PHYSICS 621 - Fall Semester 2013 - ODU

Graduate Quantum Mechanics - Problem Set 7

Problem 1)

An atom of mass $4 \cdot 10^9$ eV/c^2 has its position measured within 2 nm accuracy. Assume that it is in a Gaussian wave packet state afterwards. How much time will elapse before the uncertainty of our knowledge about its position has doubled? How about a 1 μg speck of matter that has been located to within 1 μm?

Problem 2)

A point-like particle of mass m sits in a one-dimensional potential well. The potential is infinitely high for $x < -s$ and for $x > +s$, while it is at a constant value of $V_0 > 0$ for $-s \leq x < 0$ and zero for $0 \leq x \leq s$. The particle is in the ground state (lowest energy eigenstate of the Hamiltonian) with energy $E_0 > V_0$.
Question: What is the probability that the particle can be found in the left half ($x < 0$) of the potential well? Outline how you would solve this problem step by step, without actually solving the (transcendental) equations that you encounter:

1. Write down the one-dimensional Schrödinger equation for this problem.
2. Find the generic stationary solutions in the left and right half of the potential well (you may assume $E > V_0$).
3. List all boundary conditions that must be fulfilled (there are 4 of them!)
4. Rewrite your two half-solutions from item 2. above to explicitly fulfill as many of the boundary conditions as possible.
5. Outline how you would find the lowest energy (ground state eigenvalue E) that solves the one-dimensional Schrödinger equation. No closed algebraic solution is possible or required for this part - just explain which equation needs to be solved.
6. Assuming you have E, how would you determine the normalization constants for the two half-solutions?
7. Once you have those in hand as well, how can you answer the original question?

Problem 3)

Consider the "Gaussian wave packet" from the lecture or p. 154 in Shankar. Calculate the probability current j_x for every point x at time $t = 0$. Using our result for the probability density, $\rho(x, t)$, show through explicit calculation (not by invoking general principles!!!) that the continuity equation for probability is fulfilled at time $t = 0$.