Take note of formula sheet linked to course web page for a lot of useful tidbits of information!

Systems of Units

SI [m,s,kg, C], e.g. $e = 1.6 \times 10^{-19}$ C, $c = 3 \times 10^8$ m/s.

Gauss [cm, s, g, ESU]

(Sub)Atomic units (more appropriate for small objects): nm instead of m, eV $= 1.6 \times 10^{-19}$ J instead of J; express momenta in eV/c and masses in eV/c^2. $c = 300$ nm PHz (Peta-Hertz $= 10^{15}$ Hz). Most important constant: $\hbar c = 197.33$ eV nm . Second most important: $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ [SI] $= e^2/(\hbar c)$ [Gauss] $= 1/137.036$.

SI units to Gaussian units

q, I, ... $\rightarrow q_G = \frac{q_{SI}}{\sqrt{4\pi\epsilon_0}}$

$\vec{E} \rightarrow \vec{E_G} = \vec{E_{SI}}\sqrt{4\pi\epsilon_0}$

$\vec{\nabla}\cdot \vec{E_{SI}} = \frac{\rho_{SI}}{\epsilon_0}$

$\rightarrow \vec{\nabla}\cdot \vec{E_G} = \sqrt{4\pi\epsilon_0}\vec{\nabla}\cdot \vec{E_{SI}}$

$= \sqrt{4\pi\epsilon_0}\frac{\rho_{SI}}{\epsilon_0}$

$= (\sqrt{4\pi\epsilon_0})^2\frac{\rho_G}{\epsilon_0}$

$= 4\pi\rho_G$

$q\vec{E} = \vec{F}$

$q_{SI}(\vec{v} \times \vec{B_{SI}}) = \vec{F}$

$q_G(\frac{\vec{v}}{c} \times \vec{B_G}) = \vec{F}$

$\vec{\nabla} \times \vec{E}$

$=^- \frac{\partial \vec{B}}{\partial t}$ (SI)

$=^- \frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (G)

Coulomb potential: $V = \frac{q}{4\pi\epsilon_0 r}$ SI $= \frac{q}{r}$ [Gauss].

Quantum “Weirdness”

Electrons going through ”two slits at once” interfering with themselves

Uncertainty relationship, Statistical interpretation (“Quantum theory is not about the world as it *is*, but about the kind of information we can have about the world”)

Infinitely long solenoid, no field outside (only vector potential which classically is not observable) $\Rightarrow$ passing electrons react to potential (quantum), known as the Aharonov-Boom effect

Preperation and measurement of the system is classical $\Rightarrow$ How does this transition quantum $\rightarrow$ classical occur? (Collapse of the wave function)

"Spooky action at a distance" $\rightarrow$ entanglement, coherence-decoherence

Complex Numbers

$z = Re(z) + iIm(z)$

$= r(\cos\phi + i\sin\phi)$

$\Rightarrow z = re^{i\phi}$

Rotates vector

$r = \sqrt{(Re(z))^2 + (Im(z))^2}$

$e^{i2\phi} = (e^{i\phi})^2 \Rightarrow \cos^2\theta - \sin^2\theta = \cos 2\theta$ etc.

$z^* = Re(z) - iIm(z) = re^{-i\phi}$; $r = \sqrt{z^* z}$

$\ln(z)$

Problem: $re^{i\phi} = re^{i(2n\pi+\phi)}$

$\ln(z) = \ln(r) + i(2n\pi + \phi)$

Usually, let $n = 0$

Fourier Series

Both orthogonal and a complete set

Orthogonal: $\int_0^{2\pi} \sin(nx)\sin(mx)dx = \pi\delta_{mn}$

Complete Set:

$f(x)$ is defined on $[0, 2\pi]$

$f(x) = \Sigma_{m=0}^{\infty}[a_m \sin(mx) + b_m \cos(mx)]$

$f(x) = \Sigma_{n=-\infty}^{\infty} c_n e^{inx}$

a_m, b_m are real; c_n is complex

Can be generalized to arbitrary interval $[x_1, x_2]$ (replace x with $\frac{x-x_1}{x_2-x_1}2\pi$) or even periodic functions on all of the real numbers ($f(x) = f(x+a)$).

Fourier Transform

$f(x)$ is defined on $(-\infty, \infty)$, then the Fourier Transform is

$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx'} f(x')dx'$ yielding

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} \tilde{f}(k)dk$

Plugging the 1st into the 2nd yields

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk e^{ikx} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx' e^{-ikx'} f(x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx' f(x') \int_{-\infty}^{\infty} dk e^{ik(x-x')} \tag{1}$$

Since the integral over x' simply gives the value of the function f at x, the integral $\frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')}$ must be equal to $\delta(x - x')$.

– Example: Fourier Transform of a Standard Gaussian function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$:

$$\tilde{f}(k) = \left(\frac{1}{\sqrt{2\pi}}\right)^2 \int_{-\infty}^{\infty} e^{-ikx} e^{-x^2/2} dx = \left(\frac{1}{\sqrt{2\pi}}\right)^2 \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+2ikx-k^2)} e^{-k^2/2} dx \tag{2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-k^2/2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x+ik)^2} dx \tag{3}$$

The last integral is simply an integral over a regular Gaussian, except along a line that is not the x-axis but offset by ik in the complex plane and parallel to the x-axis. Using the fact that the Gaussian does not have any poles in the complex plane and falls off to zero at large positive and negative real parts of its argument, one can invoke Cauchy's theorem to show that this integral must have the same value as that along the x-axis, which is 1 (since the Gaussian is properly normalized). Therefore, the Fourier transform of a Gaussian is again a Gaussian in the conjugate variable: $\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} e^{-k^2/2}$
BTW, similar arguments can be used to prove that

$$\int_{-\infty}^{\infty} e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}} e^c \tag{4}$$

for *any* set of **complex** numbers a, b, c as long as the real part of $\sqrt{a} > 0$. We will make use of this in class.