Lecture 9/27/16 Notes

Review:

- Quantum Mech. makes predictions about probabilities of states, observables, and measurements.
- There are two types of measurement outcomes, discrete and continuous.
- Continuous measurements – Have probability densities, $p(x)$, determines the probability of a measurement result lying in some range: $Prob(x+\Delta x) \approx p(x)\Delta x$. Every continuous measurement has inherent uncertainty (standard deviation of the predicted outcome). For example, position and momentum have uncertainties σ_x and σ_p respectively, and are related to each other by the inequality $\sigma_x \ \sigma_p \geq \frac{h}{4\pi}$, where h is Planch's constant.
- Discrete measurements – Have simple probability, $Prob(x_i)$ **for a specific outcome** x_i. Discrete measurements theoretically can be measured to infinite precision.

New material:

Examples of discrete probabilities:

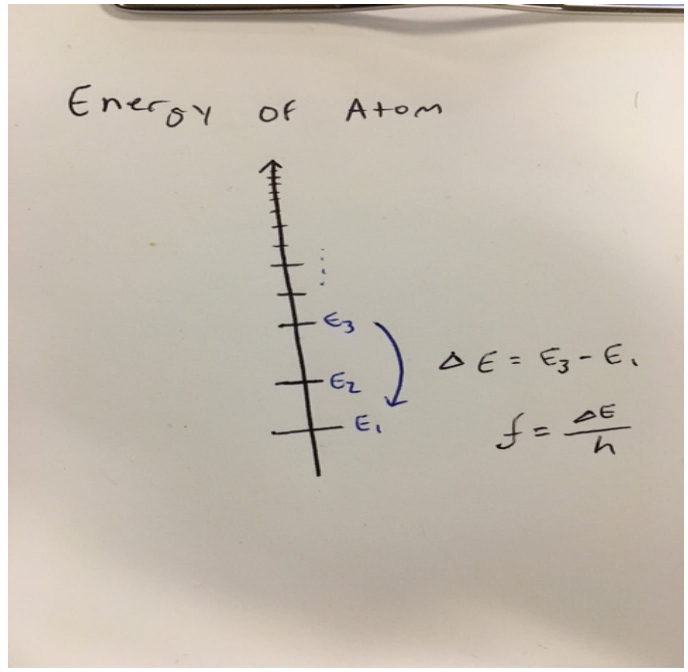
1. Binding energies
2. Angular momentum ($J_{vector} = n\frac{h}{4\pi}$; $n = 0,1,2,\ldots$)
3. Light

From Classical Mech. light is thought of as a continuous wave with wavelength, λ, and frequency, *f*, which are related by: $c = \lambda f$. **[*Note*: we also define a wave "number" $k = 2\pi/\lambda$ and a "angular frequency" $\omega = 2\pi f$, with $c = \omega/k$.)**

In Quantum Mech. light is thought of as made of discrete wave packets called a photons. Einstein theorized correctly in his theory of the photoelectric effect that these packets of light have energy, $E=hf$ **and therefore momentum** $p = E/c = h/\lambda$. Photons are generally represented with the symbol γ.

The equation of energy $E=hf$ can be used to explain blackbody radiation. An object with a temperature greater that 0 kelvin has thermal energy. This thermal energy comes from the vibration of atoms. The atomic vibrations fluctuate, and energy is passed from atom to atom or the energy is lost from the object via light. Since the energy that can be emitted via light of given frequency is quantized, Planck could explain why the blackbody spectrum does not become more and more intense at higher and higher frequency - he derived his law for blackbody radiation and also introduced "his" constant, h.

A second example is light emitted from a dilute gas of a specific type of atoms. This light comes from electrons dropping to lower allowed energies within each atom. As stated above, these energies (binding energies) are discrete. Since there can only be specific energy losses, there can only be specific wavelengths of light emitted for a particular object. In objects made of many different atoms the emitted wavelengths of light look as though they are continuous because of the various allowed energy levels in each atom. However, in a pure substance the discreteness of the light wavelengths can be seen.



What is the state vector?

The state vector is a function of time that can be used to describe the state of the system and can be used to predict the probability for measuring certain values for all observables. All of the underlined concepts require further explanation.

What is a vector?

A vector is a member of a vector space and can be described as a magnitude and direction or as a list of numbers. The list of numbers must be:

- Addable: $\vec{r_1} + \vec{r_2} = (x_1+x_2,\ y_1+y_2,\ z_1+z_2)$
- Multipliable with a scalar: $a\vec{r_1} = (ax_1,\ ay_1,\ az_1)$

A vector space has dimensions. The number of dimensions a vector space can have is any positive number n: $n = 1,2,3,\ldots,\aleph_0,\ldots,\aleph$

$\aleph_0$ = countable ∞

$\aleph$ = continuous ∞

A function f(x) is a continuous vector where $x \in \mathbb{R}$ (real numbers) and $f(x) \in \mathbb{C}$ (complex vector space). All functions f(x) form a vector space.

Special Vector Spaces: "Scalar Product"

A special vector space is one where the dot product can be defined.

EX: $\vec{r_1} \cdot \vec{r_2} = |\vec{r_1}||\vec{r_2}|\cos(\alpha)$

In a discrete vector space: $\{a_i,\ i = 1,2,3,\ldots,\infty\} \cdot \{b_i,\ i = 1,2,3,\ldots,\infty\} = \sum_{i=1}^{\infty} a_i b_i$

In a continuous vector space: $f \cdot g = \int f^*(x) g(x)\, dx$

A vector space with complex scalars and a well-defined scalar product is called a *Hilbert space*. All state vectors are members of a suitable Hilbert Space.

Example: Describe position along the x-axis. Hilbert Space is made of vectors $|\psi\rangle$ defined by complex-valued functions $\psi(x)$ that are "square-integrable" (to ensure the existence of the scalar product:

$\langle\phi|\psi\rangle = \int \phi^*(x)\psi(x)\, dx < \infty$

In particular, the probability density for finding the particle described by $|\psi\rangle$ near some position x is given by $p(x) = \psi^*(x)\psi(x)$. But the same state vector also encodes information about all other observables one could choose to measure, e.g. momentum.