Estimating Cost of Equity

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Estimation of cost of equity is required for many financial applications such as capital budgeting and performance evaluation using EVA©. A common procedure is to use the Capital Asset Pricing Model [CAPM] which involves estimation of an expected risk premium equal to beta times the expected risk premium on the "market" portfolio, the portfolio containing all assets in the world. Since the market portfolio is not observable a proxy must be used. Typically beta is estimated using a domestic index as a proxy or is purchased from a commercial provider and six to eight percent is used as an estimate for the expected market risk premium. The estimates for beta and the expected market risk premium are then multiplied together. In this paper it is shown that this procedure more than likely yields a biased estimate for the cost of equity and as a result leads to misallocation of funds and biased performance measures. It is shown theoretically that the same index must be used for estimation of beta and the expected market risk premium. Further the estimate for the market risk premium must be adjusted to take into account the correlation between the proxy used and the market portfolio. Finally, a straight forward procedure for obtaining an unbiased estimate for cost of equity is presented.

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Introduction

Estimating the cost of equity is crucial for many financial decisions such as, capital budgeting, capital structure, and performance evaluation using EVA©. In a recent survey Bruner et al. [1998] found that the most common method favoured by practitioners for doing this is the Capital Asset Pricing Model [CAPM]. According to CAPM the expected return on any asset is equal to the risk free rate plus a risk premium. The risk premium is equal to the expected systematic risk of the asset relative to the "market" portfolio (beta) times the expected risk premium on the "market" portfolio. Since the market portfolio consists of all assets in the world and it is not observable a proxy must be used in order to implement CAPM. Typically, beta is estimated using a domestic index as a proxy and an estimate of between 6% and 8% is used for the expected market risk premium since this represents the average excess return on a portfolio of stocks over the risk free rate since 1925[1]. These two values are then multiplied together to obtain an estimate for cost of equity. In this paper it is shown that this more than likely yields a biased estimate. Since biased estimates result in misallocation of funds and biased performance measures it is important to obtain unbiased estimates. For this purpose, first, the theoretical relationship between the index used as a proxy and expected return on an asset, when CAPM is used, is obtained. Second, a simple procedure for obtaining estimates which incorporate this relationship, and are therefore unbiased, is presented.

In theory, the prescription provided by CAPM for obtaining the expected return on an asset is very clear: expected return is equal to the risk free rate plus a risk premium equal to the beta for the asset, relative to the portfolio, times the expected market risk premium. The beta for the asset is equal to the covariance, over the period of interest, between the return on the asset and that on the market portfolio divided by the variance of the market portfolio. In practice, the problem is that the market portfolio is not observable so to implement CAPM it is necessary to use a proxy. This raises (at least) two questions in relation to using CAPM to obtain an unbiased estimate for the expected return on an asset: (1) Does it matter what proxy is used for estimating beta and the expected market risk premium? (2) Should the same proxy be used for estimating beta and the expected market risk premium? These questions are addressed in this paper. To obtain an unbiased estimate for the expected return on an asset, in particular for the cost of equity, the short answer to question one is no and to question two yes. Specifically, it is shown that (a) the estimate for the expected market risk premium obtained using a proxy must be adjusted to take into account the correlation between the proxy and the market portfolio and (b) the same proxy must be used to estimate beta. It is also shown that although the market portfolio cannot be observed it is possible to obtain an estimate for

[1] For a discussion of these topics see any standard textbook in Finance or Chapter 12 in Stewart [1990].

the adjusted market risk premium using a reasonably simple procedure. Finally, from the results obtained, any reasonable proxy can be used for estimation.

The rest of the paper proceeds in the following way. In the next section, section I, the implications of using different indexes for estimating beta and the expected market risk premium are examined. The data used for this purpose, and in subsequent analysis, is also described in this section. In section II an estimation procedure for obtaining an unbiased estimate for expected return using CAPM is presented. Implementation of the procedure is discussed in section III. Section IV concludes.

I. Estimation using different proxies.

In this section the implications of using different proxies for estimating beta and the expected market risk premium are examined. For this purpose four indexes (potential proxies) are considered: the Standard and Poor's Composite Index, probably the most popular index, the Morgan Stanley Capital World Index, since it appears to be the most comprehensive and therefore potentially the closest to the world market index, and two CRSP indexes (value and equal weighted) representing broad based domestic indexes containing, among others, NASDAQ and NYSE stocks[2]. The descriptive statistics for the monthly returns, for each of the indexes, are reported in Table I.

Place Table I here please.

The CRSP equal weighted index has the largest average return (1.84503%) and the Morgan Stanley Index has the lowest average monthly return (0.74927%). The average difference is over one percent per month. Part of this of course is due to the fact that dividends are included in the CRSP equal weighted index and not in the Morgan Stanley Index. Also with an equal weighted index, smaller stocks have a relatively larger weight and these stocks may have a larger return than large stocks, over parts the period examined, leading to higher returns for equal weighted indexes compared to value weighted indexes[3]. In relation to obtaining an estimate for the market risk premium, therefore, from Table I there can be up to one percent difference in monthly return depending on the index

[2] The CRSP indexes are obtained from the CRSP tapes and Standard and Poor's Composite Index and the Morgan Stanleys Capital World are retrieved from Datastream. Both CRSP indexes include dividends. The aim of this paper is not to analyse a large number of different indexes but to illustrate a set of problems and a general solution procedure for any proxy chosen. Thus only a small number of indexes are examined.
[3] This effect is also referred to as the small firm effect, see for example Banz [1981].

used. The common practice of using between six and eight percent is possibly in recognition of this[4].

For estimation of beta, for any asset, what is important is the variation in the index and covariation between the return on the index and the return on the asset. Table II shows the correlation coefficients between the indexes; these range from 0.6853 to 0.9883. The highest correlation is between the CRSP value weighted index and the Standard and Poor's Index (the return on the CRSP index without dividends is about the same as Standard and Poor's Index) so these two indexes are very similar. The lowest correlation coefficient is between the CRSP equal weighted index and the Morgan Stanley index. Thus one would expect Standard and Poor's index and the CRSP value weighted index to yield similar betas and the Morgan Stanley Index and the equal weighted index to yield very different betas. This suggests that different proxies yield different beta values for the same asset.

Place Table II here please.

To determine whether or not the different indexes do in fact yield significantly different beta estimates, individual betas were estimated for stocks on the CRSP tape, using five years of monthly data[5]. For this purpose monthly dividend adjusted returns were retrieved from the CRSP tape from 1970 to the end of 1996. To avoid thin trading problems only stocks from the NYSE that were traded more than 95% of the days were included in the sample. The sample size ranges from a low of 780 in 1975 to a high of 1308 in 1994.

To obtain beta estimates for individual stocks, starting in 1970, the following equation was estimated for each stock:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{pr,t} - r_{ft}) + \varepsilon_t$$

where r_{it} is the return on the stock, $r_{pr,t}$ the return on the index (proxy), and r_{ft} the monthly return on three month Treasury Bills. For each stock over the period 1970 to 1974 inclusive, a beta was estimated for each of the four indexes. The procedure was repeated for 1971 to 1975 inclusive, and so forth, producing a total of 22,905 estimates of beta for each index. The results from this exercise are reported in Table III. From Table III the CRSP equal weighted index has the smallest average

[4]Most of the debate surrounding estimation of the market risk premium, to date, has centred on whether an arithmetic or geometric mean should be used for calculating the return on the index and not on whether the index provides an unbiased estimate for the market risk premium.

[5]Five years of monthly data was used because this is standard procedure. The purpose of this paper is to show the implications of using different proxies for estimating cost of equity in terms of biasedness. Efficiency is dealt with in another paper.

beta (0.922) whereas the CRSP value weighted index has the largest average beta (1.2). With a sample size of 22905 one would not expect such large differences in beta estimates if they were all unbiased estimates of the true market beta[6]. In summary, therefore, using five years of monthly data different proxies yield significantly different beta estimates. What about betas provided by commercial beta providers? Here the situation is much the same, Bruner et al. found for their (smaller) sample an average beta according to Bloomberg of 1.03 whereas according to Value Line it was 1.24.

Thus obtaining an estimate for cost of equity by multiplying an estimate for beta by an estimate for the expected market risk premium using different proxies clearly results in very different estimates, all of which cannot be unbiased. This is undesirable because if such estimates are used for capital budgeting decisions and performance measurement it can lead to misallocation of funds and biased performance measures. For example using the CRSP equal weighted index for beta and the average return on the Standard and Poor's Index for the expected market risk premium gives a relatively low value for the cost of equity and therefore a relatively high value for EVA©. In particular, if the same estimate is used for the expected market risk premium then the betas obtained using different proxies will yield very different estimates for the cost of equity. For example using 8% for the expected market risk premium, the difference for the resulting estimate for the cost of equity between the Standard and Poor's Index and the CRSP equal weighted index is 1.6% on an annual basis (for a stock with a beta of one!).

The trick to solving the problem of obtaining an unbiased estimate for the cost of equity is to recognise that what is important is that the overall estimate be unbiased; the properties of the individual elements are irrelevant. In the next section, it is shown that, in contrast to the common procedure of using independent estimates for the expected market risk premium return on the market and beta, to obtain an unbiased estimate of the cost of equity, the expected market risk premium and beta must be estimated using the same proxy. Further it is shown that the estimate for the expected market risk premium must be adjusted to take into account the relation between the proxy and the market portfolio. Finally, from the results obtained any reasonable proxy will do. Thus the answer to question (1) in the introduction is no and to question (2) is yes.

[6] Due to the overlapping estimation period the 22905 beta estimates are not independent of each other.

II. Obtaining an unbiased estimate for the cost of equity.

From section I the objective is to obtain an unbiased estimate for the cost of equity, or more generally an asset, using CAPM. That is, we want the actual return on the asset to differ from the expected return by at most a random error term:

$$r^e_{i,t+1} = E_t\left[r^e_{i,t+1}\right] + \varepsilon_{i,t+1} \qquad (1)$$

where r^e_i is excess return for asset i, the return on asset i minus the risk free rate, and $\varepsilon_{i,t+1}$ is a white noise error term. For this purpose, first the theoretical relationship between the expected return on an asset and the proxy portfolio, used in place of the market portfolio, implied by CAPM is analysed. Second an estimation procedure leading to an unbiased estimate is presented.

From CAPM the expected return on asset i is given by:

$$E_t[r^e_{i,t+1}] = \beta^{wm}_{i,t+1} E_t\left[r^e_{wm,t+1}\right] \qquad (2)$$

where $E_t[r^e_{i,t+1}]$ is the expected excess return for asset or portfolio i, $E_t\left[r^e_{wm,t+1}\right]$ is the expected excess return on the market, and $\beta^{wm}_{i,t+1}$ is systematic risk for asset i relative to the market portfolio. All values are for period t+1 and expectations are taken in period t. For the theoretical analysis the time subscripts are dropped to reduce notation. From the introduction the problem associated with estimating expected return using CAPM is that the market portfolio cannot be observed so it is necessary to use a proxy. From section I different proxies generate different estimates for cost of equity. They cannot all be unbiased and since the market portfolio cannot be observed it is not possible to determine which one is unbiased. The question, therefore, is how to obtain an unbiased estimate using a proxy instead of the market portfolio. For this purpose the theoretical relationship implied by CAPM between expected return on an asset, the expected excess return on the proxy, and beta relative to the proxy is derived. That is, an expression for the expected return on an asset in terms of what is actually estimated, beta relative to the proxy and the expected excess return on the proxy, is obtained.

From CAPM, equation (2), the relationship between the return on the proxy portfolio and the market portfolio is given by:

$$E[r^e_{pr}] = \beta^{wm}_{pr} E[r^e_{wm}] \qquad (3).$$

Using (3) to substitute for $E[r^e_{wm}]$ in terms of the proxy in equation (2) yields:

$$E[r^e_i] = \frac{\beta^{wm}_i}{\beta^{wm}_{pr}} E\left[r^e_{pr}\right] \qquad (4)$$

Equation (4) gives the expected excess return on asset i in terms of the expected excess return on the proxy. Since a proxy is also used for estimation of beta this needs to be incorporated into the right hand side of (4).

Now, by definition, for any asset i (or portfolio) and any proxy portfolio x, beta for the asset (portfolio) relative to the proxy is given by:

$$\beta_i^x = \frac{Cov(r_i, r_x)}{\sigma_x^2} \qquad (5)$$

In particular, for asset i relative to the market portfolio:

$$\beta_i^{wm} = \frac{Cov(r_i, r_{wm})}{\sigma_{wm}^2} \qquad (6)$$

and for the proxy portfolio relative to the market portfolio:

$$\beta_{pr}^{wm} = \frac{Cov(r_{pr}, r_{wm})}{\sigma_{wm}^2} \qquad (7)$$

Making use of (6) and (7) it is straight forward to show (see appendix):

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2}. \qquad (8)$$

Substituting (8) into (4) gives:

$$E[r_i^e] = \beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right] \qquad (9)$$

Equation (9) provides an expression for expected return on asset i in terms of the beta for asset i relative to a proxy and the expected excess return on the *same* proxy, i.e. in terms of what we actually estimate, β_i^{pr} and $E\left[r_{pr}^e\right]$. From (9) two conclusions can be drawn. One, using the beta estimate from a proxy times the expected excess return on the proxy $\left(\beta_i^{pr} E\left[r_{pr}^e\right]\right)$ yields a biased estimate for the cost of equity. This is because doing so ignores the adjustment factor $\frac{1}{\rho_{pr,mw}^2}$. Two, the same proxy must be used for estimation of beta and the expected market risk premium[7]. Thus the common procedure of estimating beta against an index and using six to eight percent as an estimate for the expected market risk premium leads to a biased estimate for cost of equity.

To obtain an unbiased estimate of the cost of equity it is therefore necessary to obtain an estimate for the right hand side of (9). This includes the correlation between the proxy and the unobservable market portfolio - so we are back where we started? Well, not quite. The estimation procedure suggested by Fama and MacBeth [1974] provides a natural estimate for $\beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$. In particular, the procedure provides an estimate for $\frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$ so it is not necessary to estimate

[7] From above algebra, if two different proxies are used then we need an estimate for the beta for each proxy relative to the world market portfolio (!).

$\frac{1}{\rho^2_{pr,mw}}$ separately. Fama and McBeth's procedure consists of two steps. First, beta is estimated using a proxy in the following time-series regression:

$$r_{it} - r_{fr} = a_i + \beta_i^{pr}(r_{prt} - r_{ft}) + \varepsilon_{it}, t = 1,..,T, i = 1,..,N \qquad (10)$$

This estimation uses five years of monthly data, for example 1970 to 1974 inclusive, one regression for each stock in the sample (N). Second, each of the estimated betas from the first step are used in a cross section regression of the following form using stock returns for the subsequent year, for example 1975:

$$r^a_{it+1} - r^a_{fr+1} = \gamma_{0\,i} + \gamma_1 \beta^{pr}_{it} + \varepsilon_{it}, i = 1,..,N \qquad (11)$$

where the superscript "pr" on beta indicates the fact that a proxy was used to estimate beta in (10). This is done using annual data, i.e. r^a_{it+1} is the annual return for stock i for 1975 and the risk free rate of return is the annual return on the Treasury bill. A simple way to obtain an estimate for γ_0 and γ_1 is to roll the above procedure forward and then take an average of the estimated values of γ_0 and γ_1. Thus γ_1 provides an estimate for $\frac{1}{\rho^2_{pr,mw}} E\left[r^e_{pr}\right]$ which when multiplied by β^{pr}_{it} provides an unbiased estimate for $\beta^{wm}_{i,t+1} E_t\left[r^e_{wm,t+1}\right]$. From the theoretical relationship obtained above (equation (9)) γ_0 should be zero and $\beta^{pr}_i \frac{1}{\rho^2_{pr,mw}} E\left[r^e_{pr}\right]$ provides an unbiased estimate for expected return. This procedure is implemented in the next section.

III. Estimating expected return using CAPM.

From Section II, first equation (10) is used to estimate a beta for each stock and then the betas obtained from estimation of (10) are used in the cross-section regression (11) to obtain an unbiased estimate for expected return. This procedure was undertaken for each of the indexes discussed in section I. The estimated slope coefficients are provided in Table IV.

Place Table IV here please.

The shaded area indicates significance; they are in general significant for each year. From section II the average of these coefficients provides an estimate for the adjusted risk premium, the expected return on the proxy times the adjustment coefficient, $\frac{1}{\rho^2_{pr,mw}} E\left[r^e_{pr}\right]$, (provided the distribution is stable over the period). The lowest adjusted risk premium (3%) is obtained using Standard and Poor's

Index whereas the CRSP equal weighted index gives the highest (7%). This is not surprising since from Table III the Standard and Poor's Index had the second highest average beta of 1.1 whereas the average beta for the CRSP equal weighted index was 0.9.

Place Table V here please.

Table V reports the estimates for the intercept terms which, according to CAPM and equation (10) should all be zero; clearly they are not. There are several possible reasons for this. The first is that CAPM is not valid. There is an extensive academic literature supporting this view.[8] If there are more risk factors than the market index then the inclusion of an intercept term captures the average of these missing factors. This is, of course, a brute force method. A more accurate procedure would be to estimate a multifactor model. The second possible reason is that CAPM is correct but the intercept term captures drifts in beta. In the above procedure the beta used in the cross section regression is from the previous period and Blume [1975], for example, argued that there is mean reversion in beta; that is, a high beta is followed by a smaller beta. Depending on this adjustment process inclusion of an intercept term may capture part of this movement.[9] Again under this interpretation an intercept term should be included in the calculation of the expected return on an asset.

Finally, from Table IV and Table V, estimates for the cost of equity based on the four indexes examined are as follows:

Standard and Poor's:

$$\hat{r}_i = r_{ft} + 4.30 + \beta_i^{sp} \times 3.75$$

CRSP value weighted index:

$$\hat{r}_i = r_{ft} + 3.36 + \beta_i^{vw} \times 4.44$$

CRSP equal weighted index:

$$\hat{r}_i = r_{ft} + 2.22 + \beta_i^{ew} \times 7.03$$

Morgan Stanley:

$$\hat{r}_i = r_{ft} + 4.43 + \beta_i^{ms} \times 4.01$$

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[8] The Arbitrage Pricing Theory by Ross [1976] provides an example of a theoretical model and Fama and French [1992] an empirical example of a three factor model.

[9] Notice that γ_1 captures the dynamic process suggested by Blume [1975]; that is, an additional benefit of this procedure is that Blume's adjustment is implicit in the procedure.

The four indexes provide an estimate for the expected market risk premium $\left(\gamma_0 + \overline{\beta^{pr}} \times \gamma_1\right)$ of around 8.4 percent, where $\overline{\beta^{pr}}$ is the average beta from Table II. Thus the four proxies provide more or less the same estimate for the expected market risk premium when the appropriate adjustment is made. Further for an individual stock, provided the same proxy is used for beta and the expected market risk premium and the appropriate adjustment is made to the estimate for the market risk the four proxies yield the same estimate for cost of equity.

Finally, the R^2 from the cross section regressions are reported in Table VI to determine how much of the variation in actual returns can be explained by differences in beta. The results are not encouraging. From Table VI, betas calculated using the Standard and Poor's Index can, on average, explain 1.6% of the variation in stock returns. The CRSP equal weighted index fares a bit better; it can explain nearly 3%. Based on this it is surprising that CAPM is so widely used.

Place Table VI here please.

IV. Conclusion.

This paper has shown that the standard procedure of using independent estimates of beta and the expected market risk premium to obtain an estimate for cost of equity based on CAPM most likely yields a biased estimate. This is undesirable since it leads to misallocation of funds and biased performance measures. It is shown in this paper that to obtain an unbiased estimate for cost of equity, while any reasonable proxy may be used for estimation of beta, the same proxy must be used for estimating the expected market risk premium. Further, the estimate for the expected market risk premium must be adjusted to take into account the relationship between the proxy and the market portfolio. A two step procedure was suggested which does this. If a beta is obtained from a beta provider then the relevant adjusted risk premium should also be obtained because without this it is not possible to obtain an unbiased estimate for cost of equity based on the beta provided. From the analysis done here it would not be difficult for beta providers to make the relevant adjusted risk premium available.

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Appendix

In the text it is stated that it is possible to show:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2} \qquad (8)$$

where the superscript “wm” refers to the world market portfolio and “pr” to the proxy portfolio. To derive this expression start with the definition of beta for asset i relative to the proxy pr:

$$\beta_i^{pr} = \frac{Cov(r_i, r_{pr})}{\sigma_{pr}^2} \qquad (A1)$$

From CAPM the actual return on the proxy portfolio is given by:

$$r_{pr} = r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr} \qquad (A2)$$

where ε_{pr} is a random error term with an expected value of zero. If the proxy portfolio lies on the efficient set then ε_{pr} is zero. Given that the proxy portfolio contains less assets than the world market portfolio this is unlikely to be the case. However the proxy is, in general, well diversified so it is not unreasonable to assume that ε_{pr} is small and uncorrelated with the return on individual stocks.

Substituting (A1) into (A2), and recognising that the risk free rate is a constant gives:

$$\beta_i^{pr} = \frac{Cov(r_i, r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm} + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} + \frac{Cov(r_i, \varepsilon_{pr})}{\sigma_{pr}^2}$$

Assuming that the return on asset “i” is uncorrelated with a shock to the proxy portfolio the last term is zero, so we have:

$$\beta_i^{pr} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} = \frac{\beta_{pr}^{wm} Cov(r_i, r_{wm}) \sigma_{wm}^2}{\sigma_{pr}^2 \sigma_{wm}^2} = \frac{\beta_{pr}^{wm} \beta_i^{wm} \sigma_{wm}^2}{\sigma_{pr}^2}$$

$$\Leftrightarrow \frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \frac{\sigma_{pr}^2 \beta_i^{pr}}{\sigma_{wm}^2 (\beta_{pr}^{wm})^2}$$

Finally, using the definition of beta for the proxy portfolio:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{\sigma_{pr}^2}{\sigma_{wm}^2} \frac{(\sigma_{wm}^2)^2}{Cov(r_{pr}, r_{wm})^2} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2}$$

Table I.

Summary statistics

Summary statistics are for monthly returns on the indicated indexes expressed as percentages. Standard and Poor refers to the Standard and Poor's Composite Index and Morgan Stanley refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

Index	Number of observations	Mean	Standard deviation	Minimum	Maximum
Standard and Poor	323	0.7626	4.35781	-21.76304	16.30469
Morgan Stanley	323	0.74927	4.08246	-17.12423	14.26563
Value weighted CRSP index	323	1.06192	4.49597	-22.50055	16.57984
Equal weighted CRSP index	323	1.84503	5.70234	-25.08664	30.28762

Table II

Correlation coefficients between the indexes

The Pearson correlation coefficients are calculated based on monthly returns for each of the indexes. Standard and Poor (and S & P) refers to the Standard and Poor's Composite Index and Morgan Stanley (and MSCI) refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

	S & P	MSCI	CRSP value weighted	CRSP equal weighted
Standard and Poor	1	0.826	0.9883	0.785
Morgan Stanley		1	0.8216	0.6853
Value weighted CRSP index			1	0.8485
Equal weighted CRSP index				1

Table III

Summary statistics for beta estimates.

For each index, an individual stock beta is estimated using the following model:

$$r_{it} - r_{fr} = \alpha_i + \beta_i(r_{It} - r_{ft}) + \varepsilon_{it}, t = 1,..,T, i = 1,..,N \quad (10)$$

where r_{it} is the monthly return (including dividends) for stock i, r_{fr} is the monthly return on three month Treasury bills and r_{iT} is the monthly return on the index. The first estimation period is 1970 to 1974 inclusive, the second is 1971 to 1975 inclusive, and so forth up to and including 1996.

Index	Number	Average beta	Standard deviation	Minimum	Maximum
Standard and Poors	22,905	1.0982	0.4717	-1.9468	3.4652
Morgan Stanley	22,905	0.9765	0.5133	-1.5106	3.8294
CRSP value weighted	22,905	1.1173	0.4833	-1.3448	3.7249
CRSP equal weighted	22,905	0.922	0.4717	-0.4082	7.26

Table IV
Estimates of slope coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1}^{pr} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.1786	0.2315	0.4357	0.2597
1976	0.0822	0.1079	0.2172	0.0998
1977	0.0664	0.077	0.1052	0.0682
1978	0.1429	0.1563	0.1938	0.1432
1979	0.1845	0.1996	0.2019	0.1324
1980	0.1339	0.1408	0.1275	0.0808
1981	-0.0957	-0.093	-0.0654	-0.0723
1982	-0.0033	0.0056	0.0628	0.0129
1983	0.088	0.1017	0.1713	0.0859
1984	-0.1439	-0.1533	-0.1899	-0.1412
1985	-0.0788	-0.0772	-0.0721	-0.0763
1986	-0.106	-0.1159	-0.1473	-0.1245
1987	0.0104	0.0032	0.0056	0.0536
1988	0.0264	0.028	0.0284	-0.0123
1989	-0.0867	-0.1003	-0.1316	-0.1157
1990	-0.0795	-0.0916	-0.1346	-0.0686
1991	0.2516	0.263	0.2973	0.2516
1992	0.1425	0.1469	0.1807	0.17
1993	-0.0388	-0.0106	0.104	-0.0644
1994	0.0421	0.0412	0.0314	0.0055
1995	0.0674	0.0676	0.0731	0.1342
1996	0.0428	0.0444	0.0526	0.061
Mean	0.0375	0.0442	0.0703	0.0401

Table V.
Estimates of Intercept coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.2727	0.2125	0.0899	0.1445
1976	0.2121	0.1802	0.1161	0.1854
1977	-0.1283	-0.1422	-0.1426	-0.1349
1978	-0.1957	-0.2147	-0.2011	-0.211
1979	-0.0591	-0.0814	-0.0346	-0.0159
1980	-0.0858	-0.0938	-0.0277	-0.0163
1981	-0.0056	-0.0125	-0.0615	-0.0271
1982	0.1766	0.1664	0.115	0.1575
1983	0.0769	0.0662	0.019	0.0811
1984	0.0355	0.0397	0.0428	0.0347
1985	0.267	0.2629	0.2452	0.2629
1986	0.1439	0.1539	0.164	0.1545
1987	-0.1257	-0.1183	-0.1202	-0.1629
1988	0.0563	0.0541	0.0568	0.0976
1989	0.16	0.1766	0.203	0.1832
1990	-0.1785	-0.1622	-0.1207	-0.1981
1991	0.0876	0.0651	0.0269	0.1598
1992	-0.0151	-0.0253	-0.0478	-0.0014
1993	0.169	0.1414	0.0416	0.1659
1994	-0.126	-0.128	-0.1089	-0.0863
1995	0.148	0.1447	0.1589	0.1388
1996	0.0613	0.0556	0.0626	0.0627
Mean	0.043	0.0336	0.0216	0.0443

Table VI.
Mean R-squared for c ross section regressions

The R-squared is obtained from the following regression:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.0194	0.0325	0.0983	0.048
1976	0.0105	0.0188	0.0668	0.0179
1977	0.0125	0.0177	0.0315	0.0156
1978	0.0337	0.0429	0.0637	0.0434
1979	0.0201	0.0252	0.0275	0.0141
1980	0.0281	0.0323	0.0201	0.012
1981	0.0284	0.0256	0.0098	0.018
1982	0	0	0.003	0.0001
1983	0.0123	0.0151	0.0339	0.0112
1984	0.0806	0.0856	0.101	0.0752
1985	0.0125	0.0113	0.008	0.011
1986	0.0295	0.0352	0.0514	0.0367
1987	0.0001	0	0	0.0049
1988	0.0008	0.0009	0.001	0.0002
1989	0.0072	0.01	0.0179	0.0149
1990	0.0091	0.013	0.0313	0.0069
1991	0.0173	0.0205	0.0293	0.0119
1992	0.0151	0.0171	0.0255	0.0153
1993	0.0019	0.0001	0.0177	0.0023
1994	0.005	0.0054	0.0031	0
1995	0.0035	0.0038	0.0041	0.0055
1996	0.0045	0.0054	0.0073	0.0053
Mean	0.016	0.019	**0.0296**	0.0168

Estimating Cost of Equity

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Estimation of cost of equity is required for many financial applications such as capital budgeting and performance evaluation using EVA©. A common procedure is to use the Capital Asset Pricing Model [CAPM] which involves estimation of an expected risk premium equal to beta times the expected risk premium on the "market" portfolio, the portfolio containing all assets in the world. Since the market portfolio is not observable a proxy must be used. Typically beta is estimated using a domestic index as a proxy or is purchased from a commercial provider and six to eight percent is used as an estimate for the expected market risk premium. The estimates for beta and the expected market risk premium are then multiplied together. In this paper it is shown that this procedure more than likely yields a biased estimate for the cost of equity and as a result leads to misallocation of funds and biased performance measures. It is shown theoretically that the same index must be used for estimation of beta and the expected market risk premium. Further the estimate for the market risk premium must be adjusted to take into account the correlation between the proxy used and the market portfolio. Finally, a straight forward procedure for obtaining an unbiased estimate for cost of equity is presented.

Thursday, November 30, 2000

Introduction

Estimating the cost of equity is crucial for many financial decisions such as, capital budgeting, capital structure, and performance evaluation using EVA©. In a recent survey Bruner et al. [1998] found that the most common method favoured by practitioners for doing this is the Capital Asset Pricing Model [CAPM]. According to CAPM the expected return on any asset is equal to the risk free rate plus a risk premium. The risk premium is equal to the expected systematic risk of the asset relative to the "market" portfolio (beta) times the expected risk premium on the "market" portfolio. Since the market portfolio consists of all assets in the world and it is not observable a proxy must be used in order to implement CAPM. Typically, beta is estimated using a domestic index as a proxy and an estimate of between 6% and 8% is used for the expected market risk premium since this represents the average excess return on a portfolio of stocks over the risk free rate since 1925[1]. These two values are then multiplied together to obtain an estimate for cost of equity. In this paper it is shown that this more than likely yields a biased estimate. Since biased estimates result in misallocation of funds and biased performance measures it is important to obtain unbiased estimates. For this purpose, first, the theoretical relationship between the index used as a proxy and expected return on an asset, when CAPM is used, is obtained. Second, a simple procedure for obtaining estimates which incorporate this relationship, and are therefore unbiased, is presented.

In theory, the prescription provided by CAPM for obtaining the expected return on an asset is very clear: expected return is equal to the risk free rate plus a risk premium equal to the beta for the asset, relative to the portfolio, times the expected market risk premium. The beta for the asset is equal to the covariance, over the period of interest, between the return on the asset and that on the market portfolio divided by the variance of the market portfolio. In practice, the problem is that the market portfolio is not observable so to implement CAPM it is necessary to use a proxy. This raises (at least) two questions in relation to using CAPM to obtain an unbiased estimate for the expected return on an asset: (1) Does it matter what proxy is used for estimating beta and the expected market risk premium? (2) Should the same proxy be used for estimating beta and the expected market risk premium? These questions are addressed in this paper. To obtain an unbiased estimate for the expected return on an asset, in particular for the cost of equity, the short answer to question one is no and to question two yes. Specifically, it is shown that (a) the estimate for the expected market risk premium obtained using a proxy must be adjusted to take into account the correlation between the proxy and the market portfolio and (b) the same proxy must be used to estimate beta. It is also shown that although the market portfolio cannot be observed it is possible to obtain an estimate for

[1] For a discussion of these topics see any standard textbook in Finance or Chapter 12 in Stewart [1990].

the adjusted market risk premium using a reasonably simple procedure. Finally, from the results obtained, any reasonable proxy can be used for estimation.

The rest of the paper proceeds in the following way. In the next section, section I, the implications of using different indexes for estimating beta and the expected market risk premium are examined. The data used for this purpose, and in subsequent analysis, is also described in this section. In section II an estimation procedure for obtaining an unbiased estimate for expected return using CAPM is presented. Implementation of the procedure is discussed in section III. Section IV concludes.

I. Estimation using different proxies.

In this section the implications of using different proxies for estimating beta and the expected market risk premium are examined. For this purpose four indexes (potential proxies) are considered: the Standard and Poor's Composite Index, probably the most popular index, the Morgan Stanley Capital World Index, since it appears to be the most comprehensive and therefore potentially the closest to the world market index, and two CRSP indexes (value and equal weighted) representing broad based domestic indexes containing, among others, NASDAQ and NYSE stocks[2]. The descriptive statistics for the monthly returns, for each of the indexes, are reported in Table I.

Place Table I here please.

The CRSP equal weighted index has the largest average return (1.84503%) and the Morgan Stanley Index has the lowest average monthly return (0.74927%). The average difference is over one percent per month. Part of this of course is due to the fact that dividends are included in the CRSP equal weighted index and not in the Morgan Stanley Index. Also with an equal weighted index, smaller stocks have a relatively larger weight and these stocks may have a larger return than large stocks, over parts the period examined, leading to higher returns for equal weighted indexes compared to value weighted indexes[3]. In relation to obtaining an estimate for the market risk premium, therefore, from Table I there can be up to one percent difference in monthly return depending on the index

[2] The CRSP indexes are obtained from the CRSP tapes and Standard and Poor's Composite Index and the Morgan Stanleys Capital World are retrieved from Datastream. Both CRSP indexes include dividends. The aim of this paper is not to analyse a large number of different indexes but to illustrate a set of problems and a general solution procedure for any proxy chosen. Thus only a small number of indexes are examined.

[3] This effect is also referred to as the small firm effect, see for example Banz [1981].

used. The common practice of using between six and eight percent is possibly in recognition of this[4].

For estimation of beta, for any asset, what is important is the variation in the index and covariation between the return on the index and the return on the asset. Table II shows the correlation coefficients between the indexes; these range from 0.6853 to 0.9883. The highest correlation is between the CRSP value weighted index and the Standard and Poor's Index (the return on the CRSP index without dividends is about the same as Standard and Poor's Index) so these two indexes are very similar. The lowest correlation coefficient is between the CRSP equal weighted index and the Morgan Stanley index. Thus one would expect Standard and Poor's index and the CRSP value weighted index to yield similar betas and the Morgan Stanley Index and the equal weighted index to yield very different betas. This suggests that different proxies yield different beta values for the same asset.

Place Table II here please.

To determine whether or not the different indexes do in fact yield significantly different beta estimates, individual betas were estimated for stocks on the CRSP tape, using five years of monthly data[5]. For this purpose monthly dividend adjusted returns were retrieved from the CRSP tape from 1970 to the end of 1996. To avoid thin trading problems only stocks from the NYSE that were traded more than 95% of the days were included in the sample. The sample size ranges from a low of 780 in 1975 to a high of 1308 in 1994.

To obtain beta estimates for individual stocks, starting in 1970, the following equation was estimated for each stock:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{pr,t} - r_{ft}) + \varepsilon_t$$

where r_{it} is the return on the stock, $r_{pr,t}$ the return on the index (proxy), and r_{ft} the monthly return on three month Treasury Bills. For each stock over the period 1970 to 1974 inclusive, a beta was estimated for each of the four indexes. The procedure was repeated for 1971 to 1975 inclusive, and so forth, producing a total of 22,905 estimates of beta for each index. The results from this exercise are reported in Table III. From Table III the CRSP equal weighted index has the smallest average

[4]Most of the debate surrounding estimation of the market risk premium, to date, has centred on whether an arithmetic or geometric mean should be used for calculating the return on the index and not on whether the index provides an unbiased estimate for the market risk premium.

[5]Five years of monthly data was used because this is standard procedure. The purpose of this paper is to show the implications of using different proxies for estimating cost of equity in terms of biasedness. Efficiency is dealt with in another paper.

beta (0.922) whereas the CRSP value weighted index has the largest average beta (1.2). With a sample size of 22905 one would not expect such large differences in beta estimates if they were all unbiased estimates of the true market beta[6]. In summary, therefore, using five years of monthly data different proxies yield significantly different beta estimates. What about betas provided by commercial beta providers? Here the situation is much the same, Bruner et al. found for their (smaller) sample an average beta according to Bloomberg of 1.03 whereas according to Value Line it was 1.24.

Thus obtaining an estimate for cost of equity by multiplying an estimate for beta by an estimate for the expected market risk premium using different proxies clearly results in very different estimates, all of which cannot be unbiased. This is undesirable because if such estimates are used for capital budgeting decisions and performance measurement it can lead to misallocation of funds and biased performance measures. For example using the CRSP equal weighted index for beta and the average return on the Standard and Poor's Index for the expected market risk premium gives a relatively low value for the cost of equity and therefore a relatively high value for EVA©. In particular, if the same estimate is used for the expected market risk premium then the betas obtained using different proxies will yield very different estimates for the cost of equity. For example using 8% for the expected market risk premium, the difference for the resulting estimate for the cost of equity between the Standard and Poor's Index and the CRSP equal weighted index is 1.6% on an annual basis (for a stock with a beta of one!).

The trick to solving the problem of obtaining an unbiased estimate for the cost of equity is to recognise that what is important is that the overall estimate be unbiased; the properties of the individual elements are irrelevant. In the next section, it is shown that, in contrast to the common procedure of using independent estimates for the expected market risk premium return on the market and beta, to obtain an unbiased estimate of the cost of equity, the expected market risk premium and beta must be estimated using the same proxy. Further it is shown that the estimate for the expected market risk premium must be adjusted to take into account the relation between the proxy and the market portfolio. Finally, from the results obtained any reasonable proxy will do. Thus the answer to question (1) in the introduction is no and to question (2) is yes.

[6] Due to the overlapping estimation period the 22905 beta estimates are not independent of each other.

II. Obtaining an unbiased estimate for the cost of equity.

From section I the objective is to obtain an unbiased estimate for the cost of equity, or more generally an asset, using CAPM. That is, we want the actual return on the asset to differ from the expected return by at most a random error term:

$$r^e_{i,t+1} = E_t\left[r^e_{i,t+1}\right] + \varepsilon_{i,t+1} \qquad (1)$$

where r^e_i is excess return for asset i, the return on asset i minus the risk free rate, and $\varepsilon_{i,t+1}$ is a white noise error term. For this purpose, first the theoretical relationship between the expected return on an asset and the proxy portfolio, used in place of the market portfolio, implied by CAPM is analysed. Second an estimation procedure leading to an unbiased estimate is presented.

From CAPM the expected return on asset i is given by:

$$E_t[r^e_{i,t+1}] = \beta^{wm}_{i,t+1} E_t\left[r^e_{wm,t+1}\right] \qquad (2)$$

where $E_t[r^e_{i,t+1}]$ is the expected excess return for asset or portfolio i, $E_t\left[r^e_{wm,t+1}\right]$ is the expected excess return on the market, and $\beta^{wm}_{i,t+1}$ is systematic risk for asset i relative to the market portfolio. All values are for period t+1 and expectations are taken in period t. For the theoretical analysis the time subscripts are dropped to reduce notation. From the introduction the problem associated with estimating expected return using CAPM is that the market portfolio cannot be observed so it is necessary to use a proxy. From section I different proxies generate different estimates for cost of equity. They cannot all be unbiased and since the market portfolio cannot be observed it is not possible to determine which one is unbiased. The question, therefore, is how to obtain an unbiased estimate using a proxy instead of the market portfolio. For this purpose the theoretical relationship implied by CAPM between expected return on an asset, the expected excess return on the proxy, and beta relative to the proxy is derived. That is, an expression for the expected return on an asset in terms of what is actually estimated, beta relative to the proxy and the expected excess return on the proxy, is obtained.

From CAPM, equation (2), the relationship between the return on the proxy portfolio and the market portfolio is given by:

$$E[r^e_{pr}] = \beta^{wm}_{pr} E[r^e_{wm}] \qquad (3).$$

Using (3) to substitute for $E[r^e_{wm}]$ in terms of the proxy in equation (2) yields:

$$E[r^e_i] = \frac{\beta^{wm}_i}{\beta^{wm}_{pr}} E\left[r^e_{pr}\right] \qquad (4)$$

Equation (4) gives the expected excess return on asset i in terms of the expected excess return on the proxy. Since a proxy is also used for estimation of beta this needs to be incorporated into the right hand side of (4).

Now, by definition, for any asset i (or portfolio) and any proxy portfolio x, beta for the asset (portfolio) relative to the proxy is given by:

$$\beta_i^x = \frac{Cov(r_i, r_x)}{\sigma_x^2} \qquad (5)$$

In particular, for asset i relative to the market portfolio:

$$\beta_i^{wm} = \frac{Cov(r_i, r_{wm})}{\sigma_{wm}^2} \qquad (6)$$

and for the proxy portfolio relative to the market portfolio:

$$\beta_{pr}^{wm} = \frac{Cov(r_{pr}, r_{wm})}{\sigma_{wm}^2} \qquad (7)$$

Making use of (6) and (7) it is straight forward to show (see appendix):

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2}. \qquad (8)$$

Substituting (8) into (4) gives:

$$E[r_i^e] = \beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right] \qquad (9)$$

Equation (9) provides an expression for expected return on asset i in terms of the beta for asset i relative to a proxy and the expected excess return on the *same* proxy, i.e. in terms of what we actually estimate, β_i^{pr} and $E\left[r_{pr}^e\right]$. From (9) two conclusions can be drawn. One, using the beta estimate from a proxy times the expected excess return on the proxy $\left(\beta_i^{pr} E\left[r_{pr}^e\right]\right)$ yields a biased estimate for the cost of equity. This is because doing so ignores the adjustment factor $\frac{1}{\rho_{pr,mw}^2}$. Two, the same proxy must be used for estimation of beta and the expected market risk premium[7]. Thus the common procedure of estimating beta against an index and using six to eight percent as an estimate for the expected market risk premium leads to a biased estimate for cost of equity.

To obtain an unbiased estimate of the cost of equity it is therefore necessary to obtain an estimate for the right hand side of (9). This includes the correlation between the proxy and the unobservable market portfolio - so we are back where we started? Well, not quite. The estimation procedure suggested by Fama and MacBeth [1974] provides a natural estimate for $\beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$. In particular, the procedure provides an estimate for $\frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$ so it is not necessary to estimate

[7] From above algebra, if two different proxies are used then we need an estimate for the beta for each proxy relative to the world market portfolio (!).

$\frac{1}{\rho^2_{pr,mw}}$ separately. Fama and McBeth's procedure consists of two steps. First, beta is estimated using a proxy in the following time-series regression:

$$r_{it} - r_{fr} = \alpha_i + \beta_i^{pr}(r_{prt} - r_{ft}) + \varepsilon_{it}, t = 1,..,T, i = 1,..,N \qquad (10)$$

This estimation uses five years of monthly data, for example 1970 to 1974 inclusive, one regression for each stock in the sample (N). Second, each of the estimated betas from the first step are used in a cross section regression of the following form using stock returns for the subsequent year, for example 1975:

$$r^a_{it+1} - r^a_{fr+1} = \gamma_{0\,i} + \gamma_1 \beta^{pr}_{it} + \varepsilon_{it}, i = 1,..,N \qquad (11)$$

where the superscript "pr" on beta indicates the fact that a proxy was used to estimate beta in (10). This is done using annual data, i.e. r^a_{it+1} is the annual return for stock i for 1975 and the risk free rate of return is the annual return on the Treasury bill. A simple way to obtain an estimate for γ_0 and γ_1 is to roll the above procedure forward and then take an average of the estimated values of γ_0 and γ_1. Thus γ_1 provides an estimate for $\frac{1}{\rho^2_{pr,mw}}E\left[r^e_{pr}\right]$ which when multiplied by β^{pr}_{it} provides an unbiased estimate for $\beta^{wm}_{i,t+1}E_t\left[r^e_{wm,t+1}\right]$. From the theoretical relationship obtained above (equation (9)) γ_0 should be zero and $\beta^{pr}_i \frac{1}{\rho^2_{pr,mw}}E\left[r^e_{pr}\right]$ provides an unbiased estimate for expected return. This procedure is implemented in the next section.

III. Estimating expected return using CAPM.

From Section II, first equation (10) is used to estimate a beta for each stock and then the betas obtained from estimation of (10) are used in the cross-section regression (11) to obtain an unbiased estimate for expected return. This procedure was undertaken for each of the indexes discussed in section I. The estimated slope coefficients are provided in Table IV.

Place Table IV here please.

The shaded area indicates significance; they are in general significant for each year. From section II the average of these coefficients provides an estimate for the adjusted risk premium, the expected return on the proxy times the adjustment coefficient, $\frac{1}{\rho^2_{pr,mw}}E\left[r^e_{pr}\right]$, (provided the distribution is stable over the period). The lowest adjusted risk premium (3%) is obtained using Standard and Poor's

Index whereas the CRSP equal weighted index gives the highest (7%). This is not surprising since from Table III the Standard and Poor's Index had the second highest average beta of 1.1 whereas the average beta for the CRSP equal weighted index was 0.9.

Place Table V here please.

Table V reports the estimates for the intercept terms which, according to CAPM and equation (10) should all be zero; clearly they are not. There are several possible reasons for this. The first is that CAPM is not valid. There is an extensive academic literature supporting this view.[8] If there are more risk factors than the market index then the inclusion of an intercept term captures the average of these missing factors. This is, of course, a brute force method. A more accurate procedure would be to estimate a multifactor model. The second possible reason is that CAPM is correct but the intercept term captures drifts in beta. In the above procedure the beta used in the cross section regression is from the previous period and Blume [1975], for example, argued that there is mean reversion in beta; that is, a high beta is followed by a smaller beta. Depending on this adjustment process inclusion of an intercept term may capture part of this movement.[9] Again under this interpretation an intercept term should be included in the calculation of the expected return on an asset.

Finally, from Table IV and Table V, estimates for the cost of equity based on the four indexes examined are as follows:

Standard and Poor's:

$$\hat{r}_i = r_{ft} + 4.30 + \beta_i^{sp} \times 3.75$$

CRSP value weighted index:

$$\hat{r}_i = r_{ft} + 3.36 + \beta_i^{vw} \times 4.44$$

CRSP equal weighted index:

$$\hat{r}_i = r_{ft} + 2.22 + \beta_i^{ew} \times 7.03$$

Morgan Stanley:

$$\hat{r}_i = r_{ft} + 4.43 + \beta_i^{ms} \times 4.01$$

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[8] The Arbitrage Pricing Theory by Ross [1976] provides an example of a theoretical model and Fama and French [1992] an empirical example of a three factor model.

[9] Notice that γ_1 captures the dynamic process suggested by Blume [1975]; that is, an additional benefit of this procedure is that Blume's adjustment is implicit in the procedure.

The four indexes provide an estimate for the expected market risk premium $\left(\gamma_0 + \overline{\beta^{pr}} \times \gamma_1\right)$ of around 8.4 percent, where $\overline{\beta^{pr}}$ is the average beta from Table II. Thus the four proxies provide more or less the same estimate for the expected market risk premium when the appropriate adjustment is made. Further for an individual stock, provided the same proxy is used for beta and the expected market risk premium and the appropriate adjustment is made to the estimate for the market risk the four proxies yield the same estimate for cost of equity.

Finally, the R^2 from the cross section regressions are reported in Table VI to determine how much of the variation in actual returns can be explained by differences in beta. The results are not encouraging. From Table VI, betas calculated using the Standard and Poor's Index can, on average, explain 1.6% of the variation in stock returns. The CRSP equal weighted index fares a bit better; it can explain nearly 3%. Based on this it is surprising that CAPM is so widely used.

Place Table VI here please.

IV. Conclusion.

This paper has shown that the standard procedure of using independent estimates of beta and the expected market risk premium to obtain an estimate for cost of equity based on CAPM most likely yields a biased estimate. This is undesirable since it leads to misallocation of funds and biased performance measures. It is shown in this paper that to obtain an unbiased estimate for cost of equity, while any reasonable proxy may be used for estimation of beta, the same proxy must be used for estimating the expected market risk premium. Further, the estimate for the expected market risk premium must be adjusted to take into account the relationship between the proxy and the market portfolio. A two step procedure was suggested which does this. If a beta is obtained from a beta provider then the relevant adjusted risk premium should also be obtained because without this it is not possible to obtain an unbiased estimate for cost of equity based on the beta provided. From the analysis done here it would not be difficult for beta providers to make the relevant adjusted risk premium available.

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Appendix

In the text it is stated that it is possible to show:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr}\frac{1}{\rho_{wm,pr}^2} \qquad (8)$$

where the superscript "wm" refers to the world market portfolio and "pr" to the proxy portfolio. To derive this expression start with the definition of beta for asset i relative to the proxy pr:

$$\beta_i^{pr} = \frac{Cov(r_i, r_{pr})}{\sigma_{pr}^2} \qquad (A1)$$

From CAPM the actual return on the proxy portfolio is given by:

$$r_{pr} = r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr} \qquad (A2)$$

where ε_{pr} is a random error term with an expected value of zero. If the proxy portfolio lies on the efficient set then ε_{pr} is zero. Given that the proxy portfolio contains less assets than the world market portfolio this is unlikely to be the case. However the proxy is, in general, well diversified so it is not unreasonable to assume that ε_{pr} is small and uncorrelated with the return on individual stocks.

Substituting (A1) into (A2), and recognising that the risk free rate is a constant gives:

$$\beta_i^{pr} = \frac{Cov(r_i, r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm} + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} + \frac{Cov(r_i, \varepsilon_{pr})}{\sigma_{pr}^2}$$

Assuming that the return on asset "i" is uncorrelated with a shock to the proxy portfolio the last term is zero, so we have:

$$\beta_i^{pr} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} = \frac{\beta_{pr}^{wm} Cov(r_i, r_{wm})\sigma_{wm}^2}{\sigma_{pr}^2 \sigma_{wm}^2} = \frac{\beta_{pr}^{wm}\beta_i^{wm}\sigma_{wm}^2}{\sigma_{pr}^2}$$

$$\Leftrightarrow \frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \frac{\sigma_{pr}^2 \beta_i^{pr}}{\sigma_{wm}^2 (\beta_{pr}^{wm})^2}$$

Finally, using the definition of beta for the proxy portfolio:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr}\frac{\sigma_{pr}^2}{\sigma_{wm}^2}\frac{(\sigma_{wm}^2)^2}{Cov(r_{pr}, r_{wm})^2} = \beta_i^{pr}\frac{1}{\rho_{wm,pr}^2}$$

Table I.

Summary statistics

Summary statistics are for monthly returns on the indicated indexes expressed as percentages. Standard and Poor refers to the Standard and Poor's Composite Index and Morgan Stanley refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

Index	Number of observations	Mean	Standard deviation	Minimum	Maximum
Standard and Poor	323	0.7626	4.35781	-21.76304	16.30469
Morgan Stanley	323	0.74927	4.08246	-17.12423	14.26563
Value weighted CRSP index	323	1.06192	4.49597	-22.50055	16.57984
Equal weighted CRSP index	323	1.84503	5.70234	-25.08664	30.28762

Table II
Correlation coefficients between the indexes

The Pearson correlation coefficients are calculated based on monthly returns for each of the indexes. Standard and Poor (and S & P) refers to the Standard and Poor's Composite Index and Morgan Stanley (and MSCI) refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

	S & P	MSCI	CRSP value weighted	CRSP equal weighted
Standard and Poor	1	0.826	0.9883	0.785
Morgan Stanley		1	0.8216	0.6853
Value weighted CRSP index			1	0.8485
Equal weighted CRSP index				1

Table III

Summary statistics for beta estimates.

For each index, an individual stock beta is estimated using the following model:

$$r_{it} - r_{fr} = \alpha_i + \beta_i (r_{It} - r_{ft}) + \varepsilon_{it}, t = 1, .., T, i = 1, .., N \quad (10)$$

where r_{it} is the monthly return (including dividends) for stock i, r_{fr} is the monthly return on three month Treasury bills and r_{iT} is the monthly return on the index. The first estimation period is 1970 to 1974 inclusive, the second is 1971 to 1975 inclusive, and so forth up to and including 1996.

Index	Number	Average beta	Standard deviation	Minimum	Maximum
Standard and Poors	22,905	1.0982	0.4717	-1.9468	3.4652
Morgan Stanley	22,905	0.9765	0.5133	-1.5106	3.8294
CRSP value weighted	22,905	1.1173	0.4833	-1.3448	3.7249
CRSP equal weighted	22,905	0.922	0.4717	-0.4082	7.26

Table IV
Estimates of slope coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta^{pr}_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.1786	0.2315	0.4357	0.2597
1976	0.0822	0.1079	0.2172	0.0998
1977	0.0664	0.077	0.1052	0.0682
1978	0.1429	0.1563	0.1938	0.1432
1979	0.1845	0.1996	0.2019	0.1324
1980	0.1339	0.1408	0.1275	0.0808
1981	-0.0957	-0.093	-0.0654	-0.0723
1982	-0.0033	0.0056	0.0628	0.0129
1983	0.088	0.1017	0.1713	0.0859
1984	-0.1439	-0.1533	-0.1899	-0.1412
1985	-0.0788	-0.0772	-0.0721	-0.0763
1986	-0.106	-0.1159	-0.1473	-0.1245
1987	0.0104	0.0032	0.0056	0.0536
1988	0.0264	0.028	0.0284	-0.0123
1989	-0.0867	-0.1003	-0.1316	-0.1157
1990	-0.0795	-0.0916	-0.1346	-0.0686
1991	0.2516	0.263	0.2973	0.2516
1992	0.1425	0.1469	0.1807	0.17
1993	-0.0388	-0.0106	0.104	-0.0644
1994	0.0421	0.0412	0.0314	0.0055
1995	0.0674	0.0676	0.0731	0.1342
1996	0.0428	0.0444	0.0526	0.061
Mean	0.0375	0.0442	0.0703	0.0401

Table V.
Estimates of Intercept coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.2727	0.2125	0.0899	0.1445
1976	0.2121	0.1802	0.1161	0.1854
1977	-0.1283	-0.1422	-0.1426	-0.1349
1978	-0.1957	-0.2147	-0.2011	-0.211
1979	-0.0591	-0.0814	-0.0346	-0.0159
1980	-0.0858	-0.0938	-0.0277	-0.0163
1981	-0.0056	-0.0125	-0.0615	-0.0271
1982	0.1766	0.1664	0.115	0.1575
1983	0.0769	0.0662	0.019	0.0811
1984	0.0355	0.0397	0.0428	0.0347
1985	0.267	0.2629	0.2452	0.2629
1986	0.1439	0.1539	0.164	0.1545
1987	-0.1257	-0.1183	-0.1202	-0.1629
1988	0.0563	0.0541	0.0568	0.0976
1989	0.16	0.1766	0.203	0.1832
1990	-0.1785	-0.1622	-0.1207	-0.1981
1991	0.0876	0.0651	0.0269	0.1598
1992	-0.0151	-0.0253	-0.0478	-0.0014
1993	0.169	0.1414	0.0416	0.1659
1994	-0.126	-0.128	-0.1089	-0.0863
1995	0.148	0.1447	0.1589	0.1388
1996	0.0613	0.0556	0.0626	0.0627
Mean	0.043	0.0336	0.0216	0.0443

Table VI.
Mean R-squared for c ross section regressions

The R-squared is obtained from the following regression:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.0194	0.0325	0.0983	0.048
1976	0.0105	0.0188	0.0668	0.0179
1977	0.0125	0.0177	0.0315	0.0156
1978	0.0337	0.0429	0.0637	0.0434
1979	0.0201	0.0252	0.0275	0.0141
1980	0.0281	0.0323	0.0201	0.012
1981	0.0284	0.0256	0.0098	0.018
1982	0	0	0.003	0.0001
1983	0.0123	0.0151	0.0339	0.0112
1984	0.0806	0.0856	0.101	0.0752
1985	0.0125	0.0113	0.008	0.011
1986	0.0295	0.0352	0.0514	0.0367
1987	0.0001	0	0	0.0049
1988	0.0008	0.0009	0.001	0.0002
1989	0.0072	0.01	0.0179	0.0149
1990	0.0091	0.013	0.0313	0.0069
1991	0.0173	0.0205	0.0293	0.0119
1992	0.0151	0.0171	0.0255	0.0153
1993	0.0019	0.0001	0.0177	0.0023
1994	0.005	0.0054	0.0031	0
1995	0.0035	0.0038	0.0041	0.0055
1996	0.0045	0.0054	0.0073	0.0053
Mean	0.016	0.019	**0.0296**	0.0168

Estimating Cost of Equity

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Estimation of cost of equity is required for many financial applications such as capital budgeting and performance evaluation using EVA©. A common procedure is to use the Capital Asset Pricing Model [CAPM] which involves estimation of an expected risk premium equal to beta times the expected risk premium on the "market" portfolio, the portfolio containing all assets in the world. Since the market portfolio is not observable a proxy must be used. Typically beta is estimated using a domestic index as a proxy or is purchased from a commercial provider and six to eight percent is used as an estimate for the expected market risk premium. The estimates for beta and the expected market risk premium are then multiplied together. In this paper it is shown that this procedure more than likely yields a biased estimate for the cost of equity and as a result leads to misallocation of funds and biased performance measures. It is shown theoretically that the same index must be used for estimation of beta and the expected market risk premium. Further the estimate for the market risk premium must be adjusted to take into account the correlation between the proxy used and the market portfolio. Finally, a straight forward procedure for obtaining an unbiased estimate for cost of equity is presented.

Thursday, November 30, 2000

Introduction

Estimating the cost of equity is crucial for many financial decisions such as, capital budgeting, capital structure, and performance evaluation using EVA©. In a recent survey Bruner et al. [1998] found that the most common method favoured by practitioners for doing this is the Capital Asset Pricing Model [CAPM]. According to CAPM the expected return on any asset is equal to the risk free rate plus a risk premium. The risk premium is equal to the expected systematic risk of the asset relative to the "market" portfolio (beta) times the expected risk premium on the "market" portfolio. Since the market portfolio consists of all assets in the world and it is not observable a proxy must be used in order to implement CAPM. Typically, beta is estimated using a domestic index as a proxy and an estimate of between 6% and 8% is used for the expected market risk premium since this represents the average excess return on a portfolio of stocks over the risk free rate since 1925[1]. These two values are then multiplied together to obtain an estimate for cost of equity. In this paper it is shown that this more than likely yields a biased estimate. Since biased estimates result in misallocation of funds and biased performance measures it is important to obtain unbiased estimates. For this purpose, first, the theoretical relationship between the index used as a proxy and expected return on an asset, when CAPM is used, is obtained. Second, a simple procedure for obtaining estimates which incorporate this relationship, and are therefore unbiased, is presented.

In theory, the prescription provided by CAPM for obtaining the expected return on an asset is very clear: expected return is equal to the risk free rate plus a risk premium equal to the beta for the asset, relative to the portfolio, times the expected market risk premium. The beta for the asset is equal to the covariance, over the period of interest, between the return on the asset and that on the market portfolio divided by the variance of the market portfolio. In practice, the problem is that the market portfolio is not observable so to implement CAPM it is necessary to use a proxy. This raises (at least) two questions in relation to using CAPM to obtain an unbiased estimate for the expected return on an asset: (1) Does it matter what proxy is used for estimating beta and the expected market risk premium? (2) Should the same proxy be used for estimating beta and the expected market risk premium? These questions are addressed in this paper. To obtain an unbiased estimate for the expected return on an asset, in particular for the cost of equity, the short answer to question one is no and to question two yes. Specifically, it is shown that (a) the estimate for the expected market risk premium obtained using a proxy must be adjusted to take into account the correlation between the proxy and the market portfolio and (b) the same proxy must be used to estimate beta. It is also shown that although the market portfolio cannot be observed it is possible to obtain an estimate for

[1] For a discussion of these topics see any standard textbook in Finance or Chapter 12 in Stewart [1990].

the adjusted market risk premium using a reasonably simple procedure. Finally, from the results obtained, any reasonable proxy can be used for estimation.

The rest of the paper proceeds in the following way. In the next section, section I, the implications of using different indexes for estimating beta and the expected market risk premium are examined. The data used for this purpose, and in subsequent analysis, is also described in this section. In section II an estimation procedure for obtaining an unbiased estimate for expected return using CAPM is presented. Implementation of the procedure is discussed in section III. Section IV concludes.

I. Estimation using different proxies.

In this section the implications of using different proxies for estimating beta and the expected market risk premium are examined. For this purpose four indexes (potential proxies) are considered: the Standard and Poor's Composite Index, probably the most popular index, the Morgan Stanley Capital World Index, since it appears to be the most comprehensive and therefore potentially the closest to the world market index, and two CRSP indexes (value and equal weighted) representing broad based domestic indexes containing, among others, NASDAQ and NYSE stocks[2]. The descriptive statistics for the monthly returns, for each of the indexes, are reported in Table I.

Place Table I here please.

The CRSP equal weighted index has the largest average return (1.84503%) and the Morgan Stanley Index has the lowest average monthly return (0.74927%). The average difference is over one percent per month. Part of this of course is due to the fact that dividends are included in the CRSP equal weighted index and not in the Morgan Stanley Index. Also with an equal weighted index, smaller stocks have a relatively larger weight and these stocks may have a larger return than large stocks, over parts the period examined, leading to higher returns for equal weighted indexes compared to value weighted indexes[3]. In relation to obtaining an estimate for the market risk premium, therefore, from Table I there can be up to one percent difference in monthly return depending on the index

[2] The CRSP indexes are obtained from the CRSP tapes and Standard and Poor's Composite Index and the Morgan Stanleys Capital World are retrieved from Datastream. Both CRSP indexes include dividends. The aim of this paper is not to analyse a large number of different indexes but to illustrate a set of problems and a general solution procedure for any proxy chosen. Thus only a small number of indexes are examined.
[3] This effect is also referred to as the small firm effect, see for example Banz [1981].

used. The common practice of using between six and eight percent is possibly in recognition of this[4].

For estimation of beta, for any asset, what is important is the variation in the index and covariation between the return on the index and the return on the asset. Table II shows the correlation coefficients between the indexes; these range from 0.6853 to 0.9883. The highest correlation is between the CRSP value weighted index and the Standard and Poor's Index (the return on the CRSP index without dividends is about the same as Standard and Poor's Index) so these two indexes are very similar. The lowest correlation coefficient is between the CRSP equal weighted index and the Morgan Stanley index. Thus one would expect Standard and Poor's index and the CRSP value weighted index to yield similar betas and the Morgan Stanley Index and the equal weighted index to yield very different betas. This suggests that different proxies yield different beta values for the same asset.

Place Table II here please.

To determine whether or not the different indexes do in fact yield significantly different beta estimates, individual betas were estimated for stocks on the CRSP tape, using five years of monthly data[5]. For this purpose monthly dividend adjusted returns were retrieved from the CRSP tape from 1970 to the end of 1996. To avoid thin trading problems only stocks from the NYSE that were traded more than 95% of the days were included in the sample. The sample size ranges from a low of 780 in 1975 to a high of 1308 in 1994.

To obtain beta estimates for individual stocks, starting in 1970, the following equation was estimated for each stock:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{pr,t} - r_{ft}) + \varepsilon_t$$

where r_{it} is the return on the stock, $r_{pr,t}$ the return on the index (proxy), and r_{ft} the monthly return on three month Treasury Bills. For each stock over the period 1970 to 1974 inclusive, a beta was estimated for each of the four indexes. The procedure was repeated for 1971 to 1975 inclusive, and so forth, producing a total of 22,905 estimates of beta for each index. The results from this exercise are reported in Table III. From Table III the CRSP equal weighted index has the smallest average

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[4]Most of the debate surrounding estimation of the market risk premium, to date, has centred on whether an arithmetic or geometric mean should be used for calculating the return on the index and not on whether the index provides an unbiased estimate for the market risk premium.

[5]Five years of monthly data was used because this is standard procedure. The purpose of this paper is to show the implications of using different proxies for estimating cost of equity in terms of biasedness. Efficiency is dealt with in another paper.

beta (0.922) whereas the CRSP value weighted index has the largest average beta (1.2). With a sample size of 22905 one would not expect such large differences in beta estimates if they were all unbiased estimates of the true market beta[6]. In summary, therefore, using five years of monthly data different proxies yield significantly different beta estimates. What about betas provided by commercial beta providers? Here the situation is much the same, Bruner et al. found for their (smaller) sample an average beta according to Bloomberg of 1.03 whereas according to Value Line it was 1.24.

Thus obtaining an estimate for cost of equity by multiplying an estimate for beta by an estimate for the expected market risk premium using different proxies clearly results in very different estimates, all of which cannot be unbiased. This is undesirable because if such estimates are used for capital budgeting decisions and performance measurement it can lead to misallocation of funds and biased performance measures. For example using the CRSP equal weighted index for beta and the average return on the Standard and Poor's Index for the expected market risk premium gives a relatively low value for the cost of equity and therefore a relatively high value for EVA©. In particular, if the same estimate is used for the expected market risk premium then the betas obtained using different proxies will yield very different estimates for the cost of equity. For example using 8% for the expected market risk premium, the difference for the resulting estimate for the cost of equity between the Standard and Poor's Index and the CRSP equal weighted index is 1.6% on an annual basis (for a stock with a beta of one!).

The trick to solving the problem of obtaining an unbiased estimate for the cost of equity is to recognise that what is important is that the overall estimate be unbiased; the properties of the individual elements are irrelevant. In the next section, it is shown that, in contrast to the common procedure of using independent estimates for the expected market risk premium return on the market and beta, to obtain an unbiased estimate of the cost of equity, the expected market risk premium and beta must be estimated using the same proxy. Further it is shown that the estimate for the expected market risk premium must be adjusted to take into account the relation between the proxy and the market portfolio. Finally, from the results obtained any reasonable proxy will do. Thus the answer to question (1) in the introduction is no and to question (2) is yes.

[6] Due to the overlapping estimation period the 22905 beta estimates are not independent of each other.

II. Obtaining an unbiased estimate for the cost of equity.

From section I the objective is to obtain an unbiased estimate for the cost of equity, or more generally an asset, using CAPM. That is, we want the actual return on the asset to differ from the expected return by at most a random error term:

$$r^e_{i,t+1} = E_t\left[r^e_{i,t+1}\right] + \varepsilon_{i,t+1} \qquad (1)$$

where r^e_i is excess return for asset i, the return on asset i minus the risk free rate, and $\varepsilon_{i,t+1}$ is a white noise error term. For this purpose, first the theoretical relationship between the expected return on an asset and the proxy portfolio, used in place of the market portfolio, implied by CAPM is analysed. Second an estimation procedure leading to an unbiased estimate is presented.

From CAPM the expected return on asset i is given by:

$$E_t[r^e_{i,t+1}] = \beta^{wm}_{i,t+1} E_t\left[r^e_{wm,t+1}\right] \qquad (2)$$

where $E_t[r^e_{i,t+1}]$ is the expected excess return for asset or portfolio i, $E_t\left[r^e_{wm,t+1}\right]$ is the expected excess return on the market, and $\beta^{wm}_{i,t+1}$ is systematic risk for asset i relative to the market portfolio. All values are for period t+1 and expectations are taken in period t. For the theoretical analysis the time subscripts are dropped to reduce notation. From the introduction the problem associated with estimating expected return using CAPM is that the market portfolio cannot be observed so it is necessary to use a proxy. From section I different proxies generate different estimates for cost of equity. They cannot all be unbiased and since the market portfolio cannot be observed it is not possible to determine which one is unbiased. The question, therefore, is how to obtain an unbiased estimate using a proxy instead of the market portfolio. For this purpose the theoretical relationship implied by CAPM between expected return on an asset, the expected excess return on the proxy, and beta relative to the proxy is derived. That is, an expression for the expected return on an asset in terms of what is actually estimated, beta relative to the proxy and the expected excess return on the proxy, is obtained.

From CAPM, equation (2), the relationship between the return on the proxy portfolio and the market portfolio is given by:

$$E[r^e_{pr}] = \beta^{wm}_{pr} E[r^e_{wm}] \qquad (3).$$

Using (3) to substitute for $E[r^e_{wm}]$ in terms of the proxy in equation (2) yields:

$$E[r^e_i] = \frac{\beta^{wm}_i}{\beta^{wm}_{pr}} E\left[r^e_{pr}\right] \qquad (4)$$

Equation (4) gives the expected excess return on asset i in terms of the expected excess return on the proxy. Since a proxy is also used for estimation of beta this needs to be incorporated into the right hand side of (4).

Now, by definition, for any asset i (or portfolio) and any proxy portfolio x, beta for the asset (portfolio) relative to the proxy is given by:

$$\beta_i^x = \frac{Cov(r_i, r_x)}{\sigma_x^2} \qquad (5)$$

In particular, for asset i relative to the market portfolio:

$$\beta_i^{wm} = \frac{Cov(r_i, r_{wm})}{\sigma_{wm}^2} \qquad (6)$$

and for the proxy portfolio relative to the market portfolio:

$$\beta_{pr}^{wm} = \frac{Cov(r_{pr}, r_{wm})}{\sigma_{wm}^2} \qquad (7)$$

Making use of (6) and (7) it is straight forward to show (see appendix):

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2}. \qquad (8)$$

Substituting (8) into (4) gives:

$$E[r_i^e] = \beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right] \qquad (9)$$

Equation (9) provides an expression for expected return on asset i in terms of the beta for asset i relative to a proxy and the expected excess return on the *same* proxy, i.e. in terms of what we actually estimate, β_i^{pr} and $E\left[r_{pr}^e\right]$. From (9) two conclusions can be drawn. One, using the beta estimate from a proxy times the expected excess return on the proxy $\left(\beta_i^{pr} E\left[r_{pr}^e\right]\right)$ yields a biased estimate for the cost of equity. This is because doing so ignores the adjustment factor $\frac{1}{\rho_{pr,mw}^2}$. Two, the same proxy must be used for estimation of beta and the expected market risk premium[7]. Thus the common procedure of estimating beta against an index and using six to eight percent as an estimate for the expected market risk premium leads to a biased estimate for cost of equity.

To obtain an unbiased estimate of the cost of equity it is therefore necessary to obtain an estimate for the right hand side of (9). This includes the correlation between the proxy and the unobservable market portfolio - so we are back where we started? Well, not quite. The estimation procedure suggested by Fama and MacBeth [1974] provides a natural estimate for $\beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$. In particular, the procedure provides an estimate for $\frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$ so it is not necessary to estimate

[7] From above algebra, if two different proxies are used then we need an estimate for the beta for each proxy relative to the world market portfolio (!).

$\frac{1}{\rho^2_{pr,mw}}$ separately. Fama and McBeth's procedure consists of two steps. First, beta is estimated using a proxy in the following time-series regression:

$$r_{it} - r_{fr} = \alpha_i + \beta_i^{pr}(r_{prt} - r_{ft}) + \varepsilon_{it}, t = 1,..,T, i = 1,..,N \qquad (10)$$

This estimation uses five years of monthly data, for example 1970 to 1974 inclusive, one regression for each stock in the sample (N). Second, each of the estimated betas from the first step are used in a cross section regression of the following form using stock returns for the subsequent year, for example 1975:

$$r^a_{it+1} - r^a_{fr+1} = \gamma_{0\,i} + \gamma_1 \beta^{pr}_{it} + \varepsilon_{it}, i = 1,..,N \qquad (11)$$

where the superscript "pr" on beta indicates the fact that a proxy was used to estimate beta in (10). This is done using annual data, i.e. r^a_{it+1} is the annual return for stock i for 1975 and the risk free rate of return is the annual return on the Treasury bill. A simple way to obtain an estimate for γ_0 and γ_1 is to roll the above procedure forward and then take an average of the estimated values of γ_0 and γ_1. Thus γ_1 provides an estimate for $\frac{1}{\rho^2_{pr,mw}} E\left[r^e_{pr}\right]$ which when multiplied by β^{pr}_{it} provides an unbiased estimate for $\beta^{wm}_{i,t+1} E_t\left[r^e_{wm,t+1}\right]$. From the theoretical relationship obtained above (equation (9)) γ_0 should be zero and $\beta^{pr}_i \frac{1}{\rho^2_{pr,mw}} E\left[r^e_{pr}\right]$ provides an unbiased estimate for expected return. This procedure is implemented in the next section.

III. Estimating expected return using CAPM.

From Section II, first equation (10) is used to estimate a beta for each stock and then the betas obtained from estimation of (10) are used in the cross-section regression (11) to obtain an unbiased estimate for expected return. This procedure was undertaken for each of the indexes discussed in section I. The estimated slope coefficients are provided in Table IV.

Place Table IV here please.

The shaded area indicates significance; they are in general significant for each year. From section II the average of these coefficients provides an estimate for the adjusted risk premium, the expected return on the proxy times the adjustment coefficient, $\frac{1}{\rho^2_{pr,mw}} E\left[r^e_{pr}\right]$, (provided the distribution is stable over the period). The lowest adjusted risk premium (3%) is obtained using Standard and Poor's

Index whereas the CRSP equal weighted index gives the highest (7%). This is not surprising since from Table III the Standard and Poor's Index had the second highest average beta of 1.1 whereas the average beta for the CRSP equal weighted index was 0.9.

Place Table V here please.

Table V reports the estimates for the intercept terms which, according to CAPM and equation (10) should all be zero; clearly they are not. There are several possible reasons for this. The first is that CAPM is not valid. There is an extensive academic literature supporting this view.[8] If there are more risk factors than the market index then the inclusion of an intercept term captures the average of these missing factors. This is, of course, a brute force method. A more accurate procedure would be to estimate a multifactor model. The second possible reason is that CAPM is correct but the intercept term captures drifts in beta. In the above procedure the beta used in the cross section regression is from the previous period and Blume [1975], for example, argued that there is mean reversion in beta; that is, a high beta is followed by a smaller beta. Depending on this adjustment process inclusion of an intercept term may capture part of this movement.[9] Again under this interpretation an intercept term should be included in the calculation of the expected return on an asset.

Finally, from Table IV and Table V, estimates for the cost of equity based on the four indexes examined are as follows:

Standard and Poor's:

$$\widehat{r}_i = r_{ft} + 4.30 + \beta_i^{sp} \times 3.75$$

CRSP value weighted index:

$$\widehat{r}_i = r_{ft} + 3.36 + \beta_i^{vw} \times 4.44$$

CRSP equal weighted index:

$$\widehat{r}_i = r_{ft} + 2.22 + \beta_i^{ew} \times 7.03$$

Morgan Stanley:

$$\widehat{r}_i = r_{ft} + 4.43 + \beta_i^{ms} \times 4.01$$

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[8] The Arbitrage Pricing Theory by Ross [1976] provides an example of a theoretical model and Fama and French [1992] an empirical example of a three factor model.

[9] Notice that γ_1 captures the dynamic process suggested by Blume [1975]; that is, an additional benefit of this procedure is that Blume's adjustment is implicit in the procedure.

The four indexes provide an estimate for the expected market risk premium $\left(\gamma_0 + \overline{\beta^{pr}} \times \gamma_1\right)$ of around 8.4 percent, where $\overline{\beta^{pr}}$ is the average beta from Table II. Thus the four proxies provide more or less the same estimate for the expected market risk premium when the appropriate adjustment is made. Further for an individual stock, provided the same proxy is used for beta and the expected market risk premium and the appropriate adjustment is made to the estimate for the market risk the four proxies yield the same estimate for cost of equity.

Finally, the R^2 from the cross section regressions are reported in Table VI to determine how much of the variation in actual returns can be explained by differences in beta. The results are not encouraging. From Table VI, betas calculated using the Standard and Poor's Index can, on average, explain 1.6% of the variation in stock returns. The CRSP equal weighted index fares a bit better; it can explain nearly 3%. Based on this it is surprising that CAPM is so widely used.

Place Table VI here please.

IV. Conclusion.

This paper has shown that the standard procedure of using independent estimates of beta and the expected market risk premium to obtain an estimate for cost of equity based on CAPM most likely yields a biased estimate. This is undesirable since it leads to misallocation of funds and biased performance measures. It is shown in this paper that to obtain an unbiased estimate for cost of equity, while any reasonable proxy may be used for estimation of beta, the same proxy must be used for estimating the expected market risk premium. Further, the estimate for the expected market risk premium must be adjusted to take into account the relationship between the proxy and the market portfolio. A two step procedure was suggested which does this. If a beta is obtained from a beta provider then the relevant adjusted risk premium should also be obtained because without this it is not possible to obtain an unbiased estimate for cost of equity based on the beta provided. From the analysis done here it would not be difficult for beta providers to make the relevant adjusted risk premium available.

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Appendix

In the text it is stated that it is possible to show:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr}\frac{1}{\rho_{wm,pr}^2} \qquad (8)$$

where the superscript "wm" refers to the world market portfolio and "pr" to the proxy portfolio. To derive this expression start with the definition of beta for asset i relative to the proxy pr:

$$\beta_i^{pr} = \frac{Cov(r_i, r_{pr})}{\sigma_{pr}^2} \qquad (A1)$$

From CAPM the actual return on the proxy portfolio is given by:

$$r_{pr} = r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr} \qquad (A2)$$

where ε_{pr} is a random error term with an expected value of zero. If the proxy portfolio lies on the efficient set then ε_{pr} is zero. Given that the proxy portfolio contains less assets than the world market portfolio this is unlikely to be the case. However the proxy is, in general, well diversified so it is not unreasonable to assume that ε_{pr} is small and uncorrelated with the return on individual stocks.

Substituting (A1) into (A2), and recognising that the risk free rate is a constant gives:

$$\beta_i^{pr} = \frac{Cov(r_i, r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm} + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} + \frac{Cov(r_i, \varepsilon_{pr})}{\sigma_{pr}^2}$$

Assuming that the return on asset "i" is uncorrelated with a shock to the proxy portfolio the last term is zero, so we have:

$$\beta_i^{pr} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} = \frac{\beta_{pr}^{wm} Cov(r_i, r_{wm})\sigma_{wm}^2}{\sigma_{pr}^2 \sigma_{wm}^2} = \frac{\beta_{pr}^{wm}\beta_i^{wm}\sigma_{wm}^2}{\sigma_{pr}^2}$$

$$\Leftrightarrow \frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \frac{\sigma_{pr}^2 \beta_i^{pr}}{\sigma_{wm}^2 (\beta_{pr}^{wm})^2}$$

Finally, using the definition of beta for the proxy portfolio:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr}\frac{\sigma_{pr}^2}{\sigma_{wm}^2}\frac{(\sigma_{wm}^2)^2}{Cov(r_{pr}, r_{wm})^2} = \beta_i^{pr}\frac{1}{\rho_{wm,pr}^2}$$

Table I.

Summary statistics

Summary statistics are for monthly returns on the indicated indexes expressed as percentages. Standard and Poor refers to the Standard and Poor's Composite Index and Morgan Stanley refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

Index	Number of observations	Mean	Standard deviation	Minimum	Maximum
Standard and Poor	323	0.7626	4.35781	-21.76304	16.30469
Morgan Stanley	323	0.74927	4.08246	-17.12423	14.26563
Value weighted CRSP index	323	1.06192	4.49597	-22.50055	16.57984
Equal weighted CRSP index	323	1.84503	5.70234	-25.08664	30.28762

Table II
Correlation coefficients between the indexes

The Pearson correlation coefficients are calculated based on monthly returns for each of the indexes. Standard and Poor (and S & P) refers to the Standard and Poor's Composite Index and Morgan Stanley (and MSCI) refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

	S & P	MSCI	CRSP value weighted	CRSP equal weighted
Standard and Poor	1	0.826	0.9883	0.785
Morgan Stanley		1	0.8216	0.6853
Value weighted CRSP index			1	0.8485
Equal weighted CRSP index				1

Table III

Summary statistics for beta estimates.

For each index, an individual stock beta is estimated using the following model:

$$r_{it} - r_{fr} = \alpha_i + \beta_i(r_{It} - r_{ft}) + \varepsilon_{it}, t = 1, .., T, i = 1, .., N \quad (10)$$

where r_{it} is the monthly return (including dividends) for stock i, r_{fr} is the monthly return on three month Treasury bills and r_{iT} is the monthly return on the index. The first estimation period is 1970 to 1974 inclusive, the second is 1971 to 1975 inclusive, and so forth up to and including 1996.

Index	Number	Average beta	Standard deviation	Minimum	Maximum
Standard and Poors	22,905	1.0982	0.4717	-1.9468	3.4652
Morgan Stanley	22,905	0.9765	0.5133	-1.5106	3.8294
CRSP value weighted	22,905	1.1173	0.4833	-1.3448	3.7249
CRSP equal weighted	22,905	0.922	0.4717	-0.4082	7.26

Table IV
Estimates of slope coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta^{pr}_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.1786	0.2315	0.4357	0.2597
1976	0.0822	0.1079	0.2172	0.0998
1977	0.0664	0.077	0.1052	0.0682
1978	0.1429	0.1563	0.1938	0.1432
1979	0.1845	0.1996	0.2019	0.1324
1980	0.1339	0.1408	0.1275	0.0808
1981	-0.0957	-0.093	-0.0654	-0.0723
1982	-0.0033	0.0056	0.0628	0.0129
1983	0.088	0.1017	0.1713	0.0859
1984	-0.1439	-0.1533	-0.1899	-0.1412
1985	-0.0788	-0.0772	-0.0721	-0.0763
1986	-0.106	-0.1159	-0.1473	-0.1245
1987	0.0104	0.0032	0.0056	0.0536
1988	0.0264	0.028	0.0284	-0.0123
1989	-0.0867	-0.1003	-0.1316	-0.1157
1990	-0.0795	-0.0916	-0.1346	-0.0686
1991	0.2516	0.263	0.2973	0.2516
1992	0.1425	0.1469	0.1807	0.17
1993	-0.0388	-0.0106	0.104	-0.0644
1994	0.0421	0.0412	0.0314	0.0055
1995	0.0674	0.0676	0.0731	0.1342
1996	0.0428	0.0444	0.0526	0.061
Mean	0.0375	0.0442	0.0703	0.0401

Table V.
Estimates of Intercept coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.2727	0.2125	0.0899	0.1445
1976	0.2121	0.1802	0.1161	0.1854
1977	-0.1283	-0.1422	-0.1426	-0.1349
1978	-0.1957	-0.2147	-0.2011	-0.211
1979	-0.0591	-0.0814	-0.0346	-0.0159
1980	-0.0858	-0.0938	-0.0277	-0.0163
1981	-0.0056	-0.0125	-0.0615	-0.0271
1982	0.1766	0.1664	0.115	0.1575
1983	0.0769	0.0662	0.019	0.0811
1984	0.0355	0.0397	0.0428	0.0347
1985	0.267	0.2629	0.2452	0.2629
1986	0.1439	0.1539	0.164	0.1545
1987	-0.1257	-0.1183	-0.1202	-0.1629
1988	0.0563	0.0541	0.0568	0.0976
1989	0.16	0.1766	0.203	0.1832
1990	-0.1785	-0.1622	-0.1207	-0.1981
1991	0.0876	0.0651	0.0269	0.1598
1992	-0.0151	-0.0253	-0.0478	-0.0014
1993	0.169	0.1414	0.0416	0.1659
1994	-0.126	-0.128	-0.1089	-0.0863
1995	0.148	0.1447	0.1589	0.1388
1996	0.0613	0.0556	0.0626	0.0627
Mean	0.043	0.0336	0.0216	0.0443

Table VI.
Mean R-squared for c ross section regressions

The R-squared is obtained from the following regression:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.0194	0.0325	0.0983	0.048
1976	0.0105	0.0188	0.0668	0.0179
1977	0.0125	0.0177	0.0315	0.0156
1978	0.0337	0.0429	0.0637	0.0434
1979	0.0201	0.0252	0.0275	0.0141
1980	0.0281	0.0323	0.0201	0.012
1981	0.0284	0.0256	0.0098	0.018
1982	0	0	0.003	0.0001
1983	0.0123	0.0151	0.0339	0.0112
1984	0.0806	0.0856	0.101	0.0752
1985	0.0125	0.0113	0.008	0.011
1986	0.0295	0.0352	0.0514	0.0367
1987	0.0001	0	0	0.0049
1988	0.0008	0.0009	0.001	0.0002
1989	0.0072	0.01	0.0179	0.0149
1990	0.0091	0.013	0.0313	0.0069
1991	0.0173	0.0205	0.0293	0.0119
1992	0.0151	0.0171	0.0255	0.0153
1993	0.0019	0.0001	0.0177	0.0023
1994	0.005	0.0054	0.0031	0
1995	0.0035	0.0038	0.0041	0.0055
1996	0.0045	0.0054	0.0073	0.0053
Mean	0.016	0.019	**0.0296**	0.0168

Estimating Cost of Equity

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Estimation of cost of equity is required for many financial applications such as capital budgeting and performance evaluation using EVA©. A common procedure is to use the Capital Asset Pricing Model [CAPM] which involves estimation of an expected risk premium equal to beta times the expected risk premium on the "market" portfolio, the portfolio containing all assets in the world. Since the market portfolio is not observable a proxy must be used. Typically beta is estimated using a domestic index as a proxy or is purchased from a commercial provider and six to eight percent is used as an estimate for the expected market risk premium. The estimates for beta and the expected market risk premium are then multiplied together. In this paper it is shown that this procedure more than likely yields a biased estimate for the cost of equity and as a result leads to misallocation of funds and biased performance measures. It is shown theoretically that the same index must be used for estimation of beta and the expected market risk premium. Further the estimate for the market risk premium must be adjusted to take into account the correlation between the proxy used and the market portfolio. Finally, a straight forward procedure for obtaining an unbiased estimate for cost of equity is presented.

Thursday, November 30, 2000

Introduction

Estimating the cost of equity is crucial for many financial decisions such as, capital budgeting, capital structure, and performance evaluation using EVA©. In a recent survey Bruner et al. [1998] found that the most common method favoured by practitioners for doing this is the Capital Asset Pricing Model [CAPM]. According to CAPM the expected return on any asset is equal to the risk free rate plus a risk premium. The risk premium is equal to the expected systematic risk of the asset relative to the "market" portfolio (beta) times the expected risk premium on the "market" portfolio. Since the market portfolio consists of all assets in the world and it is not observable a proxy must be used in order to implement CAPM. Typically, beta is estimated using a domestic index as a proxy and an estimate of between 6% and 8% is used for the expected market risk premium since this represents the average excess return on a portfolio of stocks over the risk free rate since 1925[1]. These two values are then multiplied together to obtain an estimate for cost of equity. In this paper it is shown that this more than likely yields a biased estimate. Since biased estimates result in misallocation of funds and biased performance measures it is important to obtain unbiased estimates. For this purpose, first, the theoretical relationship between the index used as a proxy and expected return on an asset, when CAPM is used, is obtained. Second, a simple procedure for obtaining estimates which incorporate this relationship, and are therefore unbiased, is presented.

In theory, the prescription provided by CAPM for obtaining the expected return on an asset is very clear: expected return is equal to the risk free rate plus a risk premium equal to the beta for the asset, relative to the portfolio, times the expected market risk premium. The beta for the asset is equal to the covariance, over the period of interest, between the return on the asset and that on the market portfolio divided by the variance of the market portfolio. In practice, the problem is that the market portfolio is not observable so to implement CAPM it is necessary to use a proxy. This raises (at least) two questions in relation to using CAPM to obtain an unbiased estimate for the expected return on an asset: (1) Does it matter what proxy is used for estimating beta and the expected market risk premium? (2) Should the same proxy be used for estimating beta and the expected market risk premium? These questions are addressed in this paper. To obtain an unbiased estimate for the expected return on an asset, in particular for the cost of equity, the short answer to question one is no and to question two yes. Specifically, it is shown that (a) the estimate for the expected market risk premium obtained using a proxy must be adjusted to take into account the correlation between the proxy and the market portfolio and (b) the same proxy must be used to estimate beta. It is also shown that although the market portfolio cannot be observed it is possible to obtain an estimate for

[1] For a discussion of these topics see any standard textbook in Finance or Chapter 12 in Stewart [1990].

the adjusted market risk premium using a reasonably simple procedure. Finally, from the results obtained, any reasonable proxy can be used for estimation.

The rest of the paper proceeds in the following way. In the next section, section I, the implications of using different indexes for estimating beta and the expected market risk premium are examined. The data used for this purpose, and in subsequent analysis, is also described in this section. In section II an estimation procedure for obtaining an unbiased estimate for expected return using CAPM is presented. Implementation of the procedure is discussed in section III. Section IV concludes.

I. Estimation using different proxies.

In this section the implications of using different proxies for estimating beta and the expected market risk premium are examined. For this purpose four indexes (potential proxies) are considered: the Standard and Poor's Composite Index, probably the most popular index, the Morgan Stanley Capital World Index, since it appears to be the most comprehensive and therefore potentially the closest to the world market index, and two CRSP indexes (value and equal weighted) representing broad based domestic indexes containing, among others, NASDAQ and NYSE stocks[2]. The descriptive statistics for the monthly returns, for each of the indexes, are reported in Table I.

Place Table I here please.

The CRSP equal weighted index has the largest average return (1.84503%) and the Morgan Stanley Index has the lowest average monthly return (0.74927%). The average difference is over one percent per month. Part of this of course is due to the fact that dividends are included in the CRSP equal weighted index and not in the Morgan Stanley Index. Also with an equal weighted index, smaller stocks have a relatively larger weight and these stocks may have a larger return than large stocks, over parts the period examined, leading to higher returns for equal weighted indexes compared to value weighted indexes[3]. In relation to obtaining an estimate for the market risk premium, therefore, from Table I there can be up to one percent difference in monthly return depending on the index

[2] The CRSP indexes are obtained from the CRSP tapes and Standard and Poor's Composite Index and the Morgan Stanleys Capital World are retrieved from Datastream. Both CRSP indexes include dividends. The aim of this paper is not to analyse a large number of different indexes but to illustrate a set of problems and a general solution procedure for any proxy chosen. Thus only a small number of indexes are examined.

[3] This effect is also referred to as the small firm effect, see for example Banz [1981].

used. The common practice of using between six and eight percent is possibly in recognition of this[4].

For estimation of beta, for any asset, what is important is the variation in the index and covariation between the return on the index and the return on the asset. Table II shows the correlation coefficients between the indexes; these range from 0.6853 to 0.9883. The highest correlation is between the CRSP value weighted index and the Standard and Poor's Index (the return on the CRSP index without dividends is about the same as Standard and Poor's Index) so these two indexes are very similar. The lowest correlation coefficient is between the CRSP equal weighted index and the Morgan Stanley index. Thus one would expect Standard and Poor's index and the CRSP value weighted index to yield similar betas and the Morgan Stanley Index and the equal weighted index to yield very different betas. This suggests that different proxies yield different beta values for the same asset.

Place Table II here please.

To determine whether or not the different indexes do in fact yield significantly different beta estimates, individual betas were estimated for stocks on the CRSP tape, using five years of monthly data[5]. For this purpose monthly dividend adjusted returns were retrieved from the CRSP tape from 1970 to the end of 1996. To avoid thin trading problems only stocks from the NYSE that were traded more than 95% of the days were included in the sample. The sample size ranges from a low of 780 in 1975 to a high of 1308 in 1994.

To obtain beta estimates for individual stocks, starting in 1970, the following equation was estimated for each stock:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{pr,t} - r_{ft}) + \varepsilon_t$$

where r_{it} is the return on the stock, $r_{pr,t}$ the return on the index (proxy), and r_{ft} the monthly return on three month Treasury Bills. For each stock over the period 1970 to 1974 inclusive, a beta was estimated for each of the four indexes. The procedure was repeated for 1971 to 1975 inclusive, and so forth, producing a total of 22,905 estimates of beta for each index. The results from this exercise are reported in Table III. From Table III the CRSP equal weighted index has the smallest average

[4]Most of the debate surrounding estimation of the market risk premium, to date, has centred on whether an arithmetic or geometric mean should be used for calculating the return on the index and not on whether the index provides an unbiased estimate for the market risk premium.

[5]Five years of monthly data was used because this is standard procedure. The purpose of this paper is to show the implications of using different proxies for estimating cost of equity in terms of biasedness. Efficiency is dealt with in another paper.

beta (0.922) whereas the CRSP value weighted index has the largest average beta (1.2). With a sample size of 22905 one would not expect such large differences in beta estimates if they were all unbiased estimates of the true market beta[6]. In summary, therefore, using five years of monthly data different proxies yield significantly different beta estimates. What about betas provided by commercial beta providers? Here the situation is much the same, Bruner et al. found for their (smaller) sample an average beta according to Bloomberg of 1.03 whereas according to Value Line it was 1.24.

Thus obtaining an estimate for cost of equity by multiplying an estimate for beta by an estimate for the expected market risk premium using different proxies clearly results in very different estimates, all of which cannot be unbiased. This is undesirable because if such estimates are used for capital budgeting decisions and performance measurement it can lead to misallocation of funds and biased performance measures. For example using the CRSP equal weighted index for beta and the average return on the Standard and Poor's Index for the expected market risk premium gives a relatively low value for the cost of equity and therefore a relatively high value for EVA©. In particular, if the same estimate is used for the expected market risk premium then the betas obtained using different proxies will yield very different estimates for the cost of equity. For example using 8% for the expected market risk premium, the difference for the resulting estimate for the cost of equity between the Standard and Poor's Index and the CRSP equal weighted index is 1.6% on an annual basis (for a stock with a beta of one!).

The trick to solving the problem of obtaining an unbiased estimate for the cost of equity is to recognise that what is important is that the overall estimate be unbiased; the properties of the individual elements are irrelevant. In the next section, it is shown that, in contrast to the common procedure of using independent estimates for the expected market risk premium return on the market and beta, to obtain an unbiased estimate of the cost of equity, the expected market risk premium and beta must be estimated using the same proxy. Further it is shown that the estimate for the expected market risk premium must be adjusted to take into account the relation between the proxy and the market portfolio. Finally, from the results obtained any reasonable proxy will do. Thus the answer to question (1) in the introduction is no and to question (2) is yes.

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[6] Due to the overlapping estimation period the 22905 beta estimates are not independent of each other.

II. Obtaining an unbiased estimate for the cost of equity.

From section I the objective is to obtain an unbiased estimate for the cost of equity, or more generally an asset, using CAPM. That is, we want the actual return on the asset to differ from the expected return by at most a random error term:

$$r^e_{i,t+1} = E_t\left[r^e_{i,t+1}\right] + \varepsilon_{i,t+1} \qquad (1)$$

where r^e_i is excess return for asset i, the return on asset i minus the risk free rate, and $\varepsilon_{i,t+1}$ is a white noise error term. For this purpose, first the theoretical relationship between the expected return on an asset and the proxy portfolio, used in place of the market portfolio, implied by CAPM is analysed. Second an estimation procedure leading to an unbiased estimate is presented.

From CAPM the expected return on asset i is given by:

$$E_t[r^e_{i,t+1}] = \beta^{wm}_{i,t+1} E_t\left[r^e_{wm,t+1}\right] \qquad (2)$$

where $E_t[r^e_{i,t+1}]$ is the expected excess return for asset or portfolio i, $E_t\left[r^e_{wm,t+1}\right]$ is the expected excess return on the market, and $\beta^{wm}_{i,t+1}$ is systematic risk for asset i relative to the market portfolio. All values are for period t+1 and expectations are taken in period t. For the theoretical analysis the time subscripts are dropped to reduce notation. From the introduction the problem associated with estimating expected return using CAPM is that the market portfolio cannot be observed so it is necessary to use a proxy. From section I different proxies generate different estimates for cost of equity. They cannot all be unbiased and since the market portfolio cannot be observed it is not possible to determine which one is unbiased. The question, therefore, is how to obtain an unbiased estimate using a proxy instead of the market portfolio. For this purpose the theoretical relationship implied by CAPM between expected return on an asset, the expected excess return on the proxy, and beta relative to the proxy is derived. That is, an expression for the expected return on an asset in terms of what is actually estimated, beta relative to the proxy and the expected excess return on the proxy, is obtained.

From CAPM, equation (2), the relationship between the return on the proxy portfolio and the market portfolio is given by:

$$E[r^e_{pr}] = \beta^{wm}_{pr} E[r^e_{wm}] \qquad (3).$$

Using (3) to substitute for $E[r^e_{wm}]$ in terms of the proxy in equation (2) yields:

$$E[r^e_i] = \frac{\beta^{wm}_i}{\beta^{wm}_{pr}} E\left[r^e_{pr}\right] \qquad (4)$$

Equation (4) gives the expected excess return on asset i in terms of the expected excess return on the proxy. Since a proxy is also used for estimation of beta this needs to be incorporated into the right hand side of (4).

Now, by definition, for any asset i (or portfolio) and any proxy portfolio x, beta for the asset (portfolio) relative to the proxy is given by:

$$\beta_i^x = \frac{Cov(r_i, r_x)}{\sigma_x^2} \qquad (5)$$

In particular, for asset i relative to the market portfolio:

$$\beta_i^{wm} = \frac{Cov(r_i, r_{wm})}{\sigma_{wm}^2} \qquad (6)$$

and for the proxy portfolio relative to the market portfolio:

$$\beta_{pr}^{wm} = \frac{Cov(r_{pr}, r_{wm})}{\sigma_{wm}^2} \qquad (7)$$

Making use of (6) and (7) it is straight forward to show (see appendix):

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2}. \qquad (8)$$

Substituting (8) into (4) gives:

$$E[r_i^e] = \beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right] \qquad (9)$$

Equation (9) provides an expression for expected return on asset i in terms of the beta for asset i relative to a proxy and the expected excess return on the *same* proxy, i.e. in terms of what we actually estimate, β_i^{pr} and $E\left[r_{pr}^e\right]$. From (9) two conclusions can be drawn. One, using the beta estimate from a proxy times the expected excess return on the proxy $\left(\beta_i^{pr} E\left[r_{pr}^e\right]\right)$ yields a biased estimate for the cost of equity. This is because doing so ignores the adjustment factor $\frac{1}{\rho_{pr,mw}^2}$. Two, the same proxy must be used for estimation of beta and the expected market risk premium[7]. Thus the common procedure of estimating beta against an index and using six to eight percent as an estimate for the expected market risk premium leads to a biased estimate for cost of equity.

To obtain an unbiased estimate of the cost of equity it is therefore necessary to obtain an estimate for the right hand side of (9). This includes the correlation between the proxy and the unobservable market portfolio - so we are back where we started? Well, not quite. The estimation procedure suggested by Fama and MacBeth [1974] provides a natural estimate for $\beta_i^{pr} \frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$. In particular, the procedure provides an estimate for $\frac{1}{\rho_{pr,mw}^2} E\left[r_{pr}^e\right]$ so it is not necessary to estimate

[7] From above algebra, if two different proxies are used then we need an estimate for the beta for each proxy relative to the world market portfolio (!).

$\frac{1}{\rho^2_{pr,mw}}$ separately. Fama and McBeth's procedure consists of two steps. First, beta is estimated using a proxy in the following time-series regression:

$$r_{it} - r_{fr} = \alpha_i + \beta_i^{pr}(r_{prt} - r_{ft}) + \varepsilon_{it}, t = 1,..,T, i = 1,..,N \qquad (10)$$

This estimation uses five years of monthly data, for example 1970 to 1974 inclusive, one regression for each stock in the sample (N). Second, each of the estimated betas from the first step are used in a cross section regression of the following form using stock returns for the subsequent year, for example 1975:

$$r_{it+1}^a - r_{fr+1}^a = \gamma_{0\,i} + \gamma_1 \beta_{it}^{pr} + \varepsilon_{it}, i = 1,..,N \qquad (11)$$

where the superscript "pr" on beta indicates the fact that a proxy was used to estimate beta in (10). This is done using annual data, i.e. r_{it+1}^a is the annual return for stock i for 1975 and the risk free rate of return is the annual return on the Treasury bill. A simple way to obtain an estimate for γ_0 and γ_1 is to roll the above procedure forward and then take an average of the estimated values of γ_0 and γ_1. Thus γ_1 provides an estimate for $\frac{1}{\rho^2_{pr,mw}} E\left[r_{pr}^e\right]$ which when multiplied by β_{it}^{pr} provides an unbiased estimate for $\beta_{i,t+1}^{wm} E_t\left[r_{wm,t+1}^e\right]$. From the theoretical relationship obtained above (equation (9)) γ_0 should be zero and $\beta_i^{pr} \frac{1}{\rho^2_{pr,mw}} E\left[r_{pr}^e\right]$ provides an unbiased estimate for expected return. This procedure is implemented in the next section.

III. Estimating expected return using CAPM.

From Section II, first equation (10) is used to estimate a beta for each stock and then the betas obtained from estimation of (10) are used in the cross-section regression (11) to obtain an unbiased estimate for expected return. This procedure was undertaken for each of the indexes discussed in section I. The estimated slope coefficients are provided in Table IV.

Place Table IV here please.

The shaded area indicates significance; they are in general significant for each year. From section II the average of these coefficients provides an estimate for the adjusted risk premium, the expected return on the proxy times the adjustment coefficient, $\frac{1}{\rho^2_{pr,mw}} E\left[r_{pr}^e\right]$, (provided the distribution is stable over the period). The lowest adjusted risk premium (3%) is obtained using Standard and Poor's

Index whereas the CRSP equal weighted index gives the highest (7%). This is not surprising since from Table III the Standard and Poor's Index had the second highest average beta of 1.1 whereas the average beta for the CRSP equal weighted index was 0.9.

Place Table V here please.

Table V reports the estimates for the intercept terms which, according to CAPM and equation (10) should all be zero; clearly they are not. There are several possible reasons for this. The first is that CAPM is not valid. There is an extensive academic literature supporting this view.[8] If there are more risk factors than the market index then the inclusion of an intercept term captures the average of these missing factors. This is, of course, a brute force method. A more accurate procedure would be to estimate a multifactor model. The second possible reason is that CAPM is correct but the intercept term captures drifts in beta. In the above procedure the beta used in the cross section regression is from the previous period and Blume [1975], for example, argued that there is mean reversion in beta; that is, a high beta is followed by a smaller beta. Depending on this adjustment process inclusion of an intercept term may capture part of this movement.[9] Again under this interpretation an intercept term should be included in the calculation of the expected return on an asset.

Finally, from Table IV and Table V, estimates for the cost of equity based on the four indexes examined are as follows:

Standard and Poor's:

$$\hat{r}_i = r_{ft} + 4.30 + \beta_i^{sp} \times 3.75$$

CRSP value weighted index:

$$\hat{r}_i = r_{ft} + 3.36 + \beta_i^{vw} \times 4.44$$

CRSP equal weighted index:

$$\hat{r}_i = r_{ft} + 2.22 + \beta_i^{ew} \times 7.03$$

Morgan Stanley:

$$\hat{r}_i = r_{ft} + 4.43 + \beta_i^{ms} \times 4.01$$

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[8] The Arbitrage Pricing Theory by Ross [1976] provides an example of a theoretical model and Fama and French [1992] an empirical example of a three factor model.

[9] Notice that γ_1 captures the dynamic process suggested by Blume [1975]; that is, an additional benefit of this procedure is that Blume's adjustment is implicit in the procedure.

The four indexes provide an estimate for the expected market risk premium $\left(\gamma_0 + \overline{\beta^{pr}} \times \gamma_1\right)$ of around 8.4 percent, where $\overline{\beta^{pr}}$ is the average beta from Table II. Thus the four proxies provide more or less the same estimate for the expected market risk premium when the appropriate adjustment is made. Further for an individual stock, provided the same proxy is used for beta and the expected market risk premium and the appropriate adjustment is made to the estimate for the market risk the four proxies yield the same estimate for cost of equity.

Finally, the R^2 from the cross section regressions are reported in Table VI to determine how much of the variation in actual returns can be explained by differences in beta. The results are not encouraging. From Table VI, betas calculated using the Standard and Poor's Index can, on average, explain 1.6% of the variation in stock returns. The CRSP equal weighted index fares a bit better; it can explain nearly 3%. Based on this it is surprising that CAPM is so widely used.

Place Table VI here please.

IV. Conclusion.

This paper has shown that the standard procedure of using independent estimates of beta and the expected market risk premium to obtain an estimate for cost of equity based on CAPM most likely yields a biased estimate. This is undesirable since it leads to misallocation of funds and biased performance measures. It is shown in this paper that to obtain an unbiased estimate for cost of equity, while any reasonable proxy may be used for estimation of beta, the same proxy must be used for estimating the expected market risk premium. Further, the estimate for the expected market risk premium must be adjusted to take into account the relationship between the proxy and the market portfolio. A two step procedure was suggested which does this. If a beta is obtained from a beta provider then the relevant adjusted risk premium should also be obtained because without this it is not possible to obtain an unbiased estimate for cost of equity based on the beta provided. From the analysis done here it would not be difficult for beta providers to make the relevant adjusted risk premium available.

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Appendix

In the text it is stated that it is possible to show:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2} \qquad (8)$$

where the superscript "wm" refers to the world market portfolio and "pr" to the proxy portfolio. To derive this expression start with the definition of beta for asset i relative to the proxy pr:

$$\beta_i^{pr} = \frac{Cov(r_i, r_{pr})}{\sigma_{pr}^2} \qquad (A1)$$

From CAPM the actual return on the proxy portfolio is given by:

$$r_{pr} = r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr} \qquad (A2)$$

where ε_{pr} is a random error term with an expected value of zero. If the proxy portfolio lies on the efficient set then ε_{pr} is zero. Given that the proxy portfolio contains less assets than the world market portfolio this is unlikely to be the case. However the proxy is, in general, well diversified so it is not unreasonable to assume that ε_{pr} is small and uncorrelated with the return on individual stocks.

Substituting (A1) into (A2), and recognising that the risk free rate is a constant gives:

$$\beta_i^{pr} = \frac{Cov(r_i, r_f + \beta_{pr}^{wm}(r_{wm} - r_f) + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm} + \varepsilon_{pr})}{\sigma_{pr}^2} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} + \frac{Cov(r_i, \varepsilon_{pr})}{\sigma_{pr}^2}$$

Assuming that the return on asset "i" is uncorrelated with a shock to the proxy portfolio the last term is zero, so we have:

$$\beta_i^{pr} = \frac{Cov(r_i, \beta_{pr}^{wm} r_{wm})}{\sigma_{pr}^2} = \frac{\beta_{pr}^{wm} Cov(r_i, r_{wm}) \sigma_{wm}^2}{\sigma_{pr}^2 \sigma_{wm}^2} = \frac{\beta_{pr}^{wm} \beta_i^{wm} \sigma_{wm}^2}{\sigma_{pr}^2}$$

$$\Leftrightarrow \frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \frac{\sigma_{pr}^2 \beta_i^{pr}}{\sigma_{wm}^2 (\beta_{pr}^{wm})^2}$$

Finally, using the definition of beta for the proxy portfolio:

$$\frac{\beta_i^{wm}}{\beta_{pr}^{wm}} = \beta_i^{pr} \frac{\sigma_{pr}^2}{\sigma_{wm}^2} \frac{(\sigma_{wm}^2)^2}{Cov(r_{pr}, r_{wm})^2} = \beta_i^{pr} \frac{1}{\rho_{wm,pr}^2}$$

Table I.

Summary statistics

Summary statistics are for monthly returns on the indicated indexes expressed as percentages. Standard and Poor refers to the Standard and Poor's Composite Index and Morgan Stanley refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

Index	Number of observations	Mean	Standard deviation	Minimum	Maximum
Standard and Poor	323	0.7626	4.35781	-21.76304	16.30469
Morgan Stanley	323	0.74927	4.08246	-17.12423	14.26563
Value weighted CRSP index	323	1.06192	4.49597	-22.50055	16.57984
Equal weighted CRSP index	323	1.84503	5.70234	-25.08664	30.28762

Table II
Correlation coefficients between the indexes

The Pearson correlation coefficients are calculated based on monthly returns for each of the indexes. Standard and Poor (and S & P) refers to the Standard and Poor's Composite Index and Morgan Stanley (and MSCI) refers to the Morgan Stanley World Market Capital Index. The CRSP equal and value weighted indexes refer to the indexes included on the CRSP tape with dividends.

	S & P	MSCI	CRSP value weighted	CRSP equal weighted
Standard and Poor	1	0.826	0.9883	0.785
Morgan Stanley		1	0.8216	0.6853
Value weighted CRSP index			1	0.8485
Equal weighted CRSP index				1

Table III

Summary statistics for beta estimates.

For each index, an individual stock beta is estimated using the following model:

$$r_{it} - r_{fr} = \alpha_i + \beta_i (r_{It} - r_{ft}) + \varepsilon_{it}, t = 1, .., T, i = 1, .., N \quad (10)$$

where r_{it} is the monthly return (including dividends) for stock i, r_{fr} is the monthly return on three month Treasury bills and r_{iT} is the monthly return on the index. The first estimation period is 1970 to 1974 inclusive, the second is 1971 to 1975 inclusive, and so forth up to and including 1996.

Index	Number	Average beta	Standard deviation	Minimum	Maximum
Standard and Poors	22,905	1.0982	0.4717	-1.9468	3.4652
Morgan Stanley	22,905	0.9765	0.5133	-1.5106	3.8294
CRSP value weighted	22,905	1.1173	0.4833	-1.3448	3.7249
CRSP equal weighted	22,905	0.922	0.4717	-0.4082	7.26

Table IV
Estimates of slope coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1}^{pr} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.1786	0.2315	0.4357	0.2597
1976	0.0822	0.1079	0.2172	0.0998
1977	0.0664	0.077	0.1052	0.0682
1978	0.1429	0.1563	0.1938	0.1432
1979	0.1845	0.1996	0.2019	0.1324
1980	0.1339	0.1408	0.1275	0.0808
1981	-0.0957	-0.093	-0.0654	-0.0723
1982	-0.0033	0.0056	0.0628	0.0129
1983	0.088	0.1017	0.1713	0.0859
1984	-0.1439	-0.1533	-0.1899	-0.1412
1985	-0.0788	-0.0772	-0.0721	-0.0763
1986	-0.106	-0.1159	-0.1473	-0.1245
1987	0.0104	0.0032	0.0056	0.0536
1988	0.0264	0.028	0.0284	-0.0123
1989	-0.0867	-0.1003	-0.1316	-0.1157
1990	-0.0795	-0.0916	-0.1346	-0.0686
1991	0.2516	0.263	0.2973	0.2516
1992	0.1425	0.1469	0.1807	0.17
1993	-0.0388	-0.0106	0.104	-0.0644
1994	0.0421	0.0412	0.0314	0.0055
1995	0.0674	0.0676	0.0731	0.1342
1996	0.0428	0.0444	0.0526	0.061
Mean	0.0375	0.0442	0.0703	0.0401

Table V.
Estimates of Intercept coefficients

The coefficients are obtained from the following model:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year. Shaded area indicates a significant coefficient at the 5% level.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.2727	0.2125	0.0899	0.1445
1976	0.2121	0.1802	0.1161	0.1854
1977	-0.1283	-0.1422	-0.1426	-0.1349
1978	-0.1957	-0.2147	-0.2011	-0.211
1979	-0.0591	-0.0814	-0.0346	-0.0159
1980	-0.0858	-0.0938	-0.0277	-0.0163
1981	-0.0056	-0.0125	-0.0615	-0.0271
1982	0.1766	0.1664	0.115	0.1575
1983	0.0769	0.0662	0.019	0.0811
1984	0.0355	0.0397	0.0428	0.0347
1985	0.267	0.2629	0.2452	0.2629
1986	0.1439	0.1539	0.164	0.1545
1987	-0.1257	-0.1183	-0.1202	-0.1629
1988	0.0563	0.0541	0.0568	0.0976
1989	0.16	0.1766	0.203	0.1832
1990	-0.1785	-0.1622	-0.1207	-0.1981
1991	0.0876	0.0651	0.0269	0.1598
1992	-0.0151	-0.0253	-0.0478	-0.0014
1993	0.169	0.1414	0.0416	0.1659
1994	-0.126	-0.128	-0.1089	-0.0863
1995	0.148	0.1447	0.1589	0.1388
1996	0.0613	0.0556	0.0626	0.0627
Mean	0.043	0.0336	0.0216	0.0443

Table VI.
Mean R-squared for c ross section regressions

The R-squared is obtained from the following regression:

$$r_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it-1} + \varepsilon_{it} \qquad (11)$$

where beta is estimated using time series regressions for the previous periods. The dependent variable is the annual return on stock "i" minus the risk free rate of return. The equation is estimated for each year using between 780 and 1308 stocks depending on the year.

Year	S&P	Value weight	Equal weight	Morgan Stanley
1975	0.0194	0.0325	0.0983	0.048
1976	0.0105	0.0188	0.0668	0.0179
1977	0.0125	0.0177	0.0315	0.0156
1978	0.0337	0.0429	0.0637	0.0434
1979	0.0201	0.0252	0.0275	0.0141
1980	0.0281	0.0323	0.0201	0.012
1981	0.0284	0.0256	0.0098	0.018
1982	0	0	0.003	0.0001
1983	0.0123	0.0151	0.0339	0.0112
1984	0.0806	0.0856	0.101	0.0752
1985	0.0125	0.0113	0.008	0.011
1986	0.0295	0.0352	0.0514	0.0367
1987	0.0001	0	0	0.0049
1988	0.0008	0.0009	0.001	0.0002
1989	0.0072	0.01	0.0179	0.0149
1990	0.0091	0.013	0.0313	0.0069
1991	0.0173	0.0205	0.0293	0.0119
1992	0.0151	0.0171	0.0255	0.0153
1993	0.0019	0.0001	0.0177	0.0023
1994	0.005	0.0054	0.0031	0
1995	0.0035	0.0038	0.0041	0.0055
1996	0.0045	0.0054	0.0073	0.0053
Mean	0.016	0.019	**0.0296**	0.0168