PHYS323 MODERN PHYSICS Lecture Notes 9/10/2015

It follows that for any object of mass m_0, we have

$$E^2 = \left(m(u)c^2\right)^2 = m_0^2c^4 + \vec{p}^2c^2$$

$$\vec{p}^2 = E^2/c^2 - m_0^2c^2; \quad \vec{u} = \vec{p}/m(u) = \vec{p}c^2/E; \quad \beta = \frac{u}{c} = pc/E$$

$$\Gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} = \frac{m(u)}{m_0} = \frac{E}{m_0c^2} = \sqrt{1+\vec{p}^2/m_0^2c^2}$$

(this last one can be useful if one wants to find the correct Lorentz transformation into the system in which m_0 is – at least momentarily – at rest).

Even in a system with several particles, the quantity "invariant mass"

$$m_{\text{invariant}} = \sqrt{\left(\sum \frac{E_i}{c}\right)^2 - \left(\sum \vec{p}_i\right)^2} \Big/ c$$

is an invariant (since it is the same in all coordinate systems) **and** conserved (since both energy and momenta are conserved). This allows one to quickly calculate the minimum energy necessary to produce an object of that mass.

Let's turn to the interpretation of the "total relativistic energy" of some object of mass m_0. We can write it as

$$E_{tot} = m_0c^2 + T_{kin} + U_{pot} + \ldots$$

where T_{kin} is the kinetic energy (which becomes equal to the non-relativistic, Newtonian expression at small velocities, as shown earlier) and U_{pot} contains other "external" contributions to the total energy. m_0c^2 then is the "internal" or "rest" energy of the object. What does it mean?

First of all, it is a **humongous** amount of energy, even for small objects. 1 nano-gram of any material (10^{-12} kg) has a rest energy of 90,000 J! Can it be "liberated"? Note that energy conservation only requires that the sum of all energies in the initial and the final state have to be the same. So, for example, if you have an inelastic collision, the rest mass of the final composite object is larger than the sum of the rest masses of the pieces before the collision. Vice versa, an exploding fire cracker will produce fragments whose masses add up to slightly less than the fire cracker's original mass. Even warming up a piece of material (or charging up a battery, or…) will increase its mass. Of course, if the energies involved are small (everyday), the change in mass is extremely hard to observe.
Example: Warm 1 liter of water (1 kg) by 10 degree Celsius (or Kelvin) = 10,000 cal = 41,868 J. This will increase its mass by less than ½ ng, or less than ½ ppt (parts per trillion).
Another example: The binding energy (electrostatic attraction) of an electron in a hydrogen atom, -13.6 eV, reduces the effective mass of that electron by 27 ppm.

However, there are processes that involve a lot more energy, in particular nuclear and particle interactions. Example: 1 kg TNT liberates 4.2 MJ (about 1000 kcal) when it explodes. 1 ton = 1000 kg. 1 Megaton = 10^9 kg. The baddest thermonuclear bomb ever built (guess by whom), the "tsar bomba", had an explosive energy of 50 Megatons = $210 \cdot 10^{15}$ J. This was accomplished by converting large quantities of hydrogen isotopes (Deuterium,

Tritium) into heavier elements (mostly helium). Yet the net change in rest mass is only $210 \cdot 10^{15}$ J/c^2 = 2.3 kg!
Caution: This does not mean that nuclear bombs (or reactors) convert "matter" into "energy" – it simply means that, in any reaction, the reaction products add up less rest mass than the initial ingredients if lots of energy is liberated. It does explain why nuclear power (and, perhaps before you retire, nuclear fusion) are so attractive: in these reactions (fission as well as fusion), a good fraction of a PERCENT of the rest mass of the initial substance is converted into energy, so you need not all that much material to start with (maybe 500 kg for the "tsar bomba").

But CAN you convert **matter** into energy or vice versa? Einstein only says that this is possible **in principle**, but in praxis it only works under very specific circumstances (mostly in high energy nuclear and particle physics labs).

Example 1: At Jefferson lab, we take an electron (rest mass energy: 511 keV = 0.511 MeV) and accelerate it until it reaches as much as 12,000 MeV – that means
Γ = 12,000 MeV / 0.511 MeV = 23483! (It also means that it goes only 0.27 m/s slower than the speed of light, or u = 99.99999991% of c.)
We then slam it into a target made, e.g., of protons (liquid hydrogen) which have a rest mass energy of 938 MeV. Yet this is enough energy to create particles with masses up to 4000 MeV/c^2 ! (Can you guess why not even more?) This means we routinely "create" particles like pions (140 MeV), rho mesons (700 MeV) and even J/ψ mesons (3097 GeV).

Example 2: In Dan Brown's book "Angles and Demons", the bad guys threaten to obliterate the Vatican with an "antimatter bomb". While this is wildly implausible, there is some kernel of truth to this: The only way to **completely** convert the mass of some object (elementary particle) is to combine it with its "evil twin", the corresponding anti-particle. For instance, CERN does indeed produce (using an energetic accelerator) small quantities of anti-protons (nowhere near enough for a bomb), which have the same rest-mass energy (938 MeV) as protons, but opposite charge (negative). An anti-proton and a proton can literally annihilate each other, leaving nothing but pure energy (2x938 MeV) behind. This energy can either convert itself back into other particles, or it can be carried off by ultra-hard gamma radiation (which is a form of "pure energy" – see below).

How about electromagnetic radiation (from radio waves over light to gamma rays)? In fact, if we look at the same equations at the beginning of this chapter, we see that anything moving with **exactly** the speed of light (including light itself) must have $\frac{u}{c} = pc / E = 1$, therefore $E = pc$. However, according to $E^2 = \left(m(u)c^2\right)^2 = m_0^2 c^4 + \vec{p}^2 c^2$, this is only possible if m_0 is exactly zero. Therefore we conclude that particles of light (=photons) or anything else moving with the speed of light must have no (rest) mass at all! Anything with a non-zero rest mass must move with less than the speed of light, relative to **any** coordinate system. (This reinforces the basic tenet of Special Relativity; it turns out things are more complicated in GENERAL Relativity…) Vice versa, anything with rest mass zero **must**, at all times, move with the speed c – it can never slow down!