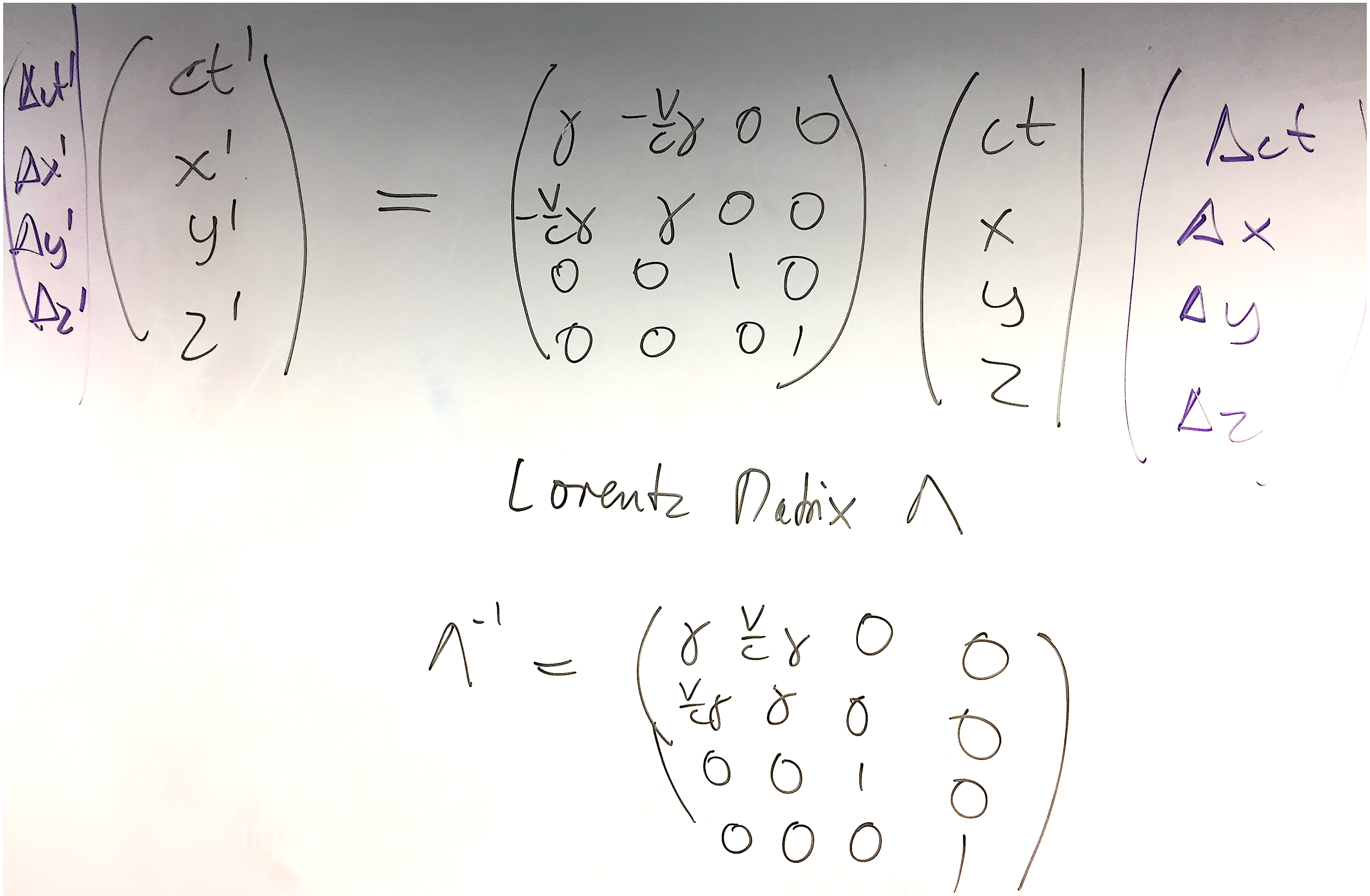
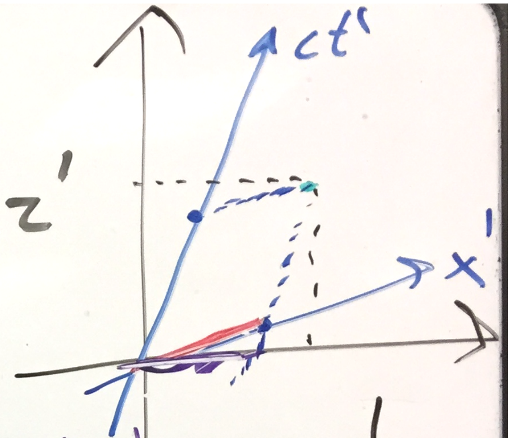


Lorentz transformation for coordinates and for the separation between two events

Lorentz Transformation

S': $(v \parallel \hat{x})$ Event ct', x', y', z'



S: $ct = \gamma\left(ct' + \frac{v}{c}x'\right)$

$x = \gamma\left(\frac{v}{c}ct' + x'\right)$ (Newton: $x = vt' + x'$)

$\boxed{y = y',\ z = z'}$

$\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

"Moving clocks go slow"

$x' = 0 \Rightarrow ct = \gamma ct'$

"Moving lengths are contracted"

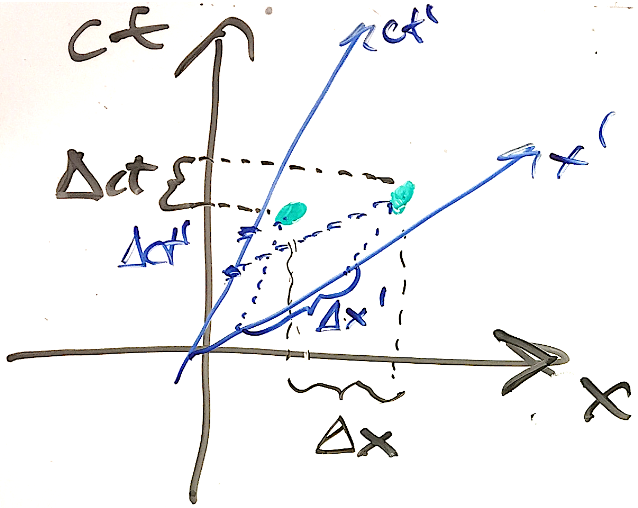
measure BOTH ends in S at $ct = 0 \Rightarrow ct' = -\frac{v}{c}x'$

$\Rightarrow x = \gamma\left(-\frac{v^2}{c^2}x' + x'\right) = \gamma\left(1 - \frac{v^2}{c^2}\right)x' = \frac{1}{\gamma}x'$

Reverse?

$ct' = \gamma\left(ct - \frac{v}{c}x\right)$

$x' = \gamma\left(-\frac{v}{c}ct + x\right)$



$$\Delta ct = \gamma\left(\Delta ct' + \frac{v}{c}\Delta x'\right)$$
$$\Delta x = \gamma\left(\Delta x' + \frac{v}{c}\Delta ct'\right)$$

Invariant : c

$$-(\Delta x'^2 + \Delta y'^2 + \Delta z'^2) + (\Delta ct')^2 = (\Delta s')^2$$
$$(\Delta ct)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (\Delta s)^2$$
$$(\Delta s')^2 = (\Delta s)^2$$

Adding velocities

1) Object moving with velocity u_x' in S'

$$\Rightarrow \Delta x' = \frac{u_x'}{c}\Delta ct'$$

in S: $$\Delta x = \gamma\left(\frac{u_x'}{c}\Delta ct' + \frac{v}{c}\Delta ct'\right)$$
$$\Delta ct = \gamma\left(\Delta ct' + \frac{v}{c}\frac{u_x'}{c}\Delta ct'\right)$$
$$\frac{\Delta x}{\Delta ct} \overset{!}{=} \frac{u_x}{c} = \frac{\frac{u_x'}{c} + \frac{v}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}}$$

2) Object moving in y-direction

u_y' $\quad \Delta y' = \frac{u_y'}{c}\cdot\Delta ct'$

$$\Delta y = \Delta y'$$

$$\frac{u_x}{c} = \frac{\gamma\frac{v}{c}\Delta ct'}{\gamma\Delta ct'} = \frac{v}{c} \quad \Bigg| \quad \frac{u_y}{c} = \frac{1}{\gamma}u_y'$$