \(n_i = 9.65 \times 10^9 \text{ cm}^{-3}\)

\(m_p = 1 \text{ m}_0\); \(m_n = 0.19 \text{ m}_0\); \(m_0 = 0.91 \times 10^{-30} \text{ kg}\); \(k = 1.38 \times 10^{-23} \text{ J/K}\); \(q = 1.6 \times 10^{-19} \text{ C}\)

\[
k_o = \frac{C_s}{C_i}, \quad k_e = \frac{C_s}{C_i} = \frac{k_o}{k_o + (1 - k_o) e^{-\varepsilon t / D}} \quad C_s = C_0 \left[ 1 - (1 - k_e)^{-k_e x / L} \right]
\]

\[
C_s = k_o C_v \left( 1 - \frac{M}{M_o} \right)^{k_o - 1} \quad C_s = k_e C_i e^{-k_e x / L}
\]

\[
F = \frac{D C_0}{x + (D / \kappa)}, \quad \chi^2 + A x = B (t + \tau), \quad A = 2D / \kappa, B = 2DC_0 / C_i, \quad \tau = (d_o^2 + 2D d_0 / \kappa) C_i / 2DC_0
\]

\[
B / A = \kappa C_0 / C_i, \quad \kappa = \sqrt{\frac{D_0}{D}}
\]
A mercury vapor plasma column fills a tube of 3 cm in diameter and 3 m in length. The average energy of the electrons in the plasma is 2 eV. The tube dissipates 100 Watts of heat evenly over its surface (neglect the ends). Assume that the only source of heat is the inelastic electron-neutral collisions, where the electron loses its average energy in each collision.

a./ How many electron-neutral collisions occur per unit volume, per second?  
(hint: compare total dissipated energy to electron energy)

b./ Calculate the mean electron velocity (hint: three degrees of freedom, 3/2 KT)

c./ If the effective diameter of the mercury atom is 4. \(10^{-10}\) m and the electron number density is \(n_e = 10^{16}\) m\(^{-3}\), calculate the density of neutral atoms and the pressure of the mercury gas in the tube.

d./ What is the mean free path of the electrons in the plasma?

e./ Calculate the electron diffusion coefficient.
Write a subroutine that calculates the nth number in the Fibonacci series in a Motorola 6811 microprocessor based system. The Fibonacci series is defined as follows. Given that $F_1 = 1$, $F_2 = 1$, then the nth Fibonacci number $F_n = F_{n-1} + F_{n-2}$, $n > 2$.

The value of $n$ is an 8-bit value found in memory location $0000$. Upon completion the nth Fibonacci number should be stored in memory location $1000$.

You may safely use memory locations $0001$ to $00FF$ for storage of any temporary variables.
<table>
<thead>
<tr>
<th>7</th>
<th>A</th>
<th>0</th>
<th>7</th>
<th>B</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>D</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- IX: Index Register X
- IY: Index Register Y
- SP: Stack Pointer
- PC: Program Counter

**Condition Codes**

- CARRY/BORROW FROM MSB
- OVERFLOW
- ZERO
- NEGATIVE
- I-INTERRUPT MASK
- HALF CARRY (FROM BIT 3)
- X-INTERRUPT MASK
- STOP DISABLE

**8-BIT ACCUMULATORS A & B**

**OR 16-BIT DOUBLE ACCUMULATOR D**
PROBLEM F2 – COMPUTER SYSTEMS

Digital System Design

You are to design a state machine that inputs a six bit binary number, and outputs a binary number whose value is 5 more than the input. Note that the binary numbers are input/output in the order of LSB to MSB. In the event of overflow, your state machine should assert an error signal.

1. (1 point) List the inputs and outputs for the state machine.

2. (5 points) Give the state machine chart and state machine table for a Moore style state machine that implements the function above.

3. (4 points) Give the implementation for the state machine given the state machine table you provided in the previous problem using a two-address microcoded controller. The figure shows the organization of a two-address microcoded controller.
PROBLEM F3 – COMPUTER SYSTEMS

Computer Architecture

1. What is the range of addresses in the instruction memory for a MIPS jump instruction (e.g., j: exit)? You need to explain why.

2. Assume that there are no pipeline (a single cycle datapath) and that the breakdown of executed instructions is as follows:

<table>
<thead>
<tr>
<th></th>
<th>add</th>
<th>Sub</th>
<th>beq</th>
<th>lw</th>
<th>sw</th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>10%</td>
<td>25%</td>
<td>25%</td>
<td>10%</td>
<td>10%</td>
<td></td>
</tr>
</tbody>
</table>

In what fraction of all cycles in the input of the sign-extended circuit needed? Why?
**PROBLEM F4 – COMPUTER SYSTEMS**

**Algorithms**

**Herding cats problem.** You have a collection of intelligent agents (autonomous entities are simulation entities that interact with their simulation environment according to some set of rules and are designed to achieve some goal. Source: Wikipedia). Each computer has two states that are designated by the colors green and red. In this problem, two agents can exist: “cats” and “herders”.

Cats interact with their environment which is a networked collection of computers whose interconnection structure is defined by a graph. In addition, cats try to avoid herders. The cat interacts in the following way:

A. It “eats” by changing the state of a computer from green to red with probability κ_{c} for a computer not occupied by a herder.

B. A cat can replicate itself after consuming some number of meals Q. The replicant moves to an adjacent green computer.

C. The cat will move to a green computer not already occupied by a cat
   
   (i) if it detects a herder agent on the same computer. Cats can detect a herder with a probability α_{h}
   
   (ii) at random with probability ν_{c}

D. The cat ceases to exist after making some number of moves μ.

Herders search for cats and try to contain them to a minimum number of computers. Herders interact in the following ways:

A. Herders move through the network under the following circumstances
   
   (i) at random with a probability ν_{h} where ν_{h} > ν_{c} to a computer that is colored red
   
   (ii) toward a computer occupied by a cat with α_{h}^{d} where d is the distance (hops) between the herder and the cat.

B. Herders can change the color of a computer not occupied by a cat from red to green with probability κ_{h}.

If a cat and a herder reside on the same computer:

A. The cat goes to sleep with probability γ_{c}.

B. The herder to goes to sleep with probability γ_{h}.

C. If a cat and a herder reside on the same computer, the cat purr can cause the herder to go to sleep with probability γ_{h}.

**Answer the following questions based on the rules described above:**

1. (6 points) Assume there is one cat and one herder and Q = 0, γ_{c} = 1, γ_{h} = 0, α_{h}^{d} = 1, κ_{c} = κ_{h} = 1, \( Q = \infty \), α_{h} = 0.5. Give the pseudocode or flow chart for an algorithm the herder can use to make a cat go to sleep and also to change all computers to the color green. Note any additional assumptions you have to make for your algorithm to work.

2. (4 points) Give the time complexity for your algorithm. Be sure to show all steps in your analysis and justify any assumptions you make.
Data Structures

1. A stack bStack contains the following items

   7
   8
   −3
   14
   5

   What is the output to the screen of the following code?

   ```java
   int x;
   while (!bStack.isEmpty()){
     bStack.pop(x);
     if (x>0 && !bStack.isEmpty())
       bStack.pop();
     cout << x << endl;
   }
   ```

2. Please provide pseudo code or diagram (explanations) for following questions

   Given the input A (2, 8, 7, 2, 0, 1, 1, 6),

   2.1 Construct a binary search tree according to the input A sequence.

   2.2 Add a node, 5, into this binary search tree?

   2.3 Delete a node, 0, from this binary search tree?
1. Given the following truth table,

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y (output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
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<td>0</td>
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<td>0</td>
<td>1</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) (5 pts) write the Boolean equation in sum-of-products form.

(b) (5 pts) Simplify the Boolean equation using Karnaugh map.