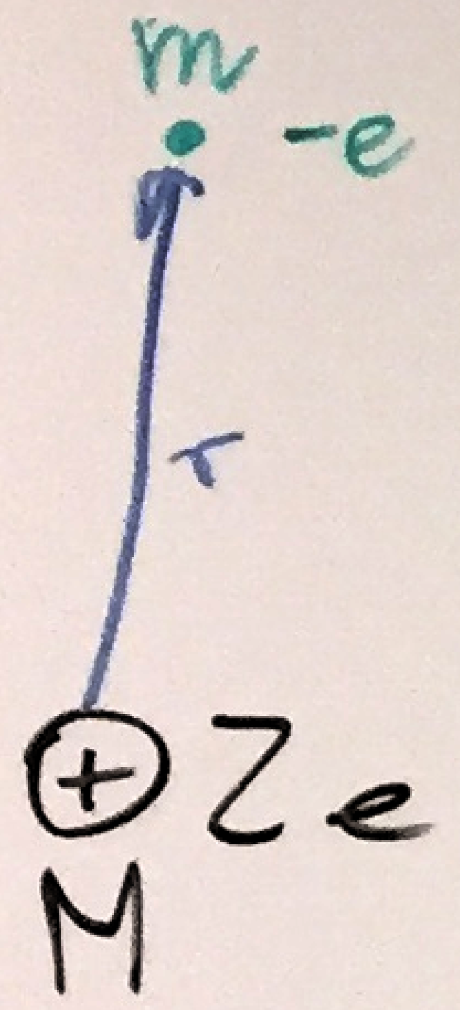


$$V(r) = \frac{Ze(-e)}{4\pi\varepsilon_0 r}$$

Reduced mass μ: 2 particles w/ m, M behave like one particle w/ $\mu = \frac{mM}{m+M}$

$$H\varphi_E(r,\theta,\varphi) = -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\varphi_E(r,\theta,\varphi) + \frac{\vec{L}^2}{2mr^2}\varphi_E(r,\theta,\varphi) + V(r)\varphi_E(r,\theta,\varphi) = E\varphi_E(r,\theta,\varphi)$$

$$\varphi_E(r,\theta,\varphi) = R_E(r)\,Y_{\ell m}(\theta,\varphi)$$

$$\vec{L}^2\,Y_{\ell m}(\theta,\varphi) = \hbar^2\ell(\ell+1)\,Y_{\ell m}(\theta,\varphi) \qquad \ell = 0, 1, \dots$$

$$L_z\,Y_{\ell m}(\theta,\varphi) = \hbar m\,Y_{\ell m}(\theta,\varphi) \qquad m = 0, \pm 1, \pm 2, \dots \pm \ell$$

$$\circledast \quad -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}R_{E,\ell}(r)\cancel{Y_{\ell m}(\theta,\varphi)} + \frac{\hbar^2\ell(\ell+1)}{2mr^2}R_{E,\ell}(r)\cancel{Y_{\ell m}} + V(r)R_{E,\ell}(r)\cancel{Y_{\ell m}} = E R_{E,\ell}(r)\cancel{Y_{\ell m}}$$

$- - -$

Solution for case $\ell = 0 \Rightarrow m = 0$ $\quad Y_{00} = \frac{1}{\sqrt{4\pi}}$ $\quad \underline{Z = 1}$

$\ell = 0, 1, 2, 3$
s p d f

$$-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} R_{E,\ell}(r) = \frac{2mZe^2}{\hbar^2 4\pi\varepsilon_0 r} R_{E,\ell}(r) + \frac{2mE}{\hbar^2} R_{E,\ell}(r)$$

$e^{-\alpha r}$ $\quad e^{-\alpha r}$

$$R(r) = e^{-\alpha r} = e^{-r/a_0}$$

$$-\alpha r^2 e^{-\alpha r}$$

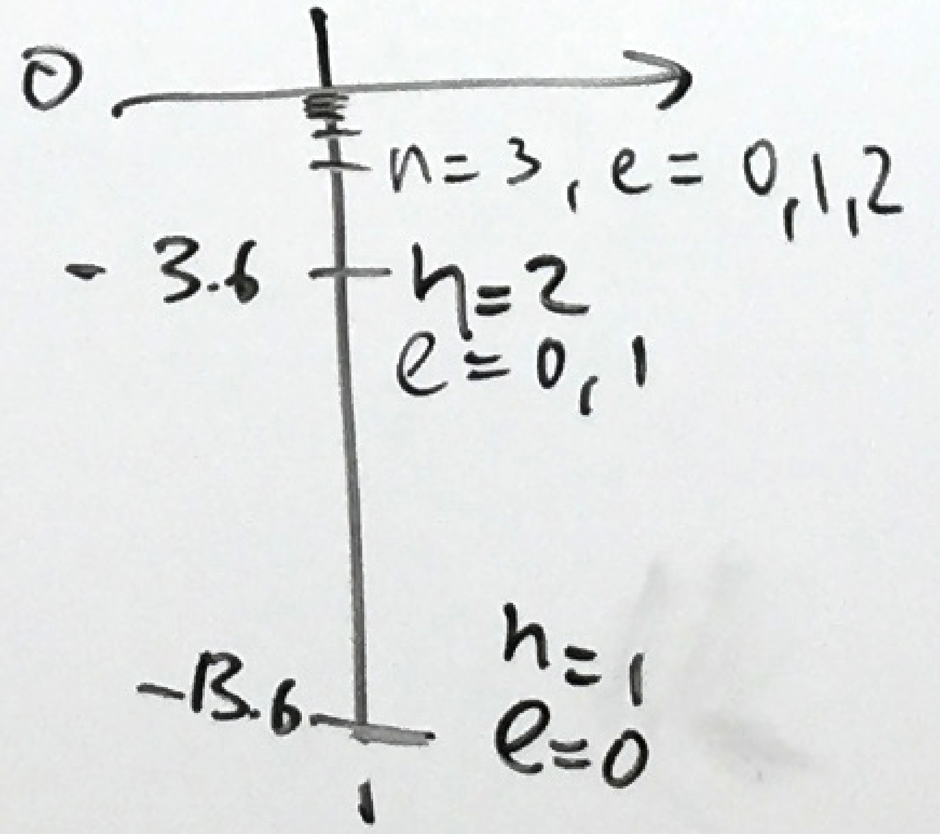
$$-\alpha^2 \cancel{r^2} e^{-\alpha r} + \frac{2\alpha}{r} e^{-\alpha r}$$

$$\Rightarrow \cancel{2}\alpha = \frac{\cancel{2} m Z e^2}{\hbar^2 4\pi\varepsilon_0} = \frac{mc}{\hbar} Z\alpha_{EM} = \frac{1}{a_0} \quad \text{Bohr radius}$$

USE: $\frac{e^2}{4\pi\varepsilon_0 \hbar c} = \alpha_{EM} = \frac{1}{137.036\ldots}$

$$a_0 = \frac{\hbar}{m_e c Z\alpha_{EM}} \simeq 0.053\,\text{nm} \cdot \frac{1}{Z} \approx \frac{1}{2}\text{Å}$$

$$\frac{\cancel{2m}E}{\cancel{\hbar^2}} = -\alpha^2 = -\frac{m^2c^2}{2\hbar^2} Z^2 \alpha_{EM}^2$$



$$E = -\frac{mc^2}{2} \cdot Z^2 \alpha_{E.M.}^2$$

electron: $m_e c^2 = 511{,}000\,\text{eV}$

$$\Rightarrow E = -13.6\,\text{eV} = -R_y$$

In GENERAL: for any ℓ, there are ∞ many solutions to ⊛

$$E_{n,\ell,m} = -R_y \cdot \frac{1}{n^2} \qquad n > \ell$$

Laguerre Polynomials: