Graduate Quantum Mechanics - Problem Set 8

Problem 1)

Consider a harmonic oscillator which is in an initial state $a|n\rangle + b|n+1\rangle$ at $t=0$, where a, b are real numbers with $a^2 + b^2 = 1$. Calculate the expectation values of $\langle X \rangle(t)$ and $\langle P \rangle(t)$ as a function of time. Compare your results to the "classical motion" $x(t)$ of a harmonic oscillator with the same physical parameters (ω, m) and the same (average) energy $E \approx (n+1)\hbar\omega$.

Problem 2)

A particle of mass m is in a one-dimensional potential of form $V(x) = \frac{1}{2}m\omega^2 x^2 + mgx$ with some real number g. (Think of this as an oscillator potential plus a constant force mg in –x direction acting on the particle). Without doing much "heavy math", can you write down the lowest energy eigenstate of this potential? (Think about the classical analog – a weight hanging on a vertical spring. How does gravity affect the equations and solution for the harmonic spring potential energy?)
XC: What is the probability that a particle starting out in the ground state of the harmonic oscillator potential only (first part of $V(x)$) ends up in the new ground state once the force is "switched on"?

Problem 3)

Find the eigenvalues and eigenstates of the one-dimensional Hamiltonian with potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 \mathbf{X}^2 , x < 0 \\ \infty, x \geq 0 \end{cases}$$

Again, nearly no math is needed – only some clever argument.