September 29, 2016

Modern Physics –Lecture 10 Notes

A state vector contains all knowable information about a state: $\mathbb{C} \ni \psi(x),\ x_{\in}\mathbb{R}$

Vectors in a vector space $|\psi\rangle(t)$ can be added $\rightarrow \underbrace{|\psi_1\rangle + |\psi_2\rangle}_{\text{superposition}}$ $\psi(x) + \varphi(x)$

can be multiplied with $z_{\in}\mathbb{C}$: $|\psi_1\rangle \rightarrow z|\psi_1\rangle$ (same state)

has a scalar product : $|\psi_1\rangle, |\psi_2\rangle \rightarrow \langle\psi_1|\psi_2\rangle$

Example: Function $\psi(x)$ represents state vector for particle moving along x-axis. If state vector changes as time passes, we can write it as a function $\psi(x,t)$.

To compare to a prototypical wave: $f(x,t) = e^{i(kx-\omega t)}, v_{phase} = \frac{\omega}{k}, (k = \frac{2\pi}{\lambda}, \omega = 2\pi v)$

$$\text{Prob}(x \ldots x\Delta x) = \begin{cases} \text{i. } \psi^*(x)\psi(x)dx \\ \text{ii. } [\psi^*(x) + \varphi^*(x)]\,[\psi(x)\varphi(x)] = \end{cases}$$

$$\psi^*(x)\psi(x) + \varphi^*(x)\varphi(x) + \psi^*(x)\varphi(x) + \varphi^*(x)\psi(x) =$$

$$||\psi(x)|^2 + |\varphi(x)|^2 + 2Re\big(\varphi^*(x)\psi(x)\big) \text{ INTERFERENCE!}$$

$c \cdot |\psi\rangle$ describes the same state as $|\psi\rangle$ and so can be normalized $\big||\psi\rangle\big|^2 = 1$.

First calculate $\big||\psi\rangle\big|^2$, then define $|\psi_{\text{new}}\rangle = \left(\frac{1}{\sqrt{\big||\psi\rangle\big|^2}}\right)|\psi\rangle$.

Example: Schr's cat *) $\big||\psi\rangle\big|^2 = (c_1^*, c_2^*)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = |c_1|^2 + |c_2|^2$, $\boxed{|\psi_{\text{new}}\rangle = \frac{|\psi\rangle}{\sqrt{|c_1|^2+|c_2|^2}}}$

Ex.) $\psi_1(x)$ $\big||\psi_1\rangle\big|^2 = \int_{-\infty}^{\infty} \psi_1^*(x)\psi_1(x)dx < \infty$

For probabilities $\psi_{new}(x) = \left(\frac{1}{\sqrt{\int_{-\infty}^{\infty} \psi_1^*(x)\psi_1(x)dx}}\right) \cdot \psi_1(x)$

*) Ex. 2-D Hilbert space:) S.s.'s cat: most general, $c_1 \uparrow + c_2 \downarrow, (c_1, c_2) = |\psi\rangle$,

$$P(\uparrow) = |c_1|^2, P(\downarrow) = |c_2|^2$$

initial state: $\uparrow (alive) \xrightarrow{t} \left(a\ prediction\left(Schrodinger\ Eqn{:}\ i\hbar\frac{d}{dt}\psi = H\psi\right)\right)$ $\frac{1}{\sqrt{2}}\uparrow + \frac{1}{\sqrt{2}}\downarrow$

$(the\ probability{:}\ projection\ on\ eigenstate\ (an\ equally\ dead\ and\ alive\ cat))$

Observables can be chosen to be measured. A *measurement* instantaneously changes the state vector (either ↑ or ↓), causing a 'collapse' of the wave function into an eigenstate of the operator.