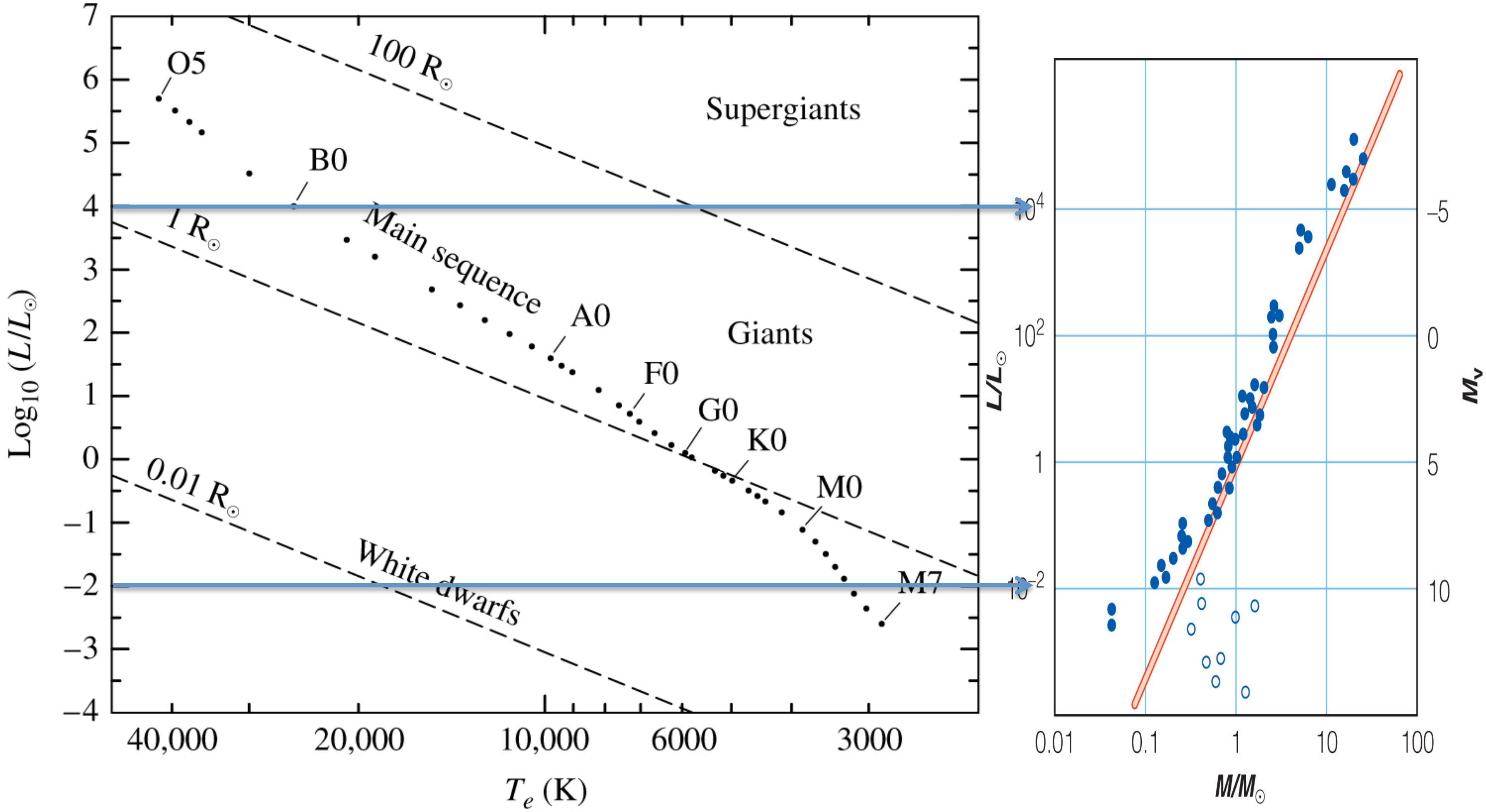
Stellar Structure

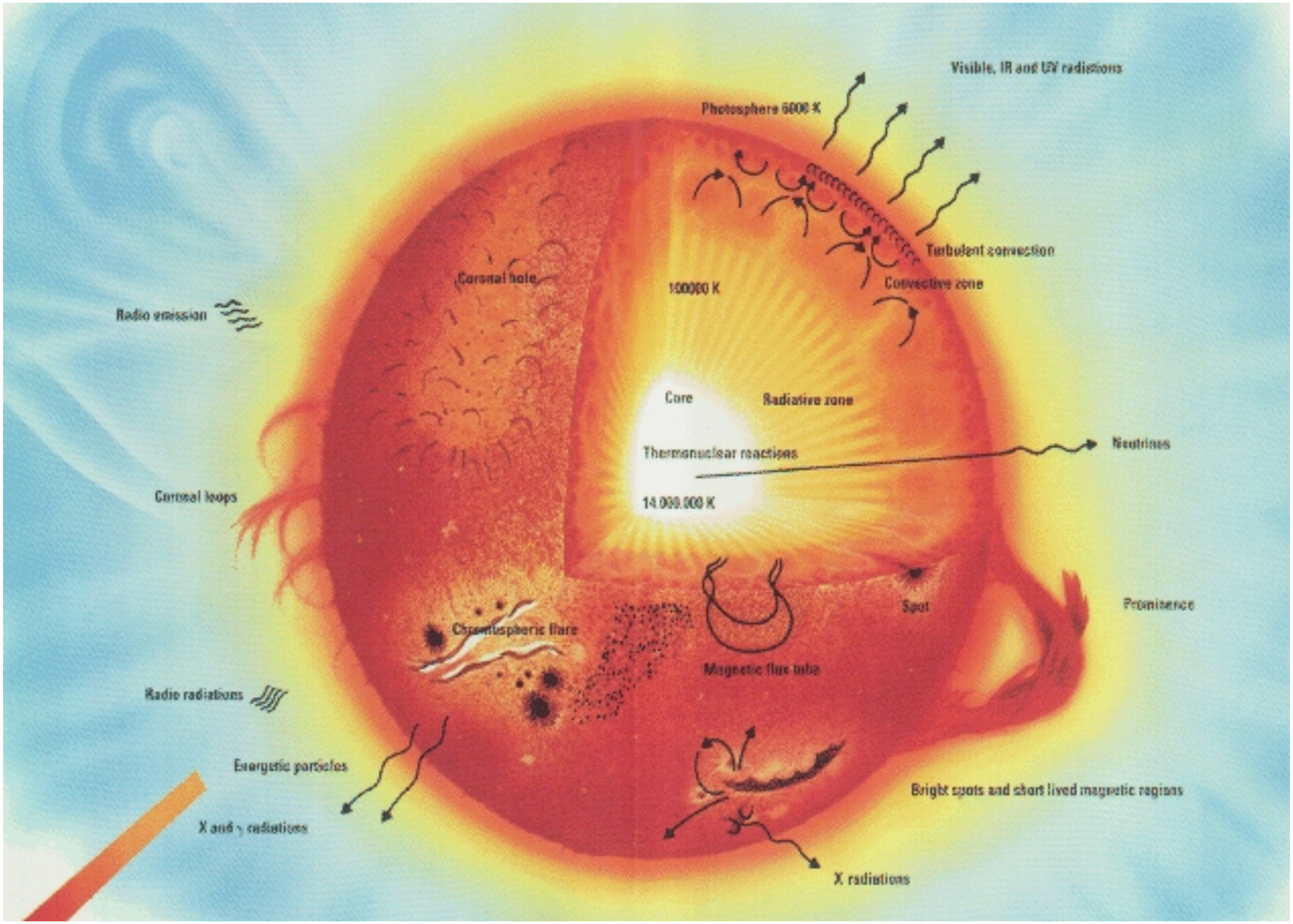
What have we learned?

- Can determine surface temperature via blackbody radiation, and absorption spectra
- Can determine relative magnitude, and after determining distance through parallax, absolute magnitude => Luminosity
- Relating black-body intensity to Luminosity yields surface and hence radius
- Binary stars: red- and blue shift of spectral lines determines absolute velocities; time dependence determines angular velocity of rotation around common center of gravity => distance of both stars from center of gravity
- Kepler's law gives sum of masses and Newton's 3rd law gives ratio of masses => individual masses
- Combining everything: average density

Hertzsprung-Russel Diagram



Question: How do we deduce interior structure of stars from these observations?



What do we need to know?

- Where does radiated energy ultimately come from?
- Need to figure out ρ, P, T as function of $r < R$
- 3 ingredients:
  - energy transport from center to surface -> $T(r)$
  - hydrostatical equilibrium (what keeps the star from further collapse -> ρ, P
  - Equation of state to relate ρ, P, T

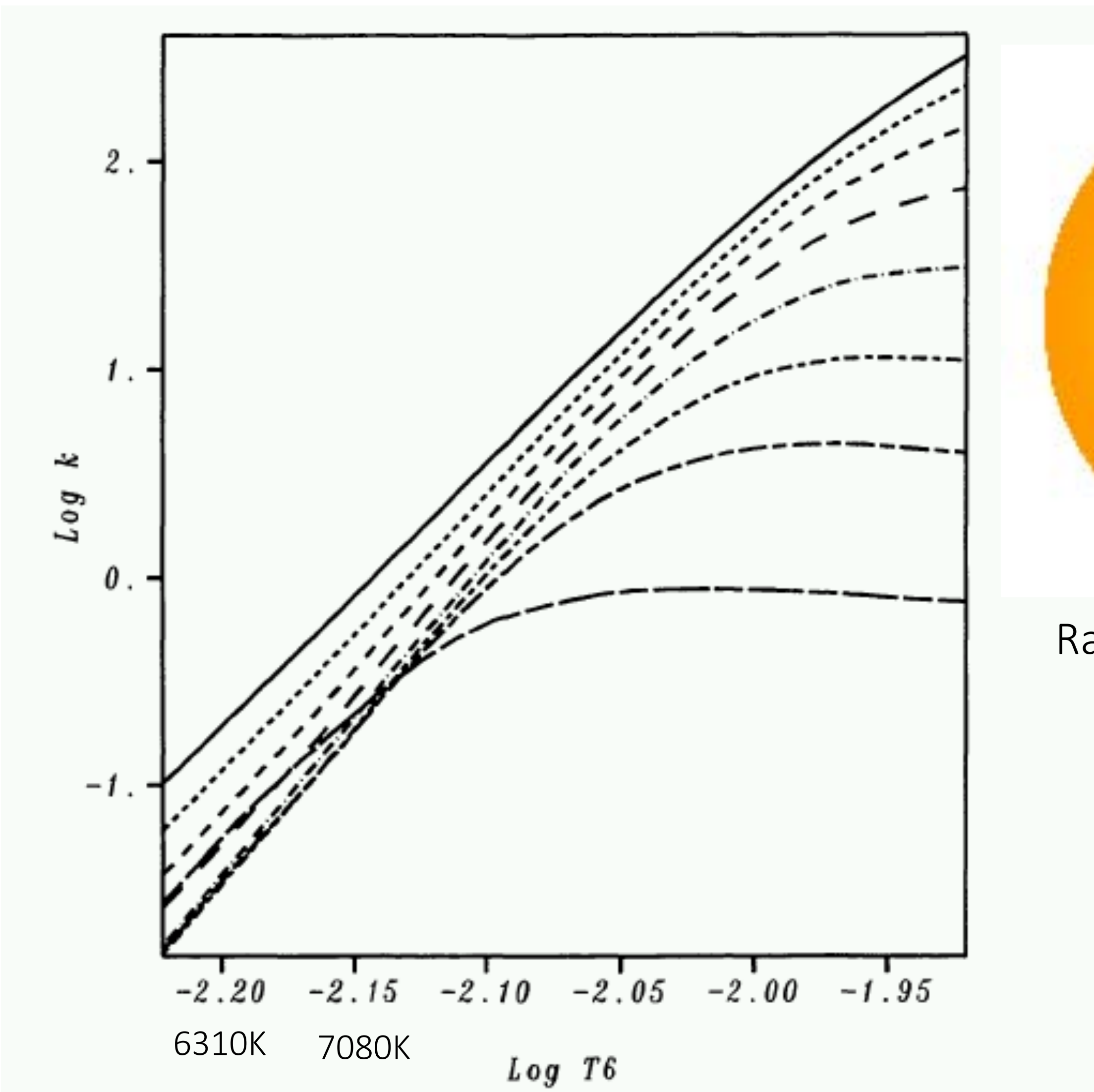
Energy Transport

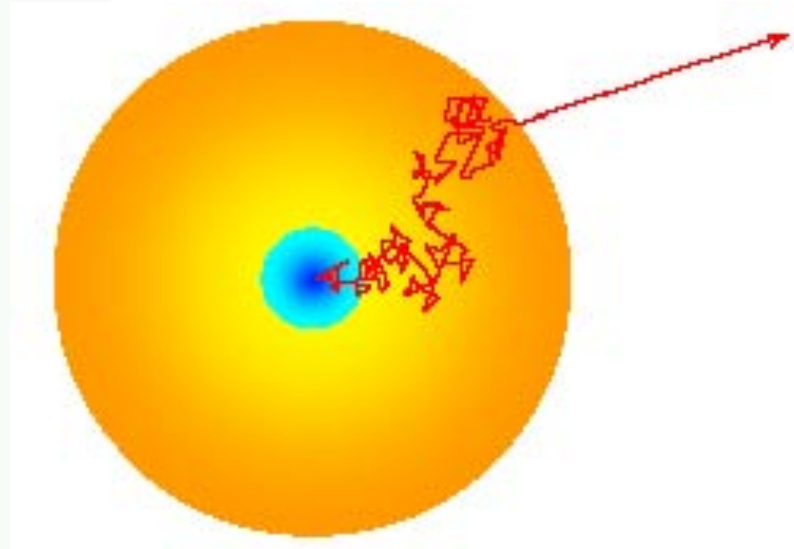
- 3 mechanisms:
  - Heat conduction (can be ignored given the humongous sizes of stars)
  - Heat convection: Does play a significant role in many stars (see later)
  - Radiation (electromagnetic)
    - Propagation (in particular net outward flow)
    - Also need to account for radiation sinks and sources
    - Also contributes to pressure!

Interaction of photons with matter

- Absorption
  - excitation of atoms from lower to higher energy eigenstates
  - ionization of atoms (electrons kicked out)
- Emission
  - Atoms going from higher to lower energy eigenstate
  - Electrons get “caught” by ions
- Scattering
  - Bremsstrahlung
  - Thompson (Compton) scattering

Opacity in Photosphere





Random Walk of photons through sun

Propagation of electromagnetic waves

Energy per volume dV in wave length interval $d\lambda$:

$$\frac{dE(\lambda\ldots\lambda+d\lambda)}{dV} = u_\lambda d\lambda \text{ ; } u_\lambda = \text{specific energy density.}$$

Example: black-body $u_\lambda = \frac{dn_\gamma}{d\lambda}\frac{hc}{\lambda} = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$

Total (integral over all wave lengths): $dE_{tot}/dV = 4\sigma/c\ T^4$

Power emitted per area dA into solid angle $d\Omega$ in wave length interval $d\lambda$:

$$\frac{dE(\lambda\ldots\lambda+d\lambda)}{dt\,dA\,d\Omega} = \cos\theta\cdot I_\lambda(\theta,\varphi)d\lambda \text{ ; } I_\lambda = \text{specific intensity.}$$

Average: $\langle I_\lambda\rangle = \frac{1}{4\pi}\iint I_\lambda(\theta,\varphi)d\Omega = \frac{c}{4\pi}u_\lambda$. Ex.: black-body: $\langle I_\lambda\rangle = \frac{2hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$

Power emitted in positive (neg.) z-direction per area dA perpendicular to z and per $d\lambda$:

Radiation flux density $F_\lambda = \int_0^{2\pi} d\varphi \int_{0\,(\pi/2)}^{\pi/2\,(\pi)} \cos\theta\cdot I_\lambda(\theta,\varphi)\sin\theta d\theta$

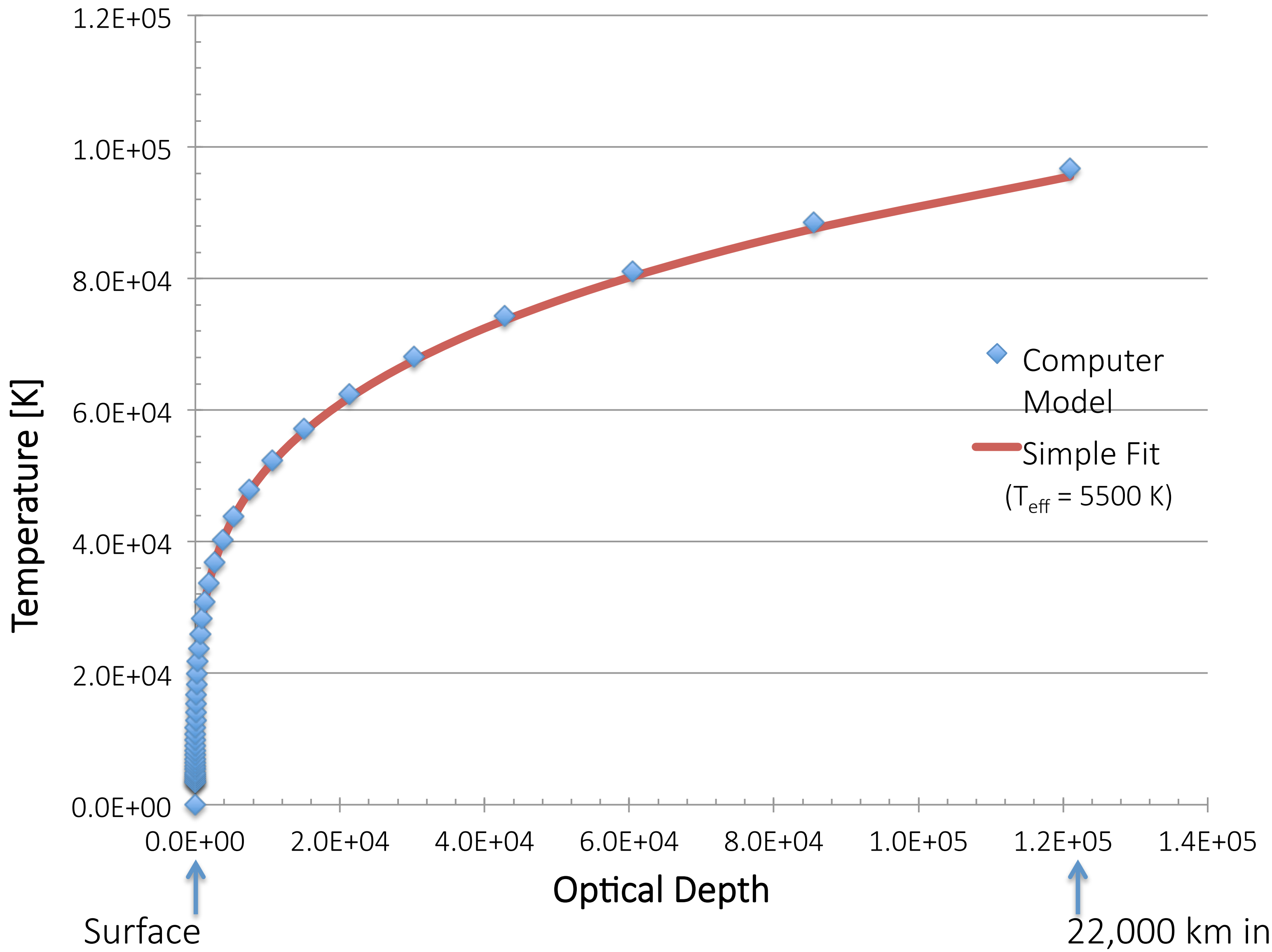
For isotropic specific intensity (for top hemisphere): $\Rightarrow F_\lambda = \pi\langle I_\lambda\rangle$

Black-body radiation: $F_\lambda d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1} = 2\pi hc\frac{f^3}{c^3}\frac{df}{e^{hf/kT}-1}$

Radiation pressure in z-direction: $dP_\lambda^z = \frac{2}{c}d\lambda\int_0^{2\pi} d\varphi \int_{0(\pi/2)}^{\pi/2(\pi)} \cos^2\theta\cdot I_\lambda(\theta,\varphi)\sin\theta d\theta$

For isotropic specific intensity in top hemisphere: $\Rightarrow dP_\lambda^z = \frac{4\pi}{3c}\langle I_\lambda\rangle d\lambda = \frac{1}{3}u_\lambda d\lambda$

Stellar Model



Interior Structure