PHYSICS 621 - Fall Semester 2012 - ODU

Graduate Quantum Mechanics - Problem Set 1

Problem 1)

Write down the total mechanical energy (kinetic plus potential) of a mass m in free fall, expressing it in terms of the momentum p and the height x above ground: $E = T_{kin} + V(x) = H(p,x)$. Take the partial derivative of the function H with respect to p and show that it is equal to the velocity, $\frac{\partial H}{\partial p} = v = \dot{x}$. Then show that the negative of the partial derivative with respect to x equals the force, i.e. the rate of change of the momentum: $-\frac{\partial H}{\partial x} = F = \dot{p}$

Problem 2)

Show that a vector potential given (in cylindrical coordinates) by $\vec{A}(r_\perp,\varphi,z) = \frac{r}{2} b\,\hat{\varphi}$ corresponds to a constant magnetic field $\vec{B} = \vec{\nabla} \times \vec{A} = b\,\hat{z}$ along the z-axis. (You may use the formula sheet)

Problem 3)

What is the force of a magnetic field of 0.1 Tesla in z-direction on an electron instantaneously moving along the x-axis with 10% of the speed of light? What is the equation of motion for this electron? What kind of motion does it describe (give all the numeric parameters, e.g. amplitude and period if the motion is periodic).

Problem 4)

For each of the following statements about Quantum Mechanics, indicate whether you believe them to be correct or wrong. Give a 1-2 sentence explanation for each of your responses:

a. If all possible information on a system is given, Quantum Mechanics can predict the outcome of any future measurement on the system accurately.
b. Quantum Mechanics cannot predict anything precisely
c. Quantum Mechanics cannot predict with certainty the result of any particular measurement on a single particle
d. The Heisenberg Uncertainty principle means that nothing can be measured precisely
e. The x- and y- components of any angular momentum cannot simultaneously be measured with arbitrary precision.
f. The time evolution of a quantum mechanical wave function is described by a unitary operator.

Problem 5)

Solve the differential equation $\frac{d^2y(x)}{dx^2} - m^2 y(x) = 0$ for real m. Make sure you find the most general solution – what are the "integration constants"?

Problem 6)

Proof that for any complex numbers c, z with $z = \exp(c)$ we have $z^* = \exp(c^*)$

Problem 7)

Find the Fourier transform $\tilde{f}(p)$ of the function

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-x^2/2\right):$$

$$\tilde{f}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \exp(-ipx)\, dx$$