$$H = \frac{-\hbar^2}{2M}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\Psi(r,\theta,\varphi) + \frac{\hbar^2 \ell(\ell+1)}{2Mr^2}\Psi + V(r)\,\Psi(r,\theta,\varphi) = E\,\Psi(r,\theta,\varphi)$$

Physics 323
10/23/14
Bound state of Hydrogen Atom & excited states
Mark Stefani

IF $\Psi(r,\theta,\varphi) = R(r)\cdot Y_{\ell m}(\theta,\varphi)$

$$(L^2)_{op}\, Y_{\ell m}(\theta,\varphi) = \ell(\ell+1)\hbar^2 Y_{\ell m}$$

$$(L_z)_{op}\, Y_{\ell m}(\theta,\varphi) = m\hbar\, Y_{\ell m}$$

ELSE General H in spherical cor.:

$$H = \frac{-\hbar^2}{2M}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\Psi(r,\theta,\varphi) + \frac{(\vec{L}^2)_{op}}{2Mr^2}\Psi(r,\theta,\varphi) + V(\vec{r})\,\Psi(r,\theta,\varphi)$$

$= V(r,\theta,\varphi)$

$\Psi(r,\theta,\varphi) = R(r)\cdot Y_{\ell m}(\theta,\varphi)$ only if $V(\vec{r})$ dependant only on $|\vec{r}| = r$

then $Y_{\ell m}$ is a multiplier ∴

$$H = -\frac{\hbar^2}{2M}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\cdot R(r) + \frac{\hbar^2 \ell(\ell+1)}{2Mr^2}\cdot R(r) + V(r)\cdot R(r) = E\cdot R(r)$$

Hydrogen atom: $V(r) = \frac{-Ze^2}{4\pi\epsilon_0 r}$

$$\therefore\ H = \frac{-\hbar^2}{2M}\left[\frac{1}{r^2}\left(2rR' + r^2R''\right) - \frac{\ell(\ell+1)}{r^2}R(r)\right] - \frac{Ze^2}{4\pi\epsilon_0 r}R(r) = E\,R(r)$$

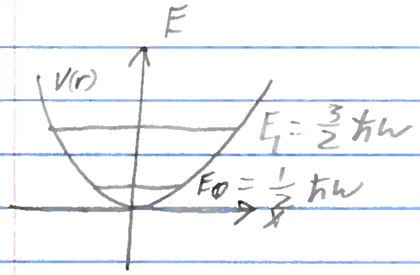
$\cancel{R(r) \sim \frac{1}{r^2}}$ $\quad r\to 0$ must remain finite

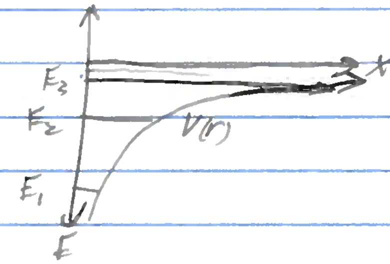
$R(r) \underset{r\to\infty}{\sim}$ quick falloff so $\int_0^\infty r^2 dr\, R(r)$ is finite

Solution: $R(r) = \text{Polynomial}(r)\, e^{-\alpha/r}$

Polynomial(r) will be a Laguerre Polynomial

Must have a minimum energy like Harmonic oscillator, or culoumb potential





$E_n \sim R_{n\ell}(r)$ $\qquad \underset{n,\ell}{E_{1,1}} = E_{2,0}$

Bound states of Hydrogen atom

$\Psi_{n,\ell,m}(r,\theta,\varphi) = R_{n,\ell}(r)\, Y_{\ell m}(\theta,\varphi)$

n ~~$R_{n\ell}(r)$~~ is called the pricipal quantum number.

limits:

$n = 1,2,3\ldots$, $\ell = 0,1,\ldots,n-1$, $m = -\ell, -\ell+1 \ldots \ell$

example: $E_1(z=1, M_E \ll M_p)$ $\qquad \frac{e^2}{4\pi\epsilon_0 \hbar c} = \alpha_{EM} = \frac{1}{137.036\ldots}$

$\frac{M_e \cdot c^2 \cdot \alpha^2}{2} \rightarrow R_y = \frac{\alpha_{EM}^2 M_e c^2}{2} = -13.6 \text{ eV}$

Ground state of Hydrogen, $E_1 = -R_y$

$R_y = -13.606$ ev $\qquad E_n = -R_y \frac{1}{n^2}$ (independent of ℓ)

Photon $hf = \Delta E = R_y\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

$\Psi_{1,0,0} = \sqrt{\frac{1}{a_0^3 \pi}}\, e^{-r/a_0}$ $\qquad a_0$ is Bohr radious

$a_0 = \frac{\hbar}{\alpha m_e c} \approx \frac{1}{2} \text{Å}$

$\Delta Prob(r,\theta,\varphi) = |\Psi(r,\theta,\varphi)|^2 \cdot \Delta\tau \qquad \Delta\tau = r^2 \sin\theta\, \Delta r\, \Delta\theta\, \underbrace{\Delta\phi}_{2\pi \text{ if integrate over all } \phi}$

Radial Probability is preportional to $r^2|\Psi|^2 \Delta r$ Maximum $\alpha\ a_0$