Graduate Quantum Mechanics - Problem Set 4

Problem 1)

Assuming a particle is described with the usual cartesian coordinates (x,y,z) and momenta (p_x, p_y, p_z). Write down the x, y and z components of the angular momentum operator in terms of these canonical variables. Calculate the Poisson brackets $\{L_x, L_z\}$ and $\{L_y, L_z\}$ explicitely. Given our interpretation of L_z as "generator" of rotations around the z axis, can you interpret your result in terms of the transformation of the vector **L** under the coordinate transformation generated by L_z?

Problem 2)

Write down the Lagrangian for two equal masses m at positions x_1 and x_2 (each measured relative to the equilibrium position), coupled to each other and (on their other sides) to two fixed walls with springs with constant k but otherwise free to move along the x-direction. If the system is in equilibrium, all three springs are relaxed (unstretched/compressed). [This is exactly the set up in Example 1.8.6 in Shankar's book, p. 46.] Set up the Lagrangian for this system. Find the generalized momenta. Then follow the explicit procedure (Legendre transformation) in the lecture to find the corresponding Hamiltonian. Write down Hamilton's canonical equations. (You don't have to solve them).

Problem 3)

Use the vector potential representation of a constant magnetic field **B** along the z-axis from our first homework problem set. The Hamiltonian for this case in cylindrical coordinates $(r_\perp, \varphi, z)$ with canonical momenta $(P_{r_\perp}, P_\varphi, P_z)$ is given by

$$H = \frac{\left(P_\varphi / r_\perp - qA_\phi \right)^2 + P_{r_\perp}^2 + P_z^2}{2m}$$

By writing down Hamilton's equations of motion, give an interpretation (in terms of the "usual" momenta or velocities) of $(P_{r_\perp}, P_\varphi, P_z)$. Which of these three are conserved? Under what condition are **all 3** conserved? Can you interpret this condition? [What does it mean physically?]